Wide angle and bi-directional beam propagation using the collocation method for the non-paraxial wave equation

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Abstract

A method, based on the collocation method, for wide angle and bi-directional propagation of non-paraxial waves is presented. The second-order wave equation is converted to a matrix ordinary differential equation, which is solved numerically requiring no correction for energy conservation or evanescent mode suppression. No approximation for the wave equation such as the Fresnel approximation, or for the square root of the propagation operator is used. Examples show the performance of the method.

Keywords: Beam propagation; Numerical methods; Optical waveguides

1. Introduction

Numerical methods for beam propagation are widely used for design and analysis of waveguides and devices. These methods directly give the total picture of the field as it propagates through a waveguide, which may have a very complicated structure involving several branches and variations in physical characteristics. Conventional methods are based on the Fresnel or paraxial approximation of the scalar wave equation and have, therefore, limited accuracy even for moderately angled beams. Several schemes have recently been proposed to treat wide-angle beam propagation [1-16]. In general, this would involve solving directly the wave equation, which involves a second-order partial differential with z (the direction of propagation) as against the first-order partial differential in the paraxial wave equation. All the methods for non-paraxial propagation discussed in the literature approach this problem iteratively, in which a numerical effort equivalent to solving the paraxial equation several times (the actual number depends on the desired accuracy and the obliquity of the beam) is required. Many of these methods neglect the backward propagating components and solve the one-way wave equation; but even methods that deal with bi-directional propagation suffer from the problem of non-conservation of energy as well as instability due to evanescent modes [6-9]. These methods include those based on the Pade approximants [2-9], Lanczos reduction [10-13] and Taylor expansion [1]. In all these methods, the square root of the propagation operator involved in the wave equation is approximated in various ways.

We have obtained direct solutions of the wave equation using the collocation method [17-19], in which the wave equation is converted into a second-order ordinary matrix differential equation, termed as the collocation equation. The collocation equation can then be solved numerically using any standard numerical method such as the Runge-Kutta method. We present in this paper, this method and give some examples to show the applicability and accuracy of this method.

2. The method

The scalar wave propagation problem is defined through the wave equation

$$\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial x^2} + k_0^2 n^2(x,z)\psi(x,z) = 0, \tag{1}$$

(z being the general direction of propagation) which is a partial differential equation and hence, is difficult to solve. In the collocation method, we seek its solution for $\psi(x,z)$ as a linear combination over a set of suitable orthogonal functions, $\phi_n(x)$: $\psi(x,z) = \sum_{n=1}^{N} c_n(z)\phi_n(x)$ where $c_n(z)$ are the expansion coefficients, n is the order of the basis functions and N is the number of basis functions used in the expansion. The choice of $\phi_n(x)$ depends on the boundary conditions and the symmetry of the guiding structure. Recently, wavelets have also been used as the basis functions [20]. For the present, we have $\phi_n(x) = \cos(m_n x)$ for $n = 1, 3, 5, \ldots, N-1$ and $\phi_n(x) = \sin(m_n x)$ for $n = 2, 4, 6, \ldots, N$. In the collocation method, we further require that the differential equation, Eq. (1), is satisfied *exactly* by the expansion at N collocation points x_j, $j = 1, 2, \ldots, N$, which are chosen such that these are the zeroes of $\phi_{N+1}(x)$. Thus, using the collocation principle and with some algebraic manipulations, one converts the wave equation, Eq. (1), into a matrix ordinary differential equation (see [17-19] for details)

$$\frac{d^2\Psi}{dz^2} + \mathbf{S}\Psi(z) = 0, \tag{2}$$

where $\mathbf{S} = \mathbf{B}\mathbf{A}^{-1} + \mathbf{R}(z)$, matrices A and B being constant matrices dependent on the chosen functions, $\phi_n(x)$, and $\Psi(z) = \text{col.}[\psi(x_1,z)\ \psi(x_2,z) \cdots \psi(x_N,z)]$, $\mathbf{R}(z) = k_0^2\,\text{diag.}[n^2(x_1,z)\ n^2(x_2,z) \cdots n^2(x_N,z)]$. We refer to Eq. (2) as the collocation equation. In deriving this equation from the wave equation, Eq. (1), no approximation has been made except that N is finite and Eq. (2) is exactly equivalent to Eq. (1) as $N \to \infty$. Thus, the accuracy of the collocation method improves indefinitely as N increases. The collocation equation is a matrix ordinary differential equation and can be solved as an initial value problem using any standard method such as the Runge-Kutta method or the predictor-corrector method. Since, we are solving the wave equation directly, the backward propagating components have not been neglected, ensuring conservation of energy [6-9].

In the methods based on standard Padé approximants, evanescent modes are not modeled properly leading to instability [6-9]. Complex coefficient Padé approximations have to be used to define and include evanescent waves properly [6-8]. Unlike these methods, our method does not require a special form of the propagator for modelling of the evanescent modes and their decay, because no square root operator has to be approximated and the wave equation is solved directly.

3. Numerical examples

In order to show the performance of the method, we include here some examples. As the first example, we consider the propagation of a Gaussian beam at 45° to the z-axis for a distance of 10 μm [2]. The width of the Gaussian beam intensity was 2.828 μm and the wavelength 1.06 μm. The computation was done with 120 collocation points and the width of the numerical window was about 80 μm. The result is shown in Fig. 1 in which the paraxially propagated beam is also included. This result matches well with that obtained by Hadley [2].

The next example is that of a tilted graded-index waveguide where $n^2(x) = n_s^2 + 2n_s\Delta n\,\text{sech}^2(2x/w)$, $n_s = 2.1455$, $\Delta n = 0.003$, $w = 5$ μm and

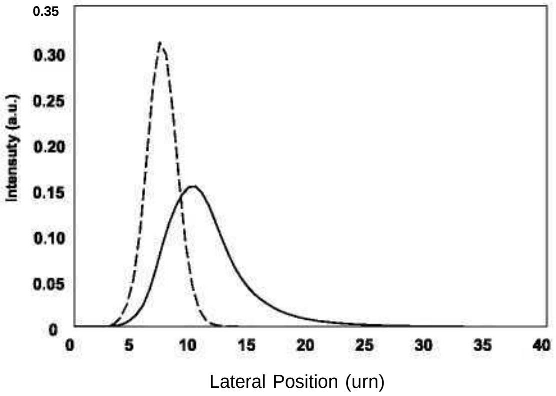

Fig. 1. Intensity (a.u.) of a Gaussian beam obtained using paraxial (dashed curve) and non-paraxial (continuous curve) propagation. The beam is phase tilted at 45° with respect to the direction of propagation.

X= 1.3 lm in which the fundamental mode is propagated [15]. The computation was done with 530 collocation points and the width of the numerical window was about 185 lm. We define here a correlation factor in Eq. (3), which measures both the dissipation in power as well as the loss of shape of the propagating mode

$$\mathrm{CF} = \frac{\left|\int \psi^*_{\mathrm{exact}}\psi_{\mathrm{calc}}\,\mathrm{d}x\right|}{\sqrt{\{\int |\psi_{\mathrm{inp}}|^2\mathrm{d}x\}\{\int |\psi_{\mathrm{exact}}|^2\mathrm{d}x\}}}, \quad (3)$$

where wi_{np}, w_{calc} and w_{exact} are the input, the propagated and the exact fields, respectively. Fig. 2 shows the error in CF vs. tilt angle after propagation to a distance of 100 lm. We have not plotted directly CF vs. tilt angle as in [15] because the CF values are all very close to 1 and the graph would just be a straight line parallel to the x-axis at 1. Our method shows much lower error in propagation as compared to the method of Shibay ama et al. [15]. Their method requires in excess of 1000 points even with an adaptive grid; in our method better results were obtained with only 530 points. At 50° with the three-step process Shibayama et al. show a CF value of about 0.96, while our method returns a value of 0.999959.

We have also considered step-index waveguides [4,21]. Comparison of the CF value is made with that obtained by Yamauchi et al. [4] in the propagation of the TE1 mode with tilt angle. For the waveguide given in [4] our calculations require 500 points with a window size of 260 lm and show lower 1 — CF values than those of Yamauchi et al. [4] with 1800 points. For the strongly guiding waveguide [21], our method obtains a CF value of 0.997 with 750 points while Yamauchi et al. use more than 1800 points to obtain CF = 0.97 only. Fig. 3 shows the 1 — CF vs. tilt angle plot for the

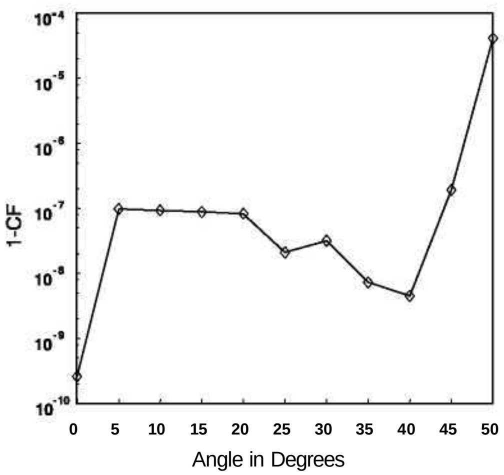

Fig. 2. Error in CF for the fundamental mode in a graded index waveguide with tilt angle.

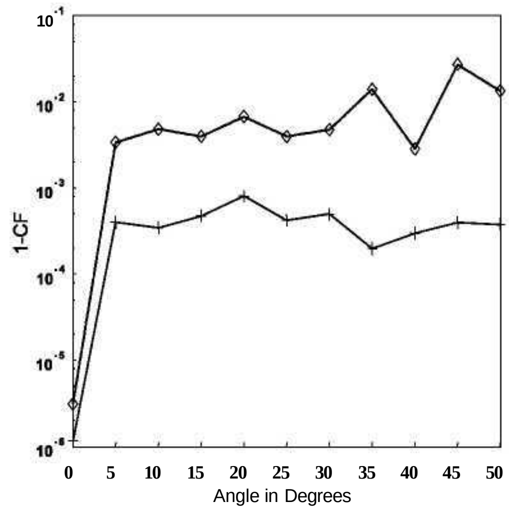

Fig. 3. Error in CF for the TE1 mode in weakly (thin line with crosses) and strongly (thick line with diamonds) guiding step index waveguides with tilt angle.

Table 1
Effort and error in different methods for propagating TE10 mode in a tilted waveguide

Method	N_x	N_z	Power in waveguide	Effort	Bi-directional
AMIGO	1311	1429	~0.95	1	No
FD2BPM	2048	1000	~0.95	1	No[a]
FTBPM	256	1000	~0.55	1	No
LETI-FD	1024	200	~0.15	1	No
Collocation	800	1000	~0.9	1	Yes

[a] The FD-BPM method has been extended for bi-directional propagation subsequently [7,8,16]; however, the result quoted here is taken from [21] and refers only to the uni-directional propagation.

TE1 modes in the weakly guiding [4] and strongly guiding [21] waveguides. As expected there is lower error for the low index difference waveguide. Higher order modes such as the TE_{10} mode were also propagated. We use 800 points for the calculation at tilt angle 20° for the benchmark test in [21]. We calculate the power loss in propagating the mode and compare it with the results obtained by others [21] in Table 1. N_x, N_z are the points in the transverse and longitudinal directions, respectively.

In the above examples the PML boundary condition was used wherever required. The computations involved in the non-paraxial wave propagation take only about 2-2.5 times that for the paraxial wave propagation and thus, in terms of computation time, our method would be more efficient than the third-order Padé approximation. However, our method does not involve any expansion of the propagation operator and here convergence check with respect to Padé order is not required. Thus, we would expect our method to be more efficient than the ones based on finite-order Padé approximation. Our method does not require any explicit correction for energy conservation [6-9] and no special form of the propagator for evanescent mode suppression [6-9]. Since the method is based on the direct solution of the wave equation, propagation in both forward and backward direction - is automatically taken into account. We are not including any examples here as we have earlier used the collocation method for the wave equation for modelling Bragg gratings and have obtained reflection spectra [19,22].

We have used the fourth-order Runge-Kutta method to solve the collocation equation. More efficient methods such as the predictor-corrector methods would make the propagation more efficient by factors upto 2. Further results on such implementations would be presented in a more detailed paper later.

4. Summary

We have presented a numerical method for non-paraxial wave propagation through waveguides and homogeneous media. The method is based on the collocation method and the resulting second-order ordinary matrix differential equation is solved using the Runge-Kutta method. The method is applicable for wide angle as well as bi-directional propagation. Examples on tilted Gaussian beam propagation through a homogeneous medium and on graded-index, tilted step-index waveguides have been included. These show the applicability and the accuracy of the method. Comparisons with available results have also been included which show that our method is capable of better accuracy and is also numerically efficient.

Acknowledgements

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