Lateral Compression of a Square or Rectangular Tube Between Two Parallel Narrow Width Indenters Placed Non-Orthogonally

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ABSTRACT

Collapse behavior of aluminum tubes of square and rectangular cross-sections, when compressed between two identical narrow width indenters placed symmetrically in parallel alignment, is examined. Experiments were performed wherein the angle between the axes of tube and indenters was varied from 0° to 90°. Load-compression curves and deformation histories of typical specimens are presented. The collapse of the tubes was seen to be generally symmetrical, though asymmetries were observed in some tubes. Considering only the symmetrical mode of deformation, an analysis is presented for constructing the load-compression curves as well as the shape of the deforming tube. The analysis considers the energy absorbed in stationary and rolling plastic hinges which are formed in the collapsing tube. Computed results thus obtained compare well with the experiments.

NOTATION

$D_1$ Depth at which first horizontal hinge is formed
$D_L$ Distance of left vertical hinge from center of tube
$D_r$ Distance of right vertical hinge from center of tube
$E_{sa}$ Energy absorbed in stationary hinges
$E_r$ Energy absorbed in rolling hinges
$H$ Height of the tube
1. INTRODUCTION

An analysis of the plastic collapse of a square tube when compressed between two short width indenters was earlier presented by Gupta and Sinha. This analysis considered that the indenters were placed orthogonal to the tube. In this paper we present an improved analysis of the problem and consider square and rectangular tubes being compressed between two narrow width indenters, which are placed in both orthogonal and non-orthogonal positions relative to the tubes. Experiments were performed on as-received tubes of aluminum, wherein these were compressed between two narrow width indenters in an Instron machine at a crosshead speed of 2.5 mm/min. The angle between the tube and indenter axes was varied from 0° to 90°. Load–compression curves and histories of deformation of the tubes were recorded. Based on experimental observations, a compatible model which considers the energy absorbed in rolling and stationary plastic hinges was developed for the analysis. The computed values of collapse load, location of hinges, plastic load–compression curve and history of deformation of the tube are presented. These results compared well with the experiments.

2. EXPERIMENTS

Square and rectangular aluminum tube specimens were compressed between two narrow width indenters which were placed in parallel alignment above and below the specimen, Fig. 1. The clockwise measured angle \( \theta \) between the normal to the tube axis and the indenter axis, Fig. 1, was varied from 0° (in the orthogonal position) to 90° (in the parallel position) in steps of 15°. Specimens were tested in their as-received condition, in an Instron machine of 5 ton capacity at a crosshead speed of 2.5 mm/min. Specimens were 150 mm long and were cut from commercially obtained tubes. This was the tube length for all the tests, with the indenters 300 mm in length.
The load–compression curves of the tubes were recorded on the machine chart recorder. The histories of deformation of the specimens were recorded by repeating experiments on several tubes and interrupting the tests at different stages of compression.

The stress–strain curves for each tube material were obtained by conducting tension tests on the Instron machine and the curves thus obtained were idealized to be perfectly plastic. Typical experimental and idealized stress–strain curves are shown in Fig. 2.

Tubes of two rectangular and one square cross-sections were employed.
TABLE I
Cross-sectional Geometry of Tube Specimen and Indenter in Different Sets of Tests and Yield Stress of Tube Material

<table>
<thead>
<tr>
<th>Set no.</th>
<th>Tube specimen, 150 mm long</th>
<th>Indenter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Height (mm)</td>
<td>Width (mm)</td>
</tr>
<tr>
<td>1</td>
<td>23.0</td>
<td>23.0</td>
</tr>
<tr>
<td>2</td>
<td>24.3</td>
<td>36.9</td>
</tr>
<tr>
<td>3</td>
<td>36.9</td>
<td>24.3</td>
</tr>
</tbody>
</table>

in the present experiments. Respective yield stress values, cross-sectional dimensions of the tubes and those of the indenters are given in Table 1.

The experimental load–compression curves for the sets 1, 2 and 3 are shown in Fig. 3. In these curves it can be seen that the magnitude of the first peak increased with an increase in θ. A second rise is also seen in these curves. The compression at which the second rise begins, reduces with an increase in θ. The load–compression curve for a typical tube of set No. 1, for θ = 30°, is given in Fig. 4 to illustrate the different stages of the collapse. The first peak in the load–compression curve is the collapse load of the tube. The section which drops after the first peak is associated with the collapse of the tube occurring with the folding of stationary plastic hinges. The second rise in the load–compression curve occurs as the two inclined hinges on the left side begin to roll (discussed later). Figure 5 shows photographs of the compressed tubes. The symmetry in the deformed shape of the tubes is clearly seen in the photographs.

3. ANALYSIS

From the experimental observations of the deformation geometries, the typical pattern of hinges assumed to have formed on all faces of the tube is shown in Fig. 6. The node numbers, considered in the analysis, have been encircled and the hinges have been marked in Roman numbers.

In a tube of sufficient length and for values of θ less than a value say θ₁, the collapse mechanism in the deforming tube is restricted to within a certain length of the tube and it extends to equal distances on either side from the center of the tube, see Fig. 5. As θ is increased beyond θ₁, and as long as it is less than a value say θ₃, plastic hinges cover the entire specimen length. In a non-orthogonal position of the indenters, the hinges are restricted to within the specimen length towards the right end of the tube. When θ > θ₃, the tube collapses in a manner similar to the collapse of a
single tube compressed between two platens with plastic hinges extending over the entire specimen length on each face. The analysis presented is for \( \theta < \theta_L \) and symmetrical mode of deformation (Fig. 6). The undeformed ends of the tube behave as rigid bodies.

In the case where \( \theta < \theta_L \), the collapse occurs when the load–compression curve begins to drop with increasing compression and plastic hinges are seen to have formed on the tube. These hinges as idealized for the analysis are shown in Fig. 6. At collapse (Fig. 4) four sets of plastic hinges are formed on the tube (Figs 5 and 6). They are:

1. A horizontal hinge, number III, under the indenter at mid-height of the tube.
2. Two vertical hinges (slightly convex outward) numbered I and V, at
a certain distance from the edges of the indenters on both the vertical faces.

(3) Four inclined hinges between the two ends of the horizontal hinge at mid-height of the tube and top and bottom of the two vertical hinges. The two inclined hinges (II and IV) in the upper half of the tube are curved and concave upward and ones in the lower half of the tube are concave downward (Fig. 5).

(4) Stationary hinges on the top and bottom surfaces and corner edges of the tube (Fig. 6). These are numbered as VI, VII, VIII, XI and XII.

In the beginning, collapse of the tubes progresses with the rotation of all plastic hinges. At some compression $\delta_1$, the faces of the indenters come into contact with the outward deforming portions of the vertical face, along the inclined hinges on the left side (II). From this point onwards the inclined plastic hinges are forced to roll by the indenters because these are in contact. The rolling hinges (II) take up positions, such that the horizontal hinge (III), at mid-height of the tube, becomes longer and the rolling inclined hinges become shorter as shown later in Fig. 9. This rolling of inclined hinges occurs for non-orthogonal positions of the indenters only.

As the indenter moves down, both ends of the two vertical hinges (I, V) move inwards towards the indenter accompanied with the rotation of hinges on the top and bottom faces (XI, XII) (Figs 5 and 6). For non-orthogonal positions of the indenter, the undeformed ends of the tubes move such that they no longer lie along the tube axis. This is due to the
Fig. 5. Deformed shapes of tubes of set No. 1 for varying $\theta$; (a) side view (b) end view (c) top view.
collapse occurs when under the indenter, four connected panels on the four faces of the tube become isolated from each other as well as from the undeformed end portions of the tube by the formation of plastic hinges I, V, VI, VII, VIII and XII at their edges. Plastic hinges II, III, IV, IX, X and XI are also formed within the panels on the sides of the tube. Their folding and rolling permits the panels on the sides to deform. This is similar to the collapse of plates under a combination of in-plane and edge loading.

The values of limiting angles, $\theta_L$ and $\theta_s$, defined earlier are dependent on the dimensions of the tube and the indenter.

When the overhang $h > H/2$ (Fig. 8) the extent of the collapse mechanism formed is within the tube length and the corresponding angle, $\theta_L$, for this sufficient overhang is given as:

$$
\theta_L = \tan^{-1}(L_1/W) - \sin^{-1}(W_i/(W^2 + L_1^2)^{1/2})
$$
(1)
where

\[ L_1 = L - 3H \tan (\beta) \]  

Correspondingly the minimum length of the tube required, for a given \( \theta \), such that \( h > \frac{H}{2} \) is:

\[ L_L = \frac{W_i}{\cos (\theta)} + W \tan (\theta) + 3H \tan (\beta) \]  

where \( L \) is the length of the specimen and \( W_i \) and \( W \) are the indenter and tube widths. The angle \( \beta \) is equal to 45°.

The collapse load of a tube, compressed between two short width indenters placed symmetrically on the top and the bottom, was found to be same as the collapse of a tube of length \( L_c \), when compressed between two platens. As shown in Fig. 8, \( L_c \) is the bottom width of a trapezium ABCD under the indenter and can be written as,

\[ L_c = \frac{W_i}{\cos (\theta)} + 2H \tan (\beta) \]  

The horizontal hinge, III, under the indenter (Fig. 6) is formed at mid-height of the tube and thus its depth \( D_I \) from the top is,

\[ D_I = \frac{H}{2} \]  

Length of this hinge is taken equal to the tube thickness. The inclined hinges, II, IV, and the horizontal hinge, III, are formed simultaneously. The inclined hinges are curved such that no distinct horizontal portion of the hinges is clearly visible under the indenter at small compressions after the elastic collapse. At large compressions (Fig. 5) it is nearly \( \frac{W_i}{\cos \theta} \) long.
In the experiments, the location of the vertical hinges, I and V, was observed as shown in Figs 6 and 8. The vertical hinges are stationary during compression of the tube. When the indenter is placed obliquely over the tube, the deformation model of the tube during its compression (Figs 6 and 8(a)) is best described by considering an increased effective width of the indenter. This increase in the width of the indenter is considered such that edge between nodes 10 and 8 (or 7 and 9) is rotated outwards about node 10 (or node 7) by an angle $\Psi$. The value of $\Psi$ was measured in all the deformed tubes and it was found to be nearly equal to $\theta$ for $\theta \leq \theta_L$ and $\theta/2$ for $\theta_s > \theta > \theta_L$.

$D_L$ and $D_R$, the distances of the left vertical hinge I and the right vertical hinge V from the tube center, are given by:

for $\theta \leq \theta_L$,

$$D_L = (W \tan(\theta)/2 + W_t/(2 \cos(\theta))) + H \tan(\beta)$$

(6)

$$D_R = D_L - W \tan(2\theta/3)$$

(7)

for $\theta_s > \theta > \theta_L$,

$$D_L = L/2$$

(8)

and for computing $D_R$ the indenter edge is rotated out by an angle $\theta/2$ about node 10 (or node 7). For $\theta > \theta_L$, node 10 may lie between nodes 12 and 13.

The four inclined hinges, II, IV (and two symmetrical), extend from the ends of the first horizontal hinge, III, and the top and bottom of the two vertical hinges, I, V, see Figs 6 and 9.

Fig. 9. Position of indenter and location of hinges at (a) commencement of rolling of left inclined hinges and (b) at some compression after rolling has begun.
The coordinates of each of the 15 nodes (Fig. 6) define the deforming geometry of the tube. Computations to locate the coordinates of these nodes are carried out by first determining the coordinates of points in contact with the indenters (7, 8, 9 and 10). The coordinates of node 4 are then determined from simple geometric considerations as shown in Fig. 10. The z coordinate remains unchanged due to symmetrical loading, the x and y coordinates of other nodes (5, 11, 12, 13, etc.) are thereafter determined by solving simultaneous equations for distances of these nodes from two other nodes, whose coordinates are already known, such that these distances remain constant during the tubes compression. For example, the coordinates of node 5 are determined by using the known coordinates of nodes 4 and 8. The distances are taken to remain constant as deformations, if any, in the panels bounded by plastic hinges are ignored.

Having computed the geometry of the deformed tube for any \( \delta \), orientation of the rigid panels, defined by the coordinates of any three nodes on it, is then determined. The change in angle between the normals to any two adjacent panels gives the change in angle of the common stationary hinge. The work done at any rotating stationary hinge is determined as:

\[
E_a = M_p L_h \theta_h
\]

where \( M_p \) is the plastic moment. \( L_h \) and \( \theta_h \) are the length of the hinge and the rotation of the hinge at a given compression.

The compression at which the two inclined hinges on the left begin to roll and the load–compression curve begins to rise is \( \delta_1 \). This is the compression at which the indenters come into contact with the inclined hinges, \( \Pi \) (and a symmetrical one), on the left (Fig. 9). The computation of \( \delta_1 \) requires determination of the distance between (the straight lines formed by) the left bottom edge of the top indenter and the inclined hinge \( \Pi \).

Knowing the initial and final positions of the ends of the rolling hinges, the area traversed by these is determined. The accuracy of this calculation is increased by taking the summation of the area traversed during small increments (\( \iota \)) of compression between the initial and the final positions.
The work done by a rolling hinge is determined from eqn (10):

\[ E_r = \frac{2M_p}{R_r} \]  

(10)

The average rolling radius \( R_r \), determined from experiments is 6.3 times the tube thickness.

The total energy, \( E_t \), absorbed in all hinges is computed over two stages of compression: (1) when \( \delta < \delta_1 \) and all hinges are stationary at their initial collapse locations; (2) when \( \delta > \delta_1 \) and the inclined hinge II dissipates energy by rolling.

The total energy, \( E_t \), is thus given by,

\[ E_t = \Sigma E_{\alpha} + \Sigma E_r \]  

(11)

The load at any \( \delta \) is given by the slope of the cumulative energy–compression curve at the compression \( \delta \). This load is determined by computing the total work done by all the hinges at two values of compression, at a small interval on either side of \( \delta \), and then determining the slope linearly between these two points. In computations this gap between the two values of compression, on either side of \( \delta \), was kept equal to \( H/1000 \).

4. RESULTS AND DISCUSSIONS

The computed load–compression curves are shown for typical specimens of set No. 1 in Fig. 11, and their energy–compression curves are shown in Fig. 12. These show very good agreement with the experiments.

The computed collapse load, based on the predicted equivalent length of a tube compressed between two platens, is compared with the experiments in Fig. 13. The computed values show very good agreement with the experiments.

In Fig. 14 the computed and the observed location of the horizontal hinge under the indenter on both the front and back faces of the tubes are compared. The predicted and experimentally observed location of the vertical hinges on the left and right sides of the tube are also given in Fig. 14, for both front and back faces of the tube. This was done to show the asymmetry due to the variations observed in the experiments. The computed results are in good agreement with the experimental values.

The compression at which a load–compression curve in an experiment began to rise, on commencement of rolling of inclined hinges, II, shows good agreement with the computed values, for all values of \( \theta \) (Fig. 15).
5. CONCLUSIONS

Square and rectangular aluminum tubes in their as-received state were compressed between a pair of parallel, orthogonally and non-orthogonally placed, short width indenters. The collapse behavior of the tubes was studied by recording the load–compression curves and observing the deforming geometry of the tubes by interrupting and terminating the tests at various stages of compression and taking photographs of the tube specimens.

Based on the experimental observations an analysis for the symmetrical mode of collapse has been presented for the computation of limiting
loading geometry, collapse load, location of hinges at collapse, deforming geometry of the tube and the load–compression and energy–compression curves. The results of the analysis compared well with those of the experiments.

REFERENCES

Fig. 15. Experimental and computed compressions at which the load–compression curves begin to rise.
