A Simple Hole Scattering Length Model for the Solution of Charge Transport in Bipolar Transistors

K. Datta and M. Jagadesh Kumar, *Senior Member, IEEE*

Abstract—Considering the small warping parabolic heavy hole model with the quasi-elastic approximation in acoustic phonon scattering, it is shown that the hole scattering length is independent of the hole energy. This result now makes it possible to solve the Boltzmann transport equation to obtain a simple analytical solution for the ballistic hole transport in a thin and uniformly doped base of a pnp transistor.

Index Terms— Ballistic transport, bipolar transistors, Boltzmann transport equation, hole scattering.

I. INTRODUCTION

GRINBERG and Luryi [1] for the first time solved the Boltzmann transport equation for electrons with appropriate boundary conditions and gave an analytical solution to the electron transport in a thin and uniformly doped base. While solving the Boltzmann transport equation, they used a simplifying assumption of an energy independent scattering length l_{BC} ^ *ISC(EK)-* This assumption is accurate for the electron acoustic phonon scattering with the quasi-elastic approximation. Based on this assumption they obtained simple analytical expressions for the electron current density and electron concentration in a short base. Following their analytical treatment of ballistic electron transport, several studies have been reported on thin base bipolar transistors [2]-[5]. However, to the best of our knowledge, no attempt has yet been made to solve the Boltzmann transport equation for obtaining an analytical solution for the hole transport in a uniformly doped thin base of a pnp transistor. This is largely due to the complexity of the hole energy band model and the energy dependent hole scattering length [6]. The aim of this paper is therefore to show that considering a small warping parabolic heavy hole model with the quasi-elastic approximation in the acoustic phonon scattering, the hole scattering length can be modeled to be independent of the hole energy. This greatly simplifies solving the Boltzmann transport equation so that the ballistic hole transport can now be treated analytically just as is done for the electron transport by Grinberg and Luryi [1].

II. MODEL

In order to develop an analytical expression for the hole scattering length, we have assumed that the hole scattering process is due to acoustic phonons with the quasi-elastic approximation and that the valence band model has a single parabolic band (heavy hole) structure with the small warping approximation. With the quasi-elastic approximation, the phonon energy e_{ph} is small compared to the energy e of the carriers. Therefore, it is important to note that phonon scattering does not significantly change the carrier energy but it can change the direction of motion [7]. Under the quasi-elastic approximation

$$(\epsilon \pm \epsilon_{ph})^{1/2} \simeq \epsilon^{1/2}\left(1 \pm \frac{\epsilon_{ph}}{2\epsilon}\right) \quad (1)$$

where e is hole energy and e_{ph} is the energy of the phonon. This approximation is sufficiently accurate down to very low temperature (10 K) [8]. The energy-wave vector relationship e = $_e$(k) for a single parabolic heavy hole valence band is given by [6]

$$e(k) = ak^2[1 - g(\vartheta, \psi)] \quad (2)$$

where *4)* and *I/J* are the polar and azimuthal angles of k with respect to crystallographic axes where *k* is the magnitude of wave-vector k. The term *g(fl, ijj)* contains the angular dependence of the degenerate band and *a* can be expressed as

$$a = \frac{Tf|A|}{2mn} \quad (3)$$

where *h* is the reduced Plank constant, *A* is the inverse valence-band parameter and m_0 is the free-electron mass.

Under the small warping approximation, we have [8]

$$a[1-g(\vartheta', \psi')] = \frac{\hbar^2}{2m_h} \quad (4)$$

where m_h is the effective heavy hole mass, *D'* and *i//* are the final angles during hole transition from state k to k' due to the acoustic phonons. the small warping approximation is valid because it substitutes the density-of-states effective mass for a warping dependent effective mass [8]. With the above approximations, the integrated scattering probability per unit time for acoustic phonon absorption and emission is [8]

$$\Gamma_{h,ac}(\epsilon) = \frac{}{2^{9/2}\pi\rho u^4\hbar^4} \times \epsilon^{-1/2}\begin{Bmatrix} F_3(x) + (K_BT_0/\epsilon)F_4(x) \\ G_3(x) - (K_BT_0/e)G_4(x) \end{Bmatrix} \quad (5)$$

where

E_{ac}	acoustic deformation potential for holes;
p	material density of the semiconductor device;
u	acoustic velocity;
K_BT_0	thermal energy of the holes.

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The authors are with the Department of Electrical Engineering, Indian Institute of Technology, New Delhi 110 016, India (e-mail: mamidala@ieee.org).

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$F_3(x)$, $F_\pm(x)$, $G_3(x)$, and $G_4(x)$ are the integral functions of x where a: is a dimensionless variable which is expressed as

$$x = \frac{2m_h^{1/2}u\epsilon^{1/2}}{K_B T_0}. \tag{6}$$

For $x \leq 3/\sqrt{2}$, the analytical expressions for $F_3(x)$, $F_4(x)$, $G_3(x)$, and $G_4(x)$ are given by [8]

$$F_3(x) = 2x^2 \left[1 - \frac{?\sqrt{2}x}{105} + \frac{x^2}{12}\right] \tag{7}$$

$$F_4(x) = x^3\left[\frac{136\sqrt{2}}{105} - x + \frac{44\sqrt{2}x^2}{315}\right] \tag{8}$$

$$G_3(x) = 2x^2 \left[1 + \frac{34\sqrt{2}x}{105} + \frac{x^2}{12}\right] \tag{9}$$

$$G_4(x) = x^3\left[\frac{136\sqrt{2}}{105} + x + \frac{44\sqrt{2}x^2}{315}\right]. \tag{10}$$

If we assume that: 1) x is less than one and 2) the hole energy ϵ is greater than $K_B T_0$, the expression for the integral scattering rate for both absorption and emission processes, given by (5), converges to

$$P_{h,\mathrm{ac}}(\epsilon) = \frac{E_{\mathrm{ac}}^2 m_h^{1/2}(K_B T_0)^3}{2^{9/2}\pi\rho u^4\hbar^4} \times \epsilon^{-1/2}2x^2. \tag{11}$$

Substituting the value of x from (6) in (11), the total scattering rate which accounts for both the absorption and emission can be expressed as

$$P_{h,\mathrm{ac},tot} = \frac{E_{\mathrm{ac}}^2 K_B T_0 m_h^2}{2\pi\rho u^2\hbar^4}\left(\frac{2\epsilon}{\ }\right)^{1/2}. \tag{12}$$

Due to the warping, $\sqrt{2\epsilon/m_h}$ is not the hole velocity any more. Hence, the expression for the scattering length of holes due to the acoustic scattering is

$$\frac{1}{l_{\mathrm{ac}}} = \frac{E_{\mathrm{ac}}^2 K_B T_0 m_h^2}{2\pi\rho u^2\hbar^4}. \tag{13}$$

This equation shows that the expression for the hole scattering length is independent of the hole energy. Equation (13) is based on the assumption that: 1) $x < 1$ and 2) $\epsilon > K_B T_0$. The validity of these two assumptions need to be examined for different mean energies of the hole as discussed below.

III. DISCUSSION

Since we are considering degeneracy in the valence band model (i.e., the small warping parabolic heavy model), the acoustic deformation potential E_{ac} is due to the transverse phonons [7]. Hence, the value of acoustic velocity for Si is $u = 5.3 \times 10^5$ cm/s [8].

A. When the Mean Hole Energy is High

Considering the average hole energy to be much more than the split-off valence band energy ϵ_{so} (= 0.044 eV), the hole effective mass $m_{\parallel}$ becomes equal to $1.26m_0$ [8]. During transport through the base region, the hole energy can increase either due to the presence of the built-in electric field if the base is nonuniformly doped or due to the ballistic transport (reduced probability of collision) if the base width is comparable to the scattering length. In either case, the maximum velocity the hole can attain is the thermal velocity (saturation velocity) which is twice the Richardson velocity [1]. Under high field conditions, the mean energy attained by the hole is within 0.2-0.3 eV when it reaches the saturation velocity. This result was confirmed using Monte Carlo simulation considering a single parabolic and warped valence band model [9], which is used in our analysis. If we assume that a similar mean energy is attained by the hole when it reaches saturation velocity by means of ballistic transport and substitute $\epsilon = 0.3$ eV for the mean hole energy in (6), we note that $x < 1$. Hence, our first assumption is valid. At room temperature, the value of $K_B T_0$ is 0.0259 eV, while the mean hole energy is one order of magnitude higher. Hence, our second assumption is also true. Therefore, (13) is valid when the mean hole energy is high.

B. When the Mean Hole Energy is Low

The mean energy consistent with the base electric fields which characterize both homo- and hetero-junction bipolar transistors may not be as high as 0.3 eV. The validity of (13) must therefore be verified when the mean hole energy is less. When the base field is low, let us assume that the lowest mean energy () that the hole can attain is the thermal equilibrium value of $(3/2)K_B T_0$. At room temperature, this mean hole energy is 0.039 eV and therefore, $\epsilon > K_B T_0$. When the mean hole energy is less than the split-off valence band energy t_{so} (= 0.044 eV), the corresponding hole effective mass is $m_h = 0.53m_0$ [8]. Substituting the values of m_h and $\epsilon = 0.039$ eV in (6), we note that $x < 1$. Hence, (13) is valid even for a low mean hole energy.

It is clear that our analytical model for hole scattering length is physically tenable at low as well as high mean hole energies. Therefore, (13) is well applicable for solving the Boltzmann transport equation to obtain a simple analytical solution for the ballistic hole transport in *pnp* bipolar transistors.

IV. CONCLUSIONS

In conclusion, we have shown that considering a small warping parabolic heavy hole model with the quasi-elastic approximation in the acoustic phonon scattering, the hole scattering length can be modeled to be independent of the hole energy. Using this model, the ballistic hole transport in short base pnp bipolar transistors can now be treated just as is done for the ballistic electron transport in npn transistors by Grinberg and Luryi [1].

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K. Datta, photograph and biography not available at the time of publication.

M. **Jagadesh Kumar** (SM'99) was born in Mamidala, Nalgonda District, Andhra Pradesh, India. He received the M.S. and Ph.D. degrees in electrical engineering from the Indian Institute of Technology, Madras.

From 1991 to 1994, he did his Postdoctoral research in modeling and processing of high-speed bipolar transistors with Prof. David J. Roulston at the University of Waterloo, Waterloo, Ont, Canada. During his stay at Waterloo, he also collaborated with Prof. Savvas G. Chamberlain on amorphous silicon TFT's. From July 1994 to December 1995, he first taught at the Indian Institute of Technology, Kharagpur, and later moved to the Indian Institute of Technology, Delhi, where he was made an Associate Professor in the Department of Electrical Engineering in July 1997. His research interests are in VLSI device modeling and simulation, IC technology, and power semiconductor devices.