Characterization of Mobility Patterns based on Cell Topography in a Cellular Radio System

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Abstract

Teletraffic modelling of users is crucial for cellular system layout and planning, and to evaluate tradeoffs in system design issues. In order to be useful, the model should capture mobility under the most generalized conditions. In this paper we develop models that incorporate general mobility patterns that are induced by the cell topography (e.g., road layout, street orientation, density of intersections). In particular, we study the effect of the different mobility patterns on the handoff probability. The mobility patterns are induced by common terrains such as Random, Manhattan, Circular and Highway. The results in this paper suggest that we may be able to estimate the handoff probabilities from a knowledge of the cell terrain, the mobile trajectory, and the vehicular movements in a cellular network, and thus may serve as a useful guide for cellular network engineering.

1 Introduction

Teletraffic and mobility modelling is essential for studying various design and tradeoff issues in a cellular system. Since most of the parameters of interest in a cellular system are stochastic quantities, one would expect the path of a mobile to be a random trajectory. These parameters could be, for example, the mobile speed, mobile direction or the call holding time. In reality, however, the cell topography stratifies certain paths and thereby constrains the trajectory of the mobile. This has a significant impact on important parameters in a cellular system, such as, the handoff probability.

Earlier models have assumed the mobile trajectories to be straight lines [4]. The authors in [9] show that there exists an emperical mapping of the general motion of a mobile to a straight line path in terms of the cell size. Several researchers suggest that the individual mobile trajectories are unimportant in a cellular system, and instead, lay stress on the *cell dwell time,* i.e., the residence time of a mobile in cell. The distribution of microcell dwell time for mobiles is calculated in [6]. Even though the cell dwell time is an important measure in the computation of the handoff probability, in literature the distribution of cell dwell time has been found to be general [9], [5], [6]. We believe this limits the usefulness of the cell dwell time. Further, the models studied earlier do not capture the effect of the cell topography on the mobility patterns, and it is with this in mind that we study the mobile trajectory in some detail. We provide analytic proofs for some of the results in this paper.

In this paper, we study various mobility patterns and their effect on the handoff probability. These mobility patterns are, in turn, induced by the topography of the cell, such as the road layout, street orientations, density of intersections etc.

The organization of this paper is as follows: In Section 2, we introduce the terminology and state the assumptions that are used in the simulation study. In Section 3, we study and analyze the effect of the number of turns of a mobile in a cell on the handofF probability. Through simulations, we characterize mobility patterns induced by some common cellular terrains, such as, Random, Manhattan, Highway and Circular. We study the difference in the handoff probability among cells with different mobility patterns. We conclude in Section 4.

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2 Mobility Model for Cellular Systems

This section introduces terminology that is essential to the understanding of mobility modelling in later sections. We state the assumptions that are used in the simulation of a cellular system.

2.1 Mobile Trajectory

Denote by a to be the current direction of a mobile and v to be the current velocity. Note that both these quantities are random variables. We define the following terms:

Decision Instance: This is the point at which a vehicle changes its mobility parameters v and a.

Termination Instance: This is a decision point where the mobile terminates its call and frees the channel allocated to it.
Arc: This is a circular path defined by a center and an angular movement with respect to the center.
Decision time: This time interval defines the time between two decision instances. In this time interval the mobile continues to move in a straight line, or follows a defined arc (in a circular terrain; this will be explained in detail in a later section)
Direction Set: This defines the feasible set of angles that the mobile can follow at a decision instance.

The cell topography (i.e., road layout, density etc.) constrains the mobility pattern and hence defines the decision instance, termination instance, decision time and the direction set. These variables in turn define the trajectory of the mobile.

2.2 The Simulation Model

Discrete Event Simulations are performed in the JAVA programming language to study the performance of the cellular system.

To simplify the overall simulation problem, we consider a single cell system. This assumption is justified when all cells in the system generate the same traffic and each cell has the same average rate of call initiation (see [3]).

In the simulations, we consider only active or calling mobiles, i.e., the mobiles always have calls in progress. The arrival process of new calls in a cell is assumed to be a homogeneous Poisson process with rate A. At call initiation, a mobile is assigned a random initial position from a uniform distribution over the cell area. The call is also assigned a random total duration (called the call holding time) sampled from an exponential distribution with rate *fi*. The direction of the active mobile depends upon the cell topography; this will be elaborated in the next section.

The discrete random variable N defines the number of decision points (turns) of a mobile. It is assumed to have a geometric distribution with parameter p. This assumption may seem simplistic but it ensures that the distribution of the call holding time is exponential (a geometric sum of exponential random variables is also exponentially distributed [8]). The decision times, T,, $i = 1,...,N$, (i.e., the random time until a change in the mobile direction) are independent and identically distributed (i.i.d.) random variables. We assume that the Tj's are exponentially distributed with a rate equal to w defined by ([8]) $u> = -\frac{\wedge}{\rho}$.

At each decision instance, we sample the velocity (u) and the current direction (a) of the mobile. The mobile velocity is sampled from a truncated Normal distribution with a given mean and variance. In Table 1, we show the parameters that have been used in the simulation study. Note that we have used the *fixed* channel assignment strategy [7].

Parameter	*Value*
Number of channels in cell	50
Call arrival rate	30 calls/min
Call holding time	120 seconds
Cell Radius	700 m
Mean of Mobile Velocity	10 m/s
Variance of Mobile Velocity	5 m/s

Table 1: Simulation Parameters

3 Mobility Characterization

In an urban area, the road layout, street orientation and the density of road intersections constrain the degrees of freedom of mobile vehicles in that region. They also determine the nature of the mobile trajectory and the feasible direction and velocity range of the mobile. This determines the traffic flow in a cell. It is therefore unrealistic to assume that the mobile speed is uniformly distributed or remains constant, and that the mobile paths follow straight lines throughout the call holding time. In developing countries, sections of the urban areas are characterized by multiple road intersections over small area, and it is reasonable to assume that the mobile trajectory has a large number of turns.

3.1 Handoff Probability versus Road Intersections

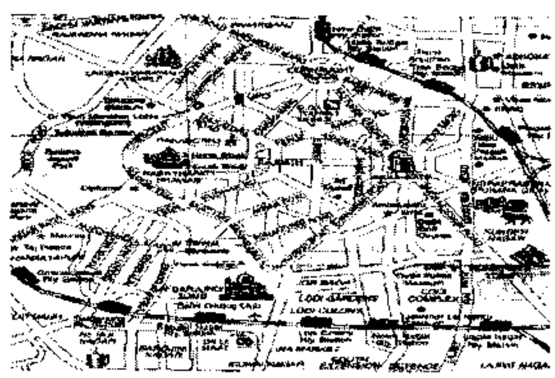

Figure 1: Road Map of a city showing different Road Patterns

The density of road intersections influence the frequency of direction change by the mobile. Typically, one expects that the greater the number of intersections encountered by a mobile, greater the expected number of direction changes by the mobile. The problem that is of interest to us is the dependence of the handoff probability on the density of road intersections in a cell. We study this behavior through discrete event simulations.

In this paper, we define *handoff probability* (P^) as the ratio of the number of calls that cross the cell boundary to the total number of calls that are generated (and accepted) in the cell. *Ph* is therefore equal to the fraction of calls that are handed over to the neighbouring cells. Since we simulate only one cell, we consider those handover calls that move out of the representative cell to the neighbouring cells. Note that if the cellular system has a homogeneous traffic across the cells, then, the fraction of incoming calls to a cell is equal to the fraction of outgoing calls from the cell and simulation of a single cell seems justified [3].

The dependence of the handoff probability on the average number of turns of a mobile, (or the density of road intersections, or the number of change of direction decisions of a mobile) obtained through *simulation* of a cell with a *random mobility pattern* is shown in Figure 2. Note that, for a fixed call holding time (and therefore for a fixed total length traversed by the mobile), the handoff probability decreases with an increase in the average number of turns. Similar behaviour is observed for the manhattan mobility pattern in a cell. An outline of the proof is as follows: consider a mobile placed at the center of a circular cell of radius r. For a mobile trajectory of length r and no turn in the mobile path, the mobile crosses the cell boundary w.p. 1. Now consider a mobile that takes a single turn in its path. The mobile starts at the center and changes its direction (at a point P) after a random length *{ar;* $0 \leq a \leq 1$). Since the shortest path from a point (P) in a circle to the circumference is the normal, it follows that the mobile now crosses the cell boundary w.p. 0. The argument is easily extended when (i) the mobile is initially placed at any point in the cell, (ii) when the length of the trajectory is greater than r, and, (iii) when the number of direction changes are greater than or equal to 2.

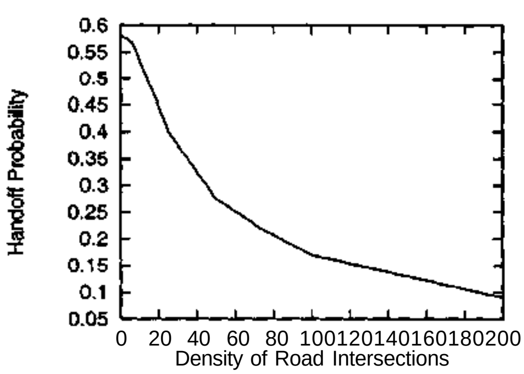

Figure 2: Handoff Probability versus Number of Turns of a Mobile

In the Appendix, we analytically prove the *asymptotic result* that as the number of change of direction decisions of a mobile in a cellular system increase to infinity, the handoff probability decreases to zero.

3.2 Comparison of Handoff Probabilities

We first describe the different cell terrain types in detail. We consider Random, Manhattan, Highway and Circular terrains in a cellular system: these are not exhaustive and other cases may arise in cellular systems. For each of these terrain types, we define a feasible direction set for a mobile. The choice of a feasible direction set precludes a mobile taking unrealistic turns when moving in a particular direction. Figure 1 shows the road map of a section of a city. The different road patterns are clearly seen in the figure.

In a cell with a random terrain, there are very few mobility constraints. The direction set, *9,* for the mobile is uniformly distributed between [-f + a, \ + a], where a is the current direction of the mobile. Initially, at the first decision instance, the mobile can select an angle uniformly between [0, 2TT). In a manhattan terrain, the typical layout would be roads that are perpendicular to each other. Hence, a mobile vehicle would be moving on a straight road until an intersection, where it then takes a decision whether to continue moving on the same road (if that is possible), or take one of the orthogonal roads. The direction set *6* in this case would be limited to *{a-^,a,a+%},* where each direction is equally likely. At the first decision instance, the mobile vehicle can choose from any one of {0, f ,7r, ^ } , each with equal probability.

Since intersections are not very common on a Highway, the direction set *0* on a highway is equal to the current direction of motion of the mobile (a). A mobile initially chooses from {0, TT} with both options equally likely. This implies that a mobile moves in either positive or negative *x* direction. The mobile then continues on that path till either (i) the call holding time is complete, or, (ii) the mobile crosses the cell boundary, in which case a handoff is initiated. The decision instances act as junctions for the mobile to change its velocity.

An urban or suburban area may have a few junctions. A junction point would typically be the center of a circular terrain. The road layout at these junctions would appear as a number of roads converging to the junction point and circular concentric roads at varying radial distances around the junction. At the initial decision point, the mobile could choose one of the four directions in a circular terrain: towards the junction point, away from the junction point, or along the two arcs of a circle around the junction point. The direction set at any other decision point would exclude the current direction: it being highly unlikely that a mobile moving in a direction takes a u-turn!

We now compare the handoff probabilities for different road layouts. For this comparison, we keep all the simulation parameters to be the same. These parameters are the call holding time, cell radius, distribution of mobiles in a cell, mobile velocity distribution and decision time distribution.

The mean length travelled by a mobile is the product of the average mobile velocity and the mean call holding time. Thus for a fixed mobile velocity, the mean length is directly proportional to the average call holding time.

In Figure 3 and Figure 4, for each of the terrains, we plot the handoff probability versus the mean length travelled by a mobile. Figure 3 corresponds to p = 0.99 (i.e., 100 decision points), Figure 4 corresponds to p = 0.92.

In Figures 3 and 4, we see that for all terrain types,

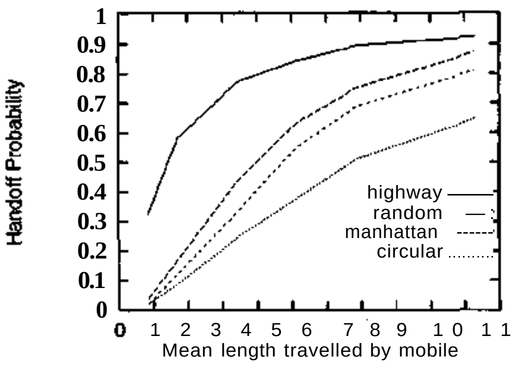

Figure 3: Handoff Probability versus Mean Call Holding Time for p = 0.99

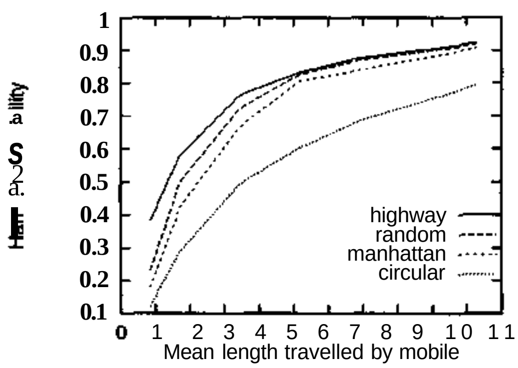

Figure 4: Handoff Probability versus Mean Call Holding Time for p = 0.92

the handoff probability increases with an increase in the expected length travelled by the mobile, and this is as expected. Observe also that as the mean length increases, the plots for different terrain types, excluding the circular terrain, converge. Beyond a threshold, a further increase in the expected length travelled by the mobile (which essentially is the mean call holding time) does not seem to effect the handoff probability significantly.

In Figure 3, we observe that for the highway terrain the handoff probability is the maximum and that the circular terrain has the least handoff probability. These observations are not surprising and we we give an explanation for the above. In a highway cell, mobiles can travel only in the positive or the negative x direction. If the call holding time is comparable to the diameter of the cell (i.e., the highway length), handoffs will be large. On the other hand, in a circular terrain, a large fraction of mobiles move either (i) along circular paths at a fixed distance from the center of the cell, or, (ii) towards the interior of a cell, (i.e., radially and towards the center). This results in low boundary crossing rates and hence low values of handoff probability.

What is interesting is that when the number of mobile turns are very large, and this is the case in Figure 3, the random terrain handoff probability bounds from above the handoff probability in the manhattan terrain. Work is in progress to prove this behaviour analytically using the techniques in [1].

It is seen that as the number of intersections decrease, (i.e., as we decrease p), the handoff probabilities in the random, manhattan and the highway terrain converge, whereas, the circular terrain continues to be characterized by the least handoff probability. This implies that, for the manhattan, random and highway layout, the difference in the handoff probability is insignificant when the number of change of direction decisions are small.

The results in this section are interesting: they suggest that we may be able to estimate the handoff probabilities from a knowledge of the terrain, the mobile trajectory, and the vehicular movements in a cellular network.

4 Conclusion

We have formulated a mobility model that includes general mobility patterns in a cellular network. A study of change of direction decisions of a mobile on the handoff probability revealed an interesting behaviour: the handoff probability decreases with an increase in the number of turns of a mobile in a cell. Further, we have studied different terrains that may arise in a cellular network, viz, Manhattan, Random, Highway and Circular. A comparison of the mobility pattern in the different terrains revealed that the circular terrain is characterized by the least handoff probability while the highway is characterized by a large handoff probability. For the random and manhattan terrains, the handoff probabilities lie between circular and highway; the handoff probability for random being greater than that for manhattan. The different handoff probabilities for different road patterns can be a useful guide for cellular network engineering.

This paper is a significant attempt to study how the handoff probability in a cell depends upon the mobility patterns that are in turn induced by the cell terrain. The results in this paper are new and interesting, and show that the individual mobile trajectories in a cellular system are also significant in that they govern the boundary crossing rate of mobiles.

Appendix: Proof of Theorem

In this section, we prove that as the number of change of direction decisions of a mobile in a cellular system increase to infinity, the handoff probability decreases to zero. To prove this, we use theorems from stochastic processes.

Recall that in this paper we consider only active mobiles. Consider a mobile initially placed at the co-ordinates (X_o,Yo) at which a call originates (Figure 5). The mobile moves through the cell for some duration and then may cross the cell boundary; this motion continues till the call holding time completion.

As in Section 2, we define the random variables Tj, 9i and H. For a mobile, Ti is the ith decision time, 6i is the direction taken by the mobile at the ith decision instance and H is the call holding time. Recall that the Tj's are i.i.d. exponentially distributed with mean $UJ\sim^{1}$, #i's are i.i.d. uniformly distributed between [0, 2TT) and H is exponentially distributed with mean /i"[1].

Denote the distance travelled in T» by *L{*. It is easy to see that if the mobile velocity *(v)* is a constant, then L; = *vT{*. That the L;'s are i.i.d., exponentially distributed follows from the fact that the T;'s are i.i.d., exponentially distributed and, v is a constant. Without loss of generality, we consider $v = 1$. This implies that $Li = Ti$ V *i*.

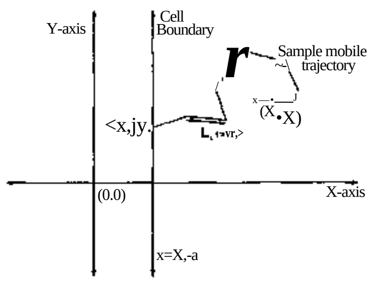

Figure 5: Sample Path of a Mobile Trajectory in a Cell

Consider a boundary of a cell at a distance a, $a > 0$, from X_o, i.e., the boundary is given by $x = X_o - a$. This boundary is parallel to the Y axis in the two dimensional plane (See Figure 5). Then, for any $a > 0$, define

$$N_a = inf\{n : a + \sum_{i=1}^{n} Ti \cos Si \leq 0\} \quad (1)$$

Define Xf as the x co-ordinate at the first decision instance of the mobile just after it crosses the line $x = Xo - a$. Thus, N_a is the number of decisions at Xf. We are interested in showing that

Theorem: For any arbitrary $a > 0$,

$$P\{\backslash Xf - X_Q |> O\} \to 0 \text{ as } u> \to oo$$

Proof: Observe that (Figure 5)

$$| X_f - XQ |< a + \max_{K i < N} T» \quad (2)$$

where the random variable N is the total number of decision points (turns) of a mobile. N is assumed to have a geometric distribution with parameter p.

Since maxi<j<jv $T_t \leq H$, $E[H] = \frac{1}{\mu} < oo$, then by the *Dominated convergence theorem* [1],

$$E\{ \lim \max TA = \lim E\{ \max TA \quad (3)$$

We will later show that the right hand side of Equation 3 is equal to zero, i.e.,

$$\lim_{w \to oo} E\{ \max_{\leq i < N} IU = 0$$

From Equation 3, therefore,

$$E\{ \lim \max T_{\{}\} = 0 \quad (4)$$

From Equations 2, 3 and 4, it follows that for any $a > 0$,

$$£\{ \lim | X/ - X_o |\} < a$$

But since a is an arbitrary positive number, we have

$$E\{ \lim | X_f - X_o |\} = 0$$

$$\Rightarrow \lim | Xf - X_o | = 0 \ a.s.$$

where a.s. stands for *almost surely* or tuif/j *probability one.*

We now show that

$$\lim_{UI-K3O} E\{ \max_{I \leq i \leq N} Ti\} = 0$$

Note that the Tj's are i.i.d. random variables, exponentially distributed with mean $w\sim^{x}$. It is easy to see that [8]

$$E\{ \max Ti\} = \frac{1}{}H_k$$

Note that the number of decision points, N, is a random variable with mean $E[N] = 1/(1 - p)$, and upon unconditioning,

$$E\{ \max Ti\} = \frac{(1-\rho)}{\mu}E\{H_N\}$$

Note that $H_N = O(logN)$ ([2], page 264). From the *Jensen's Inequality* [8] and the fact that - log(a;) is a convex function [8], $E\{log N\} \leq \log j \frac{1}{-\rho}$, which implies that

$$E\{ \max TJ = \frac{(1-\rho)}{\mu}O\left(\log\frac{1}{1-\rho}\right)$$

Finally, note that $w \to oo$ implies that /9 -> 1, and therefore $\lim_{\omega \to \infty} E\{\max_{1 \leq i \leq N} T_i\} = 0$. This completes the proof.

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