Upper Hybrid Wave Pumped Free Electron Laser

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Abstract—**A large amplitude upper hybrid wave has potential to be employed as a wiggler for the generation of high frequency coherent radiation via free electron laser (FEL) instability. At a density fluctuation level of a few percent, due to the upper hybrid wave, the growth time of FEL instability, with electron beam current of a few kilo ampere, turns out to be of the order of a nanosecond. The growth rate of FEL instability depends sensitively on** wi **and this dependence comes through** v_{osc}**, the beam oscillatory velocity.** v_{OBC} **acquires large values at** k_{Oz} ≃ [wo - (wc/7§)]/f6 =s w_c[l - (l/$_{7o}$°)]/«;$_6$. **At this value of** k_{Oz}, wi ≃ 2TO^{2}W$_C$[2 - (I/70^o)]. **However, this scheme of FEL operation suffers from a severe limitation due to the parametric instability of the upper hybrid wave. The process of parametric decay into lower hybrid and upper hybrid waves may have growth rate comparable to that of the FEL instability.**

I. INTRODUCTION

THE electrostatic waves have been proposed as wigglers for the generation of high frequency coherent radiation in a free electron laser [l]-[4]. The scheme requires a high density plasma in the interaction region of the device to sustain the electrostatic waves. Reference [1] examined a Compton regime free electron laser using Langmuir wave as wiggler. Reference [2] proposed an explosive mode of free electron laser operation in the collective Raman regime, using Langmuir wave as wiggler. In this case the negative energy beam mode feeds energy into the wiggler and radiation waves leading to an explosive instability. In [3], we have examined the possibility of employing lower hybrid waves as wigglers for the generation of millimeter waves. Recently, [4] has proposed the concept of a compact ion ripple laser for generating short wave lengths. In this scheme a large amplitude ion density ripple with fc-vector at an angle to the relativistic electron beam velocity is required.

The employment of electromagnetic [5]-[14] waves as wigglers has also drawn considerable interest over the years. Reference [5] has studied a gyrotron pumped free electron laser in the Compton regime. The generated frequency wi is given by wi = 47^2_QWO (where 70 is the relativistic gamma factor, *UJQ* is the pump wave frequency). The instability grows exponentially with time in the linear stage and saturates via particle trapping and other nonlinear effects. Reference [8] has examined the Raman regime operation of an electromagnetic wave pumped free electron laser. They have shown that it is possible to operate FEL in the explosive mode if one employs a slow wave dispersive medium. The pump and the radiation signal take energy from the negative energy space charge mode and grow. Reference [15] has demonstrated experimentally the frequency upconversion of a TM mode via free electron laser mechanism in a backward wave oscillator (BWO). The BWO is tuned to operate at 8.4 GHz. At the 8.4 GHz TM mode acquires high power (~1.2 MW) it is frequency upconverted into a 130 GHz wave.

In this paper, we examine the concept of an upper hybrid wave pumped free electron laser in the presence of a guide magnetic field. In Section II, we study the three wave nonlinear coupling, involving a upper hybrid wave wiggler, a negative energy space charge mode, and an electromagnetic mode in the collective Raman and Compton regimes. The upper hybrid wave may also undergo decay instability in the plasma, that may adversely affect the prospects of the FEL instability. In Section III, we study the parametric decay instability of a upper hybrid wave. A discussion of results is given in Section IV. The mathematical approach in the present analysis has been followed from the [3], [9], [17], [18].

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II. FREE ELECTRON LASER INSTABILITY

Consider a homogenous plasma of equilibrium density n_0^0 in a static magnetic field $\vec{B}_s \| \hat{z}$. An upper hybrid wave (ω_0, $\vec{k}_0$) propagates through it in the *x-z* plane with electrostatic potential ϕ_0

$$\phi_0 = \phi_0 e^{-i(\omega_0 t - \vec{k}_0 \cdot \vec{x})},$$

$$\omega_0^2 = \frac{1}{2}\left[(\omega_p^2 + \omega_c^2) + \sqrt{(\omega_p^2 + \omega_c^2) - 4\omega_p^2\omega_c^2 \frac{k_{0z}^2}{k_{0\pm}^2 + k_{0z}^2}}\right] \quad (1)$$

where ω_p, ω_c and ω_0 are the plasma, cyclotron and pump wave frequencies respectively. A relativistic electron beam with density n_b^0 and velocity $v_b\hat{z}$ is launched into the plasma. The beam electrons acquire an oscillatory velocity $\vec{v}_0$

$$v_{0x} = -\frac{ieE_{0x}(\omega_0 - v_b k_{0z})}{m\left[(\omega_0 - v_b k_{0z})^2 - \frac{\omega_c^2}{\gamma_0^2}\right]\gamma_0},$$

$$v_{0y} = -\frac{eE_{0x}\omega_c}{m\left[(\omega_0 - v_b k_{0z})^2 - \frac{\omega_c^2}{\gamma_0^2}\right]\gamma_0^2},$$

$$v_{0z} = \frac{eE_{0z}}{im(\omega_0 - v_b k_{0z})\gamma_0^3} \quad (2)$$

where $\gamma_0 = (1 - v_b^2/c^2)^{-1/2}$, $-e, m$ are the equilibrium relativistic gamma factor, electron charge, mass, respectively.

We perturb this equilibrium by coupled electromagnetic wave (o>i, $\vec{k}_1$ci) and the beam space charge mode (o>, $\vec{k}$) perturbations

$$\vec{E}_x \;\hat{}\; = xExe^{i\{u\}lt\sim kl z}\backslash$$

$$\phi = \phi e^{-i(\omega t - \vec{k}\cdot\vec{x})},$$

$$\omega_1 \cong k_1 c$$

$$o) = k_z v_b - \frac{\omega_{pb}}{7o^{0^{3/2}}} \quad (3)$$

where

$$\omega_{pb} = \left(\frac{4\pi n_{0b}^0 e^2}{}\right)^{1/2}$$

is the beam plasma frequency.

The phase matching conditions are

$$\vec{k}i = \vec{k} + \vec{k}_0.$$

They yield the frequency of the electromagnetic radiation

$$o>i = 27_0^{02} (k_{Oz}v_b + o>_0 - Wpb/l0^{03/2}) \cdot \quad (4)$$

wi has indirect dependence on $u>_p$. As w_p is increased o>o goes up, for a given k_{Oz}, hence $u>\backslash$ is enhanced. The frequency of the radiation wave has a rather weak dependence on beam density.

The linear response of beam electrons to the (o>i, $\vec{k}\backslash$) wave can be written as

$$\vec{v}_1 = \frac{e\vec{E}}{i\omega_1 m\gamma_0^0}. \quad (5)$$

The wiggler and the radiation wave exert a ponderomotive force on the beam electrons at (w, $\vec{k}$)

$$\vec{F}_p = -\frac{m}{2}[\vec{v}\cdot\nabla(\gamma\vec{v})] - \frac{e}{2c}(Ox3)$$

$$= -\frac{m}{2}[\vec{v}_b\cdot\nabla(\gamma_0^*\vec{v}_1 + \gamma_1\vec{v}_0^*) + \vec{v}_0^*\cdot\nabla(\gamma_0^0\vec{v}_1 + 71 \ll_6)$$

$$+ \vec{v}_1\cdot\nabla(\gamma_0^0\vec{v}_0^* + \gamma_0^*\vec{v}_b)] - \frac{e}{Y\wedge o} \times \vec{B}_1) \quad (6)$$

where 70,71 are the first and the second order perturbations of relativistic gamma factor, respectively.

However, only z component of $\vec{F}_p$ is effective in the vicinity of Cerenkov resonance $u> \simeq k_z v_b$- One may simplify (6) to write $F_{pz} = eik_z(f>_p$, where

$$(t>P = \frac{e<t>_o E i_x k_{Ox}}{2im\omega_1\gamma_0^0 k_z}\left[\frac{k_{0z}}{(\omega_0 - v_b k_{Oz})} - \frac{(o)_0 - v_b k_{Oz})k_{lz}}{\left[(\omega_0 - v_b k_{0z})^2 - \frac{\omega_c^2}{\gamma_0^{02}}\right]}\right]. \quad (7)$$

The ponderomotive force and the self consistent field $\vec{E}$ = -V(/> produce oscillatory electron velocity and density perturbation at (o), $\vec{k}$)

$$v_z = -\frac{ek_z(\phi + \phi_p)}{m\gamma_0^{03}(o) - k_z v_b)} \quad (8)$$

$$n = -\frac{n_{0b}^0 e k_z^2(\phi + \phi_p)}{m\gamma_0^{03}(\omega - k_z v_b)^2} \quad (9)$$

Using the density perturbation in Poisson's equation we obtain

$$e4> = -Xe0_P \quad (10)$$

where

$$e = 1 + Xe = 1 - \frac{\omega_{pb}^2}{\gamma_0^{03}(\omega - k_z v_b)^2},$$

χe is beam susceptibility.

The nonlinear current density at (u>i, £1) can be written as

$$\vec{J}\backslash = -\backslash nev\vec{Q}. \quad (11)$$

Using (11) in the wave equation

$$\nabla^2 E_1 + \frac{w_1^2}{c^2}\vec{E}_1 = -\frac{4\pi}{c^2}i\omega_1\vec{J}_1$$

we obtain

$$(\omega_1^2 - \kappa_1^2 c^2)E_{1x} = -\frac{\omega_{pb}^2 i\omega_1 k_z^2(\phi + \phi_p)v_{0x}}{2\gamma_0^{03}(\omega - k_z v_b)^2}. \quad (12)$$

From (10) and (12) we obtain nonlinear dispersion relation

$$(w_1^2 - f\varsigma^2 c^2)(o; - k_z v_b f = \frac{\omega^2 e^2\phi_0^2 k (1 - \nu/e)k^2 p'}{4m^2\gamma_0^5} \quad (13)$$

where

$$\frac{k}{\left[(\omega_0 - v_b k_{0z})^2 - \frac{\omega_c^2}{\gamma_0^{02}}\right]} - \frac{(wo - v_b k_{0z})^2 k_{iz}}{\left[(\omega_0 - v_b k_{0z})^2 - \wedge_{\,lo}\right]^2}.$$

We solve (13) in two limits

A. Compton Regime

In the case when beam current is small, $\backslash e < 1$, self consistent potential of the beam mode can be neglected as compared to ponderomotive potential ($cf> <C </>_p$), and (13) can be rewritten as

$$(\omega_1^2 - k_1^2 c^2)(\omega - k_z v_b)^2 = \frac{e^2\phi_0^2 k_z k_{0x}^2 p'\omega_{pb}^2}{4m^2\gamma_0^{05}}. \quad (14)$$

The two factors on the left hand side when equated to zero, $wi - k\backslash c = 0$,OJ - A;^ = 0, give radiation and beam modes respectively. We solve (14) around the simultaneous zeroes of the left hand side by expanding o>i as

$$\omega_1 = k_1 c + \delta$$

$$= k_z v_b + 6$$

Then, (14) gives

$$\wedge . \wedge \frac{{}^2\phi_0^2 k_z k_{0x}^2 p' e^{i2n\pi}}{8m^2\omega_1\gamma_0^{05}}. \quad (15)$$

The growth rate turns out to be

$$7 = \mathrm{Im}\, 6 = \left[\frac{\omega_{pb}^2 e^2\phi_0^2 k_z k_{0x}^2 p'}{8m^2\omega_1\gamma_0^{05}}\right]^{1/3}\frac{\sqrt{3}}{2}. \quad (16)$$

The growth rate scales as one third power of beam density and one third power of the wiggler wave intensity.

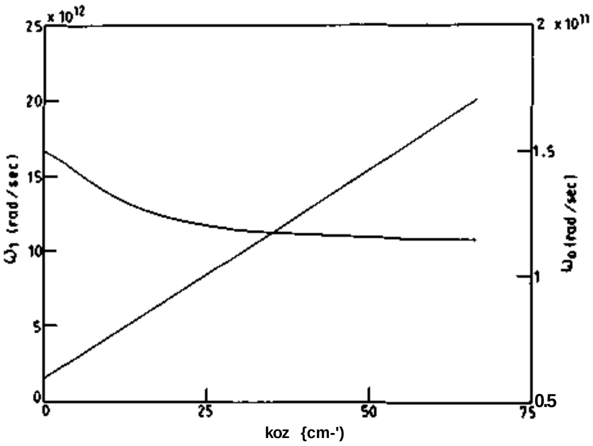

Fig. 1. Dispersion curves of an upper hybrid wave and the frequency of the radiation wave ui as a function of axial pump wave number.

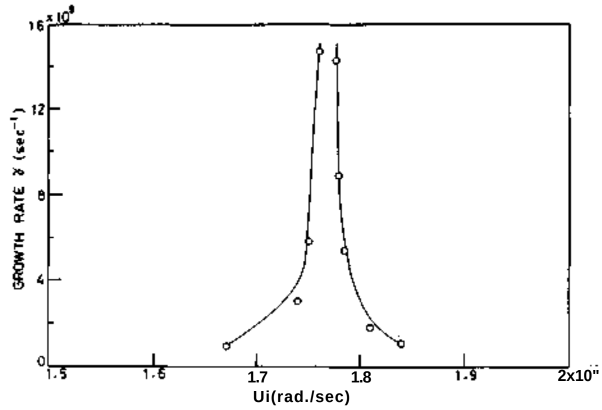

Fig. 2. Growth rate of the FEL instability in Raman regime as a function of radiation frequency *ui*.

B. Raman Regime

At large beam currents one may have e « 0. In this case self consistent potential far exceeds the ponderomotive one *((/> > 4>p)*. Then one looks for a solution of (13) around the simultaneous zeroes of the left hand side. The first factor when equated to zero, gives radiation mode while the other gives the beam space charge mode. We write

$$\text{wi} = k\backslash c + 6$$
$$= k_z v_b - \frac{\omega_{pb}}{\gamma_0^{0^{3/2}}} + \delta$$

and solve (13) for *8*. The growth rate turns out to be

$$\gamma = \text{Im}\ \delta = \frac{1}{4}\left[\frac{\omega_{pb} e^2 \phi_0^2 k_{0x}^2 k_z p'}{m^2 (\gamma_0^0)^{7/2} \omega_1}\right]^{1/2} \quad (17)$$

$$\simeq \frac{1}{2}\ \frac{v_{osc}}{c}\omega_0 \left[\frac{\text{kt7o}^{n^{5/2}}}{\omega_1}\right]^{1/2} \quad (18)$$

where v_{osc} is a measure of beam oscillatory velocity due to the pump wave in the *x*- direction. The dependence of 7 on plasma density appears through *wi*. As UJ_P is raised, u>i increases, hence, the growth rate falls down. In Fig. 1 we have plotted the dispersion curve of an upper hybrid wave, using (1) and the frequency of the radiation wave *UJ* as a function of axial pump wave number, using (4) for the following typical parameters: w_p^2/w_c^2 = 0.8, plasma density *UQ* = 3.1 x 10^{12} cm"^{3}, static magnetic field B_s = 6 kG, beam density ng_t = 2.3 10^{11} cm"^{3}, beam current I_b = 1 KA, beam velocity v_b = 0.9 c, beam cross section A_b = 1 cm^2, beam energy = 0.65 MeV, relativistic gamma factor 7$ « 2.3, beam plasma frequency w_{pb} = 2.7 x 10^{10} rad/sec. The frequency of the pump wave w_0 decreases from WUH (upper hybrid branch) to UJ_C (cyclotron frequency) as the axial pump wave number increases.

Following (18) we have plotted in Fig. 2 the growth rate of the FEL instability in Raman regime as a function of radiation frequency *wi* for the pump field amplitude E_o ~ 100 esu and following parameters given above. The growth rate of FEL instability depends sensitively on wi. This dependence comes through v_{osc}. The beam oscillatory velocity v_{osc} acquires large values at

$$k \sim \left[\ldots - \frac{\omega_c}{7o}\right] / \ldots \sim \ldots \left[1 - \frac{1}{7o}\right] / v_b.$$

At this value of k_{oz}

$$\omega_1 \simeq 2\gamma_0^{0^2}\omega_c\left[2 - \frac{1}{\gamma_0^0}\right].$$

The growth rate scales as uiJ_b^{2},w_o,$7o^{1/4}$. The power density P_o of the upper hybrid pump wave = *u>o(deo/du)o)(E'o/8Tr)vgZ*. For the same parameters as mentioned earlier and for the electron temperature ~2 eV, the power density turns out to be ~3 KW cm"^{2}. The choice of high density plasma helps in propagating beams of higher currents because of partial charge and current neutralization [16].

III. Parametric Decay Instability

The upper hybrid pump wave may also excite a parametric decay instability involving a lower hybrid wave *(f>(u>,k)* and a upper hybrid side band ^1(^1, fci)

$$(j> \bullet=. \text{(pe}^{\ldots} \quad (19)$$

$$</,! = j_{>1}e^{i(\ldots)}. \quad (20)$$

The response of plasma electrons to the pump wave is governed by the equation of motion and continuity, which on linearization yields

$$\vec{v}_0 = \frac{eE_{0x}}{m(\omega_0^2 - \omega_c^2)}(-\hat{x}i\omega_0 \ldots \text{v}, -\frac{eE_{Oz}}{imujo} \quad (21)$$

$$n_0 = -\frac{n_0^0 e\phi_0}{m}\left[\frac{k_{0x}^2}{\omega_0^2 - \omega_c^2} + \frac{k_{0z}^2}{\omega_0^2}\right]. \quad (22)$$

Now we examine the parametric decay of a upper hybrid pump wave into two other waves:

1) High frequency sideband wave <^i(wi,fci) and
2) a low frequency lower hybrid wave *(/>(w, k)*.

The phase matching conditions demand that

$$\omega_1 = ui - LJO, \vec{k}\backslash = \vec{k} - \vec{k}_0.$$

The linear response of electrons at *(wi,k\)* can be obtained from (21) and (22) by replacing 0 by 1: The pump and sideband waves exert a low frequency ponderomotive force on the electrons

$$F_p = \frac{}{2}[(\vec{vo} \bullet V)\vec{w}i + (\vec{vi} \bullet V)\vec{u}b]. \quad (23)$$

The response of the electrons at *(w, k)* due to $\vec{F_p}$ and the self-consistent field *—V<f>* is obtained by solving the equations of motion and continuity

ek$_x$d>$_,$uj u>

m{w*-ul) 2i(w'-u>£)

· [(^) · *7W$_{1x}$ + (v$_1$ · V7^v$_{0x}$]

$$-\frac{\omega_c}{2(w^2 - w^2_c)}[(*> \bullet VK + (?! \bullet VW \quad (24)$$

«z = ——h — («0 • VjWu + («i • V)V$_{Oz}$\ (25)

mu) 2iu>

$$\wedge e = \wedge d - \frac{pk^2th}{(9 \quad 9\backslash} - \frac{pk^2ih}{9} \frac{k_t}{r»-/ \; 9 \quad \omega_c^2)}$$

$$\cdot ((\vec{v}_0 \cdot \nabla)v_{1x} + (\vec{v}_1 \cdot \nabla)v_{0x}) - \frac{\omega_c k_x}{2\omega(\omega^2 - \omega_c^2)}$$

$$\cdot [(\vec{v}_0 \cdot \nabla)v_{1y} + (\vec{v}_1 \cdot \nabla)v_{0y}] + \frac{k_z}{2i\omega^2}$$

$$\cdot [(\vec{VO} \bullet V)v_{lz} + (\vec{V!} \bullet V)v_{Oz}] \quad (26)$$

$$ni = n_0 \left[\frac{\rho k_x^2 4>}{m-(w^2 - w^2)_{ci}} + \frac{\rho k^2 \phi}{m\wedge J^2}\right]' \quad (27)$$

Using (26) and (27) in the Poisson's equation we obtain

/ 2 .2 ,2 j.2 ,2 . 2\

$$\left(1 - \frac{}{\omega^2 - W^2_c} - \frac{}{A^2} - \frac{}{w^2} - \frac{}{A^2} - \frac{}{w^2 - uA\; fc^2}\right)$$

$$= -\frac{\omega_p^2 e\phi_0\phi_1\omega_0 k_x k_{1x}}{2mw(w^2-w_c^2)(w_1^2-w^2)fc^2}(\omega\omega_1 + WC^2)$$

$$\times \left[\frac{k_{0x}k_{1x}}{ZT^\wedge Zl} + \frac{k_{0z}k_{1z}}{T}\right] - \frac{u)_p^2 ed)()d>tk_z}{2mw^2k^2}$$

$$\cdot \left[\frac{\omega_0}{\omega_1}\frac{k_{0x}k_{1z}k_{1x}}{(\omega_0^2 - \omega_c^2)} + \frac{\omega_1}{\omega_0}\frac{k_{1x}k_{0z}k_{0x}}{(\omega_1^2 - \omega_c^2)} + \frac{k_{1z}k_{0z}k_z}{\omega_0\omega_1}\right]$$

$$-\frac{\omega_p^2 e\phi_0\phi_1\omega_1 k_x k_{0x}(\omega\omega_0 + w_c^2)}{2m\omega(\omega^2 - \omega_c^2)(\omega_0^2 - \omega_c^2)k^2}$$

$$\cdot [rf^\wedge + ^\wedge rJ - k_{0z}] \quad (28)$$

The nonlinear response of electrons at the sideband wave is governed by the equation of continuity.

$$\frac{\partial n_1}{at} + v \cdot \left(n_0^0\vec{v}_1 + \frac{1}{2}n\vec{v}_0^*\right) = 0 \quad (29)$$

which on solving yields the perturbed electron density

$$n_{1e} = -\frac{n_0^0 e\phi_1}{m}\left[\frac{k_{1x}}{wj - wj} + \frac{}{wj}\right] - \frac{1}{2}\frac{n_0^0 e^2\phi\phi_0\omega_0}{m^\wedge wi}$$

$$\cdot \left[\frac{1.2}{\omega_9 - \omega_e} \frac{j.2}{1 \; \omega_9}\right]\left[\frac{r\,1.\,1,}{\omega_9 - \omega_e} \frac{}{\omega_e}\right] \quad (30)$$

Using (30) in the Poisson's equation we obtain

$$\left[1 - \frac{U)^2}{3} - \frac{h^2}{12} - \frac{U^2 k^2}{2\;12}\right]$$

$$-\frac{1}{2}\frac{e\#pw_0w_p^2}{mwifc_1^2}\left[\frac{fc^2}{UJ^2 - u_c^2} + \frac{fcl}{w^2 j}\right]\left[\frac{fox^\wedge u}{W_0^2 - w_c^2} + \frac{fco_2fc_u}{w^2}\right]. \quad (31)$$

Combining (28) and (31) we obtain the nonlinear dispersion relation

$$ei(wi)e(w) = \frac{1}{4}\frac{e^2(j)^2 uj_0 u_p^2 R}{ma, w_1 pp} \quad (32)$$

where

$$ei(wi) = 1 - \frac{n}{2\;w_i - w_c} - \frac{9}{fc_i} - \frac{2}{w_i} \quad TT>$$

$$\epsilon(w) = V\left[1 + \frac{\wedge}{w?} - \frac{\wedge}{w2} * \frac{1}{fc2} - \frac{\wedge}{W^2}\frac{1}{}\right],$$

$$R = \left[\frac{W_p^2}{m}\left(\frac{k_x^2}{\omega^2 - \omega_c^2} + \frac{k_z^2}{\omega^2}\right)\left(\frac{K_{Ox}Ki_x}{\omega_0^2 - \omega_c^2} + \frac{K_{Oz}Ki_z}{\omega_0^2}\right)\right]$$

$$\cdot \left[\frac{k_x k_{1x}\omega_0(\omega\omega_1 + \omega_c^2)}{(\omega^2 - \omega_c^2)(\omega_1^2 - \omega_c^2)}\left(\frac{k_{0x}k_{1x}}{\omega^2 - \omega_c^2} - \frac{k_{0z}k_{1z}}{\omega^2}\right)\right.$$

$$-\frac{k_x k_{0x}\omega_1(\omega\omega_0 + \omega_c^2)}{(\omega^2 - \omega_c^2)(\omega_0^2 - \omega_c^2)}\left(\frac{k_{1x}k_{0x}}{\omega_1^2 - \omega_c^2} + \frac{k_{1z}k_{0z}}{\omega_1^2}\right)$$

$$-\frac{\omega_0 k_{0x}}{WiUJ}\frac{k_{1z}k_{1x}k_z}{(w_0^2 - w_c^2)} - \frac{\omega_1}{w w_o}\frac{k_{1x}k_{0z}k_{0x}k_z}{(w_1^2 - {}^\wedge 2_c)}$$

$$\left.-\frac{k_{0z}k_{1z}k_z^2}{\omega_0\omega\omega_1}\right].$$

£1(w1) and e(w) when equated to zero, gives the upper hybrid wave of frequency u/i and lower hybrid wave of frequency

$$.2 \;\; ">! = 7i\backslash\backslash v\sim c)\sim\left\{(l\backslash p\sim i)^2 - 4\omega_p^2\omega_c^2\right\}^{1}$$

where wi is different for parametric instability as well as for FEL instability

$$\omega = \omega_{LH}\left(\frac{U^\wedge kj}{VmA;^2}\right)^{1/2}$$

where

$$\omega_{LH} = \frac{\omega_{pi}}{\left(i^\wedge i\right)^{1/2}}$$

is lower hybnd frequency and $u>_{vi}$ = ion plasma frequency, We express

$$\omega_1 = \omega_{1r} + i\gamma$$

$$\omega_r + i\gamma.$$

Now expand ei(wi) and e(w) as

$$\epsilon_1(\omega_1) = \frac{\partial\epsilon_1(\omega_1)i\gamma}{\partial\omega_1}$$

$$\epsilon(\omega) = \frac{\partial\epsilon(\omega)i\gamma}{duj}$$

$$\gamma = \frac{1}{4}\left[\frac{e^2\phi_0^2\omega_0\omega_p^2 R}{m\omega\omega_1 k^2 k_1^2\left(\frac{\omega_p^2\omega_1}{(\omega_1^2-\omega_c^2)^2}\,\frac{k_{1x}^2}{k_1^2} + \frac{\omega_p^2}{U\backslash}\,\frac{\wedge}{k_1^2}\right)\left(\backslash tO^3 \quad k^2 + LJ^3\right)}\right]^{1/2}$$

$$R \approx \frac{\omega_p^2}{m}\;\frac{k_x^2}{(\omega^2 - \text{a,?})}\;\frac{k_{0x}k_{1x}}{(\text{wg} - wl)}\;\frac{k_x^2 k\backslash_X\,\omega_0}{(\text{w? - wfXu}_c^2\text{. - }_{\text{w}}\text{?})}$$

The growth rate turns out to be (see the unnumbered equation shown at the top of this page).

Then, the growth rate can be rewritten as

$$\gamma \simeq \frac{1}{4}k_x v_{osc} \simeq \frac{\omega}{v_p h}\,\frac{v_{osc}}{4} \tag{33}$$

where v_{osc} is a oscillatory velocity of plasma electrons due to the pump wave in the x-direction. By solving (33) numerically for the parameters mentioned earlier, the growth rate of the parametric decay instability of the upper hybrid wave is comparable to that of free electron laser instability. It is also sensitive to the frequency and the phase velocity of the lower hybrid wave. However, the upper hybrid wave pumped FEL suffers from a severe limitation due to the parametric instability of the upper hybrid wave. The large amplitude upper hybrid wave may decay into an upper hybrid wave and a lower hybrid wave.

It may be mentioned here that the scheme of upper hybrid wave wiggler FEL is feasible only at high plasma densities, n 8 > n o'V When plasma density is low, rfQ<n^b, the frequency of the upper hybrid wave is very close to LO_C from the relation

$$\omega_0^2 \approx \omega_c^2 - \omega_p^2\,\frac{Z\text{-}2_{\,n0z}}{k_{0z}^2 + k_{01}^2}.$$

The wave would be strongly cyclotron damped when wo — w_c ~ fcoz^th, where v_th is the electron thermal velocity. Since k_Oz Z ft/Li, where L\ is the length of system, the condition for the weak damping demands $Li(uj^2/2cj_c v_t h) > n$.

IV. RESULTS AND DISCUSSION

The operating frequency of an upper hybrid wave pumped FEL depends on fcOi, k_{Oz} and beam energy, *koi* is usually fixed by the transverse extent of the plasma column. In order to generate shorter wavelengths with moderate energy beam, one requires higher ko_z, which can be achieved by choosing w_0 close to $u>_c$. However, one can not have co_0 too close to $u)_c$ otherwise it would be heavily cyclotron damped when wo - w_c ~ $k_{Oz}v_t h$, where w_{th} is the electron thermal velocity. The growth rate of FEL instability depends sensitively on *UJ*. This dependence comes through v_{osc}, the beam oscillatory velocity. v_{osc} acquires large values at

$$k_{0z} \simeq \left[\omega_0 \quad \frac{OJc\,\text{I}\ /}{7o\,\rfloor /} \quad \text{I},\ \frac{1}{\gamma_0^0}\right] \Big/ Vb\text{-}$$

At this value of ko_z

$$\omega_1 \simeq 2\gamma_0^{0^2}\omega_c\left[2 - \frac{1}{\gamma_0^0}\right].$$

At large values of 7Q,WI scales as $7_0^{o^2}$. It has sensitive dependence on ko_z. For $ko_z \simeq 2.35$, the oscillatory beam velocity acquires a large values and the growth rate increases. The frequency of radiation at this point is $u>\backslash = 1.771 \times 10^{12}$ rad/s. The growth rate of upper hybrid wave pumped FEL scales as one fourth power of beam current. The upper hybrid wave wigglers appear to have some promise for the generation of higher frequency in a free electron laser. The operation of FEL in the collective Raman regime is more attractive and should result in the generation of millimeter waves.

The upper hybrid wave pumped FEL suffers from a severe limitation due to the parametric instability of the upper hybrid waves. The large amplitude upper hybrid wave may decay into an upper hybrid wave and a lower hybrid wave. The growth rate of the parametric decay instability of the upper hybrid wave is comparable to that of free electron laser instability.

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