Effect of moisture transfer across building components on room temperature

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Abstract

Analytical closed-form solutions for the heat and moisture distribution given by the Luikov system of partial differential equations have been obtained by using a periodic solution approach. The energy balance of a model room has been considered with different thermophysical properties produced by a change in the moisture content of the space walls, along with the effect of vapor diffusion. The results show that the humidity present in ambient air and room air affects room temperature and in the hygroscopic region it can alter the room temperature by 2–3°C depending upon the amount and direction of temperature and moisture gradients, which is in conformity with the earlier results of Hall.

Keywords: Heat and Moisture; Temperature; Building

1. Introduction

Substantial work has been done for prediction of thermal environment inside a building considering only the heat transfer [1,2] and for simulation, many programs [3] and codes [4] are available. Literature on moisture transfer in buildings is also available [5–7] but the study of combined heat and moisture transfer affecting the temperature and humidity of room air is still lacking. Any porous building material consists of a solid matrix containing pores. The pores may contain moisture (water vapor, absorbed or capillary-bound water, and ice) and dry air. In the rainy season the rainwater pouring on the walls and roof penetrates inside the fabric and causes uncomfortable, unhygienic conditions and changes heating/cooling loads. In other days also, the porous building material absorbs the water vapor present in the surrounding air according to its equilibrium sorption moisture content. In turn, the equilibrium sorption relation depends on the characteristics of the porous system, such as the porosity, the specific surface of the solid matrix, and the pore size distribution [8]. The heat transfer in the micro-capillaries of the capillary porous building material occurs mainly by molecular conduction through the framework of the body and the material bound within the pores (vapor, gas, and liquid) [9]. The moisture accumulated either by rain or absorbed from the atmosphere is normally transferred from the pores of the building material to the surrounding air by, unsaturated flow of liquid within the porous solid, the liquid—vapor phase change, and convective diffusive transfer of vapor from the surface of the solid to the surroundings [10]. The flow of moisture and heat is dependent on each other such that the solution to the thermal part of a problem should be accompanied by a simultaneous solution to the moisture part [11].

Luikov [9] has given the governing equations for simultaneous heat and mass (moisture) transfer but the solutions are either numerical or complicated involving complex eigenvalues [12]. In an earlier paper [13] we have used the periodic solutions of the heat and mass (moisture) transfer equations to obtain closed-form solutions for the temperature and moisture distribution subjected to the boundary condition that there was no moisture at the boundary in contact with the room air. This condition is not usual: normally in most of the developing countries, there is no vapor barrier, and the moisture diffuses through at a rate depending upon the building material and the surrounding conditions. In an earlier paper [14] we have considered those usual boundary conditions and obtained the closed-form solutions for the temperature and moisture distribution across the building fabric considering the periodic variation of ambient temperature on one
Nomenclature

\( u_g \) coefficient of thermal diffusivity, \( \text{m}^2/\text{s} \)
\( A_{\text{II}} \) total area of all building elements, \( \text{m}^2 \)
\( A_p \) area of the element, \( \text{m}^2 \)
\( A_w \) area of the window, \( \text{m}^2 \)
\( \delta_m \) mass transfer coefficient for vapor and liquid inside the body, \( \text{m}^2/\text{s} \)
\( C \) moisture content, \( \text{kg}/\text{kg} \)
\( C_m \) moisture capacity, \( \text{kg (moisture)}/\text{kg (dry body)} \) \( \text{M} \)
\( C_q \) heat capacity of constituent, \( \text{J/kg K} \)
\( C_r \) heat capacity of room air, \( \text{J/kg K} \)
\( g \) solar gain factor of single glazed window
\( GOC \) gain due to occupancy, \( \text{W} \)
\( h_{\text{ins}} \) inside surface heat transfer coefficient, \( \text{W}/\text{m}^2\text{K} \)
\( h_{\text{out}} \) outside surface heat transfer coefficient, \( \text{W}/\text{m}^2\text{K} \)
\( I \) solar radiation, \( \text{W/m}^2 \)
\( K \) constant coefficient \( (\varepsilon K_m / \rho C_q) \)
\( K_m \) moisture conductivity coefficient, \( \text{kg/m s M} \)
\( K_t \) thermal conductivity coefficient, \( \text{W/m k} \)
\( L \) thickness of wall, \( \text{m} \)
\( U \) moisture potential, \( ^\circ\text{M} \)
\( U_m \) moisture potential of ambient air, \( ^\circ\text{M} \)
\( U_r \) moisture potential of room air, \( ^\circ\text{M} \)
\( N \) number of air changes per hour
\( T_{\text{a}} \) temperature of ambient air, \( ^\circ\text{C} \)
\( T_r \) temperature of room air, \( ^\circ\text{C} \)
\( T_m \) solar temperature, \( ^\circ\text{C} \)
\( t \) time, \( \text{s} \)

Greek symbols

\( a \) absorption coefficient of outside surface
\( \lambda \) outside convective mass transfer coefficient, \( \text{kg/m}^2\text{s M} \)
\( \lambda \) inside convective mass transfer coefficient, \( \text{kg/m}^2\text{s M} \)
\( \delta \) thermogradiant coefficient, \( ^\circ\text{M/K} \)
\( \varepsilon \) ratio of vapor diffusion coefficient to total moisture diffusion coefficient
\( \rho \) heat of phase change, \( \text{J/kg} \)
\( \phi \) dry body density, \( \text{kg/m}^3 \)
\( \omega \) relative humidity, \( \% \)

Subscripts and Superscripts

ambient air
avg average
east
ins inside
m mass (moisture) transfer
n harmonic part
nr north
avg average part
out outside
p building element
q heat transfer
r room
R roof
S south
w west

side of the wall and assuming a fixed temperature on the other side.

In this paper, we have analyzed the periodic variation of inside temperature of a room over a day's period, considering the simultaneous heat and moisture transfer across the building element. Diffusion equations governing diffusion under moisture and temperature gradients with periodic boundary conditions were formulated and solved by an approximate steady analytical method by Cunningham [7] under conditions of constant thermal conductivity. In this work we have given closed-form solutions for the temperature and moisture distribution by periodically solving a set of simultaneous heat and mass transfer equations for each building component, followed by an energy balance in the room air and considering moisture transport properties of the building material.

2. Mathematical modelling

Considering homogeneous walls and roof in plane geometry and assuming one-dimensional movement of heat and moisture the Luijkv equations as given by Liu and Cheng [12] can be written as follows:

\[
\rho C_r \frac{\partial T}{\partial t} = K_r \frac{\partial^2 T}{\partial x^2} + \varepsilon \lambda \rho C_m \frac{\partial U}{\partial t}.
\]
\[ \rho C_v \frac{\partial U}{\partial t} = K_m \frac{\partial^2 T}{\partial x^2} + K_w \frac{\partial^2 U}{\partial x^2}. \]  

(2) 

Consider a room with four walls and roof elements each of thickness \( L \), exposed on one side to ambient conditions, with hourly variable temperature \( T_a \) and constant (assumed) relative humidity \( \phi_{a0} \). The solar radiation on each wall and roof has been taken into account in the form of solar temperature of the medium [9], \( T_{m} = T_s + u \phi_{h0} \), where \( u \phi \) is the radiant energy flow absorbed by the surface and \( h_{out} \) is the total coefficient of heat transfer between outside air and the surface of wall/roof. The moisture content \( C = C_m U \), where \( U \) is the mass transfer potential (analogues to the heat transfer potential temperature). The mass transfer potential is some function of the moisture content of thermodynamic equilibrium, must be the same in all parts of the body or system of bodies [9]. In a state of equilibrium the moisture content corresponding to relative humidity equal to 100\%, is known as the maximum septional moisture content or maximum hygroscopic humidity. The maximum hygroscopic moisture content of any body is significantly less than the maximum moisture content, which the body can acquire by absorbing water (wettability of the body). In the hygroscopic region, the mass transfer potential \( U \) is a single-valued function of relative humidity, at any temperature as given by Lukov [9]. The mass transfer potential and in turn the relative humidity \( \phi_{a0} \) of room air are also assumed constant and the room temperature is determined by solving the above equations.

Assuming the periodic varies of the input conditions, temperature, and solar radiation, the moisture and temperature distribution across the fabric and the room temperature can also be assumed periodic with the same frequency, i.e.

\[ T(x,t) = T_{0}(x) + \sum_{n=1}^{\infty} T_{n}(x) \cos(n \omega t), \]  

(3) 

\[ U(x,t) = U_{0}(x) + \sum_{n=1}^{\infty} U_{n}(x) \cos(n \omega t), \]  

(4) 

\[ T_{0}(t) = T_0 + \sum_{n=1}^{\infty} T_{nm} \exp(n \omega t), \]  

(5) 

where \( T_{n}(x), U_{n}(x) \) and \( T_{n}(t) \) are the steady parts and \( T_{nm}(x), U_{nm}(x) \) and \( T_{nm}(t) \) are the harmonic components of the temperature and the moisture. \( \omega \) is the frequency of variation normally over a 24 h cycle, i.e. \( \omega = \frac{2\pi}{24 \text{ h}} \).

Eqs. (1) and (2) can be written as

\[ \frac{\partial T}{\partial t} = a_{\gamma} \frac{\partial^2 T}{\partial x^2} + K \frac{\partial^2 U}{\partial x^2}, \]  

(6) 

\[ \frac{\partial U}{\partial t} = a_{m} \frac{\partial^2 U}{\partial x^2} + a_{n} \frac{\partial^2 T}{\partial x^2}, \]  

(7) 

where

\[ a_{\gamma} = K_q + \frac{\varepsilon}{\rho} K_m \frac{\partial}{\partial x} \]  

\[ K = \frac{\varepsilon}{\rho} K_m \frac{\partial}{\partial x}, \]  

\[ a_{n} = K_m / C_w. \]  

The transfer coefficients are assumed constant initially. Their dependence on moisture and temperature is considered during the calculation at each hour. Substituting for \( T(x,t) \) and \( U(x,t) \) in Eqs. (6) and (7) and separating the steady state and harmonic parts one gets

(a) For steady part

\[ a_{\gamma} \frac{\partial^2 T}{\partial x^2} + K \frac{\partial^2 U}{\partial x^2} = 0, \]  

(8) 

\[ a_{n} \frac{\partial^2 U}{\partial x^2} + a_{m} \frac{\partial^2 T}{\partial x^2} = 0. \]  

(9) 

(b) For fluctuating part

\[ a_{\gamma} \frac{\partial^2 T}{\partial x^2} + K \frac{\partial^2 U}{\partial x^2} = \text{im} \omega T_{m}, \]  

(10) 

\[ a_{n} \frac{\partial^2 U}{\partial x^2} + a_{m} \frac{\partial^2 T}{\partial x^2} = \text{im} \omega U_{m}. \]  

(11) 

The solution of the above equations should satisfy the following boundary conditions for free diffusion as given by Lukov:

\[ -K_{q \text{ out}} \left( \frac{\partial T}{\partial x} \right)_{x=0} + h_{\text{out}} (T_{x=0} - T_{s}) \]

\[ + (1 - \varepsilon_{\text{out}}) \lambda_{\text{out}} \theta_{\text{out}} (U_{x=0} - U_{s}) = 0, \]  

(12) 

\[ K_{m \text{ out}} \left( \frac{\partial U}{\partial x} \right)_{x=0} + h_{\text{out}} \theta_{\text{out}} \left( \frac{\partial T}{\partial x} \right)_{x=0} \]

\[ + \lambda_{\text{out}} (U_{x=0} - U_{s}) = 0, \]  

(13) 

\[ -K_{q \text{ in}} \left( \frac{\partial T}{\partial x} \right)_{x=L} + h_{\text{in}} (T_{x=L} - T_{\text{mhd}}) \]

\[ - (1 - \varepsilon_{\text{in}}) \lambda_{\text{in}} \theta_{\text{in}} (U_{x=L} - U_{t}) = 0, \]  

(14) 

\[ K_{m \text{ in}} \left( \frac{\partial U}{\partial x} \right)_{x=L} + K_{m \text{ in}} \theta_{\text{in}} \left( \frac{\partial T}{\partial x} \right)_{x=L} \]

\[ + \lambda_{\text{in}} (U_{x=L} - U_{t}) = 0. \]  

(15) 

The energy balance equation for the room temperature can be written as

\[ W_{c} \frac{\partial T_{r}}{\partial t} = \sum_{p=1}^{5} h_{\text{in}} A_{p} (T_{x=L} - T_{t}) \]

\[ + \sum_{p=1}^{5} (1 - \varepsilon_{\text{in}} \lambda_{\text{in}} \theta_{\text{in}}) (U_{x=L} - U_{t}) \]

\[ - \sum_{p=1}^{5} u_{p} A_{p} (T_{t} - T_{p}) - u_{p} A_{p} (T_{t} - T_{p}) \]

\[ - 0.33 W_{c} (T_{t} - T_{h}) + g A_{p} T_{h} + \text{GOC}. \]  

(16)
The solutions of Eqs. (8) and (9) are obtained as:

\[ U_{\alpha(x)} = F_p X + F_p. \]  \hspace{1cm} (17)

\[ T_{\alpha(x)} = F_p X + F_p. \]  \hspace{1cm} (18)

where \( p = 1, \ldots, 4 \) for East wall, \( p = 5, \ldots, 8 \) for West wall, \( p = 9, \ldots, 12 \) for North wall, \( p = 13, \ldots, 16 \) for South wall, \( p = 17, \ldots, 20 \) for roof. From the separated steady part of boundary conditions (12)–(15) four equations will be available for each building element, giving rise to 20 equations for given surfaces. Eq. (16) combines the effect of heat and moisture transfer from all surfaces to room air temperature. The separated steady part of this equation gives one combined equation including \( T_{\alpha} \). Solving these 21 equations simultaneously by the matrix solution method, one obtains the values of \( F \). The matrix solution formed is as follows:

\[ [F]_{21 \times 1} = [D]_{21 \times 2} [E]_{2 \times 1}, \]  \hspace{1cm} (19)

where the elements of the matrices are as defined in the appendix. For the fluctuating part, from Eqs. (10) and (11) one can write

\[ T_{\alpha(x)} = \frac{a_m}{a_m \omega} U_{\alpha(x)} + \left( \frac{\partial K - a_q}{i \omega} \right) \left( \frac{\partial^2 U_{\alpha(x)}}{\partial X^2} \right). \] \hspace{1cm} (20)

Substituting for \( T_{\alpha(x)} \) in Eq. (11) one obtains the following fourth-order equation for \( U_{\alpha(x)} \):

\[ a_m \delta K - a_m a_q \frac{\partial^4 U_{\alpha(x)}}{\partial X^4} + (a_m + a_q) \frac{\partial^2 U_{\alpha(x)}}{\partial X^2} - i \omega a_m U_{\alpha(x)} = 0. \] \hspace{1cm} (21)

Solution of Eq. (21) is found as

\[ \sum U_{\alpha(x)} = \sum_{j=1}^{4} \frac{c_j \exp(z_j X)}{-i \omega c_j} \] \hspace{1cm} (22)

\( z_j \)'s are the roots given by the following equation:

\[ E_1 \alpha^4 + E_2 \alpha^3 + E_3 \alpha^2 + E_4 \alpha + E_5 = 0, \] \hspace{1cm} (23)

where \( E_1 = (a_m \delta K - a_m a_q)/i \omega c_j, \) \( E_2 = 0.0, \) \( E_3 = a_m + a_q, \) \( E_4 = 0.0, \) and \( E_5 = -i \omega a_m c_j. \)

The coefficient \( c_j \) (14) are obtained by using the separated fluctuating parts of the boundary conditions (12)–(15) giving rise to 20 equations for five surfaces. The separated fluctuating part of Eq. (16) gives a combined equation including \( T_{\alpha} \). The solution of these 21 equations is made by the following matrix equation:

\[ [C] = [A]_{21 \times 2} [B]_{2 \times 1}, \] \hspace{1cm} (24)

where the elements of the matrices are defined in the appendix. Substitution of \( U_{\alpha(x)}, T_{\alpha(x)}, T_{\alpha}, \) and \( U_{\alpha(x)}, T_{\alpha} \) in Eqs. (3)–(5) yields the moisture and temperature distribution in respective building elements and room air temperature.

3. Numerical example and discussion

The thermo-physical and mass transport properties play a major role in the analysis. These properties are dependent on temperature and moisture contents. For building materials the effect of temperature variation is less significant than the moisture variation in the usual range of variations. However, the data are scarce, so on the basis of the data given by Liu and Cheng [12] the thermo-physical properties of spruce were taken and modified for different humidities (moisture contents) in accordance with Lutkov [9]. Solar radiation and relative humidity data for composite climatic conditions of New Delhi (29°N) were taken for typical months of June and January from Bansal and Minke [15]. As an example we have considered a single model room of 3 m x 3 m x 3 m size with walls and roofs made of 30 cm thick spruce. The room contains a single glazed window of dimensions 1.5 m x 1.0 m in the South wall. Numerical calculations have been done for room temperature variations for typical days in the months of June and January in Delhi to assess the effects of the following alternative conditions:

1. The effect of moisture transfer is considered compared with the corresponding case of no moisture transfer (outside humidity is constant corresponding to the daily average of the day).
2. The effects of no air change and three air changes per hour are compared.
3. The case of high humidity of about 96% (86°M) outside as well as inside is evaluated.

The numerical results obtained are graphically shown in Figs. 1–4. Fig. 1 corresponds to an average humidity

Fig. 1. Room temperature variation with time, for average outside humidity and without humidity, in June.
outside (r.h. = 50%, 24.7°C) and compares it with the corresponding case of no humidity. It is clearly shown that consideration of moisture transfer results in higher temperature fluctuations inside the room.

Increasing the air changes per hour increases fluctuations of inside temperature further as expected. Fig. 2 plotted for three air changes shows that the higher differences in the inside and outside relative humidity increase the temperature fluctuation even more, though a higher humidity inside decreases the peak of temperature slightly.

Figs. 3 and 4 plotted for the month of January show that the changes in ventilation rate have a greater effect on temperature variation in winter than in summer. Fig. 4 shows that the moisture differences between outside and inside increase the temperature fluctuations further.

The change in temperature due to moisture effect is due to the combined effect of changed conduction and diffusion of vapor. Hall et al. [10] have found that the moisture affects the heat transfer from walls by 2–3°C, so the above results
Fig. 6. Heat flux through different building components without moisture, in June.

Fig. 7. Heat flux through different building components with moisture, in January.

are in good agreement with those of Hall et al. [10]. This analysis has been done for the hygroscopic region only. Similar analysis may be done for moist conditions with modified boundary conditions and material properties. The results of temperature distribution across the fabric were used to calculate the flux from different building elements. Figs. 5–8 show that for average outside humidity conditions the heat flux is 40% more in comparison to the case when no moisture effect is considered. It is also evident that East and West walls, taken together contribute almost the same heat flux as the roof in June where as the heat flux through South wall is maximum in January. There is no net heat flux through the North, East and West walls in January because of no or low solar radiation on these surfaces.

4. Conclusion

The results presented in this paper show that moisture transfer should necessarily be considered especially in hygroscopic media (true for most building materials) in order to have a true assessment of the room air temperature variations in a building. The periodic solutions provide a fairly good estimate about these variations. Future work would concentrate on the temperature as well as humidity variations of room air inside a building.

Appendix

The elements of the matrices in Eqs. (19) and (24) are zero except those defined as follows: Here $R = (1 - \varepsilon \lambda)$. Subscript $p$ represents the building element, i.e. East, West, North, South walls and roof:

\[
D(1, 2) = R\delta_{m, \text{out}}, \\
D(1, 3) = -K_{\ell, \text{out}}, \\
D(1, 4) = h_{\text{out}}, \\
D(2, 3) = K_{m, \text{out}}, \\
D(2, 2) = \delta_{m, \text{out}}, \\
D(2, 3) = K_{m, \text{out}}, \\
D(3, 1) = R\delta_{m, \text{in}}, L_{p}, \\
D(3, 2) = R\delta_{m, \text{in}}, \\
D(3, 3) = h_{\text{in}}, L_{p}, + K_{m, \text{in}}, p, \\
D(3, 4) = h_{\text{in}}, \\
D(3, 21) = -h_{\text{in}}, \\
D(4, 1) = \delta_{m, \text{in}}, L_{p}, + K_{m, \text{in}}, p, \\
D(4, 2) = \delta_{m, \text{in}}, \\
D(4, 3) = K_{m, \text{in}}, p, + \delta_{m, \text{in}}, p.
\]

The above elements are written for $p = \text{East wall}$. On similar lines the elements $D(5, 21)$–$D(8, 21)$ for West wall, $D(9, 21)$–$D(12, 21)$ for North wall, $D(13, 21)$–$D(16, 21)$ for South wall and $D(17, 21)$–$D(20, 21)$ for roof can be written:

\[
D(21, 1) = R\delta_{m, \text{in}}, A_{p}L_{p}, \\
D(21, 2) = R\delta_{m, \text{in}}, A_{p}.
\]
\[ D(21,3) = A_p h_{ins} I_p. \]
\[ D(20,4) = A_p h_{ins}. \]

The above elements are written for \( p = \) East wall. On similar lines the elements \( D(21,5) - D(21,8) \) for West wall, \( D(21,9) - D(21,11) \) for North wall, \( D(21,13) - D(21,16) \) for South wall, \( D(21,17) - D(21,20) \) for roof can be written:

\[ D(21,21) = -A_T h_{ins} - 0.33 NV - u_r A_p - u_r A_TL \]

The elements of matrix \( E \) are as follows:

\[ E(1) = R_{ins} U_{i3} + h_{out} T_{o3} \]
\[ E(2) = U_{i3} \delta_{m i} \]
\[ E(3) = R_{ins} U_{i3} \]
\[ E(4) = u_r U_{i3} \]

The above elements are written for \( p = \) East wall. On similar lines \( E(5) - E(8) \) for West wall, \( E(9) - E(12) \) for North wall, \( E(13) - E(16) \) for South wall, and \( E(17) - E(20) \) for roof can be written:

\[ E(21) = (-0.33 NV - u_r A_p - u_r A_TL) T_{o3} \]
\[ -A_T I_p + R_{ins} \delta_{m i} A_T L U_{i3} = GOC, \]

where \( T_{o3}, T_{o3} \) are the average parts of the solar and ambient temperatures, respectively. \( I_{oc} \) is the average part of the solar radiation on South wall.

The elements of matrix \( A \) are zero except those defined as follows: Here \( Q_{out} = \alpha_f q_{out}/\alpha_m q_{out} \delta_{oc}, Z_{out} = \delta_{oc}/\delta_{oc} K_{m} \delta_{oc} \delta_{oc} \).

\[ A(1,1,4) = Q_{out} / \delta_{oc} + R_{ins} \delta_{oc} - K_{m} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2}, \]
\[ A(2,1,4) = \frac{K_{m} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2}}{\frac{Q_{out} \delta_{oc}}{\delta_{oc}^2}} - K_{m} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2} + K_{m} \delta_{oc} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2} + K_{m} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2}, \]
\[ A(3,1,4) = \exp(\alpha_f L_p) R_{ins} \delta_{oc} + R_{ins} \delta_{oc} - K_{m} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2} + K_{m} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2}, \]
\[ A(4,1,4) = -h_{ins} E_{5}, \]
\[ A(5,1,4) = \exp(\alpha_f L_p) \delta_{oc} + K_{m} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2} + K_{m} \frac{Q_{out} \delta_{oc}}{\delta_{oc}^2}, \]

The above elements are written for \( p = \) East wall. On the similar lines \( A(5,21) - A(8,21) \) for West wall, \( A(9,21) - A(12,21) \) for North wall, \( A(13,21) - A(16,21) \) for South wall, \( A(17,21) - A(20,21) \) for roof, can be written:

\[ A(21,1,4) = A_p h_{ins} \exp(\alpha_f L_p) [Q_{ins} + Z_{ins} \delta_{oc}], \]
\[ A(21,1,4) = A_p h_{ins}. \]

The above elements are written for \( p = \) East wall. On the similar lines \( A(21,5,8) \) for West wall, \( A(9,12) \) for North wall, \( A(13,16) \) for South wall and \( A(17,20) \) for roof can be written:

\[ A(21,21) = (-W, C_T, \ln \omega - h_{ins} A_T L - u_r A_T L - 0.33 NV - u_r A_p) E_{5}. \]

The elements of matrix \( B \) are zero except those defined as follows:

\[ B(1) = h_{oc} T_{oc} E_{5}, \]
\[ B(5) = h_{oc} T_{ins} E_{5}, \]
\[ B(9) = h_{oc} T_{ins} E_{5}, \]
\[ B(13) = h_{oc} T_{ins} E_{5}, \]
\[ B(17) = h_{oc} T_{ins} E_{5}, \]
\[ B(21) = (-u_r A_T L, T_{oc} - u_r A_T L - 0.33 NV T_{oc} - u_r A_p) E_{5}. \]

where \( T_{oc}, I_{oc} \) are the harmonic parts of the ambient temperature and solar radiation (on South wall), respectively.

References