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TRANSIENT FREE CONVECTION IN A FLUID SATURATED POROUS MEDIA WITH TEMPERATURE DEPENDENT VISCOSITY

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ABSTRACT

The effect of temperature dependent viscosity on the heat transfer rate for a transient free convection in a porous medium adjacent to a vertical flat surface is studied. Solutions are obtained numerically by a suitable similarity transformation. It is shown that time taken to reach steady state is more for cooled wall than for heated wall. Also it is found that decrease in the viscosity ratio parameter increases the heat transfer rate and decreases the temperature.

Introduction

Study of natural convection heat transfer in fluid-saturated porous media has attracted considerable attention because of its numerous applications in many fields such as in gas production, grain storage, geothermal energy, petroleum reservoirs, packed-bed catalytic reactors and porous insullation [1]. It is important to consider the variable fluid properties in the study of natural convection when the heat transfer takes place under the conditions where there are large temperature differences within the fluid.

Analysis of natural convection with variable viscosity in a viscous fluid has been performed rather extensively [2]. Kassoy and Zebib [3] considered variable viscosity effects on the onset of convection in a horizontal, isotropic, water-saturated porous medium. Straus and Schubert [4] and Horne and O'Sullivan [5] have also considered the onset of convection of water as a non-Boussinesq fluid with viscosity and thermal expansivity dependence. Dona and Stewart [6] studied the varible viscosity effects on natural convection heat transfer in a short vertical cylinder for a volumetrically heated porous medium. Lai and Kulacki [7] investigated the effect of variable viscosity on mixed convection boundary layer flow around a vertical surface in a saturated porous medium. Ramirez and Saez [8] studied the forced convection heat transfer over a flat surface imbedded in a saturated porous medium by considering the temperature dependence of viscosity. Many studies [7,8] assume the linear variation of viscosity. Recently, Jang and Leu [9] presented steady state buoyancy-induced flow of liquids in a porous medium by taking the exponential variation of viscosity with temperature. Similarity solution for the study of the variable viscosity effect on transient natural convection flow in a porous medium seems not have been carried out.

The present study examines the effect of temperature dependence of viscosity on transient development of the natural convection in liquids from a vertical heated flat surface embedded in a fluid-saturated porous medium. Similarity solutions are obtained for the case of isothermal boundary condition.

Mathematical Formulation

Consider the relation between temperature and viscosity given by the equation [9]

$$\mu = \mu_f \exp [A (\bar{T} - \bar{T}_f)] \tag{1}$$

in a fluid saturated porous medium adjacent to a vertical flat plate [Fig.1]. Where μ_f is the absolute viscosity at the film temperature $\bar{T}_f = (\bar{T}_w + \bar{T}_\infty) / 2$

Under the assumption of constant fluid properties except the density, ρ, and absolute viscosity, μ, along with the Boussinesq and boundary layer approximations, the governing equations based on Darcy's law in non-dimensional form given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u = T \left\{ \nu^* \right\}^{\frac{1}{2} - T} \tag{3}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} \tag{4}$$

Where $\nu^* = \left\{ \frac{\nu_w}{\nu_\infty} \right\} = \exp [A (\bar{T}_w - \bar{T}_\infty)]$ is the wall to ambient viscosity ratio parameter.

It should be noted that $\nu^* = 1$ corresponds to the case of constant viscosity. For liquids the values of $\nu^* < 1$ corresponds to heated wall and the values of $\nu^* > 1$ corresponds to cooled wall.

In the above equations u and v are the Darcy velocity components in the x,y directions, t is the time and T is the temperature.

The non-dimensional quantities t, x, y, u, v, T are related to their dimensional counterparts $\bar{t}$, $\bar{x}$, $\bar{y}$, $\bar{u}$, $\bar{v}$, $\bar{T}$ by

$$t = \frac{\bar{t}\ Ra\ \alpha}{\sigma\ L^2},\ x = \frac{\bar{x}}{L},\ y = \frac{\bar{y}\ Ra^{1/2}}{L},\ u = \frac{\bar{u}\ L}{\alpha\ Ra},\ v = \frac{\bar{v}\ L}{\alpha\ Ra^{1/2}},\ T = \frac{(\bar{T} - \bar{T}_\infty)}{(\bar{T}_w - \bar{T}_\infty)} \tag{5}$$

Where subscripts w, ∞ refer to the condition at the surface wall and ambient fluid and Ra is the Rayleigh number given by

$$Ra = \frac{\rho\ K\ g\ \beta\ |\bar{T}_w - \bar{T}_\infty|\ L}{\alpha\ \mu_f} \tag{6}$$

The quantities σ, g, β, α, K denote heat capacity ratio, acceleration due to gravity, coefficient of thermal expansion, thermal diffusivity and permeability of the porous medium respectively.

The appropriate initial and boundary conditions for the problem are

$$t = 0 \;:\quad u, v = 0 \;,\quad T = 0 \;,\quad 0 \le y \le \infty \tag{7}$$

$$\left.\begin{array}{ll} y = 0: & v = 0 \;,\; T = 1 \\ y \to \infty: & u \to 0 \;,\; T \to 0 \end{array}\right\} \quad t > 0 \tag{8}$$

Method of Solution

In order to seek a solution to equations (2)-(4) we introduce the following similarity transformation

$$\tau = \frac{t}{x} \;,\quad \eta = \frac{y}{\sqrt{x}} \;,\quad \psi = \sqrt{x}\; f(\eta) \;,\quad T = \theta(\tau, \eta) \tag{9}$$

In terms of the new variables, it can be shown that the velocity components are given by

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial f}{\partial \eta} \tag{10}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{1}{2\sqrt{x}} \left\{ \eta \frac{\partial f}{\partial \eta} - f \right\} \tag{11}$$

Substituting for T, u, v from equations (9)-(11) the governing equations (2)-(4) and initial and boundary conditions (7) and (8) yield the following coupled system of differential equations:

$$\frac{\partial f}{\partial \eta} = \left\{ \nu^* \right\}^{\frac{1}{2} - \theta} \theta \tag{12}$$

$$\left\{ 1 - \tau \frac{\partial f}{\partial \eta} \right\} \frac{\partial \theta}{\partial \tau} - \frac{1}{2} f \frac{\partial \theta}{\partial \eta} = \frac{\partial^2 \theta}{\partial \eta^2} \tag{13}$$

with the following initial and boundary conditions

$$f(0, \eta) = \theta(0, \eta) = 0 \qquad 0 \le \eta \le \infty \tag{14}$$

$$f(\tau, 0) = \theta(\tau, 0) - 1 = \theta(\tau, \infty) = 0, \;\; \tau > 0 \tag{15}$$

Numerical Procedure

It can be noticed that equation (12) can be integrated easily to yield

$$f = \int_0^{\eta} \theta \, [\, \nu^*]^{\frac{1}{2} - \theta} d\eta \qquad (16)$$

Now we descretise equation (13) with a simple backward difference formula for the τ-derivative and central difference formula for η-derivatives to reduce the equation into a tridiagonal algebraic system

$$A_i \, \theta_{i-1,j+1} + B_i \, \theta_{i,j+1} + C_i \, \theta_{i+1,j+1} = D_i \qquad (17)$$

where

$$A_i = \frac{\Delta\tau}{4\,\Delta\eta} f_{i,j+1} - \frac{\Delta\tau}{\Delta\eta^2} \;,$$

$$B_i = 1 - \frac{\Delta\tau \;(j+1)\; [f_{i+1,j+1} - f_{i-1,j+1}]}{2\;\Delta\eta} + 2\,\frac{\Delta\tau}{\Delta\eta^2}$$

$$C_i = -\frac{\Delta\tau}{4\,\Delta\eta} f_{i,j+1} - \frac{\Delta\tau}{\Delta\eta^2} \qquad \text{and}$$

$$D_i = \theta_{i,j}\left\{ 1 - \frac{\Delta\tau \;(j+1)\; [f_{i+1,j+1} - f_{i-1,j+1}]}{2\;\Delta\eta} \right\}$$

The nodal point identified by a double index i,j defines its position η_i, τ_j.

The tridiagonal system (17) is solved by using Thomas algorithm (see Schlichting [10] page. 187) coupled with an interative scheme. The values of previous time step are assigned as an approximate values of f and θ before starting iterations. In each iteration f was kept constant and after obtaining the values of θ, f was replaced by new values obtained by numerical integration of the equation (16). In this case the trapizoidal rule of integration was used. This method is chosen for its simplicity and wide range of possible applications.

Discussion

The step sizes $\Delta\eta = 0.1$ and $\Delta\tau = 0.05$ were used while computations with four decimal accuracy as the criterion for convergence. The main results of the present study are displayed in Figures 2-4.

The first quantity of interest is the local heat transfer rate, which is given by

$$\frac{Nu_x}{\sqrt{Ra_x}} = -\left\{\frac{\partial\theta}{\partial\eta}(\tau,0)\right\} \tag{18}$$

The effect of viscosity ratio parameter ν^* on the heat transfer rate is shown in Fig. 2. It can be seen from the figure that increase in the value of ν^* reduces the heat transfer rate. The variation of heat transfer rate is not much significant for larger values of ν^*.

Temperature profiles for different values of ν^* at different time levels are plotted in Fig. 3. From the figure it is clear that at any location η temperature is more for cooled wall than for heated wall. It can be noticed that for larger ν^* variation of temperature is not much significant for small time.

Time taken to reach steady state (τ_s) with viscosity ratio parameter ν^* is shown in Fig. 4. It is evident from the figure that τ_s is an increasing function of ν^*, That is, time taken to reach steady state is lesser for heated wall than for cooled wall.

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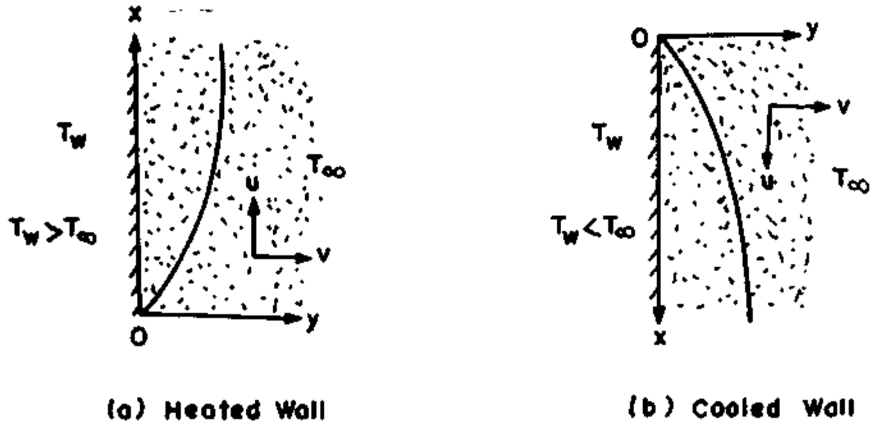

FIG. 1
Physical Model and Coordinate System

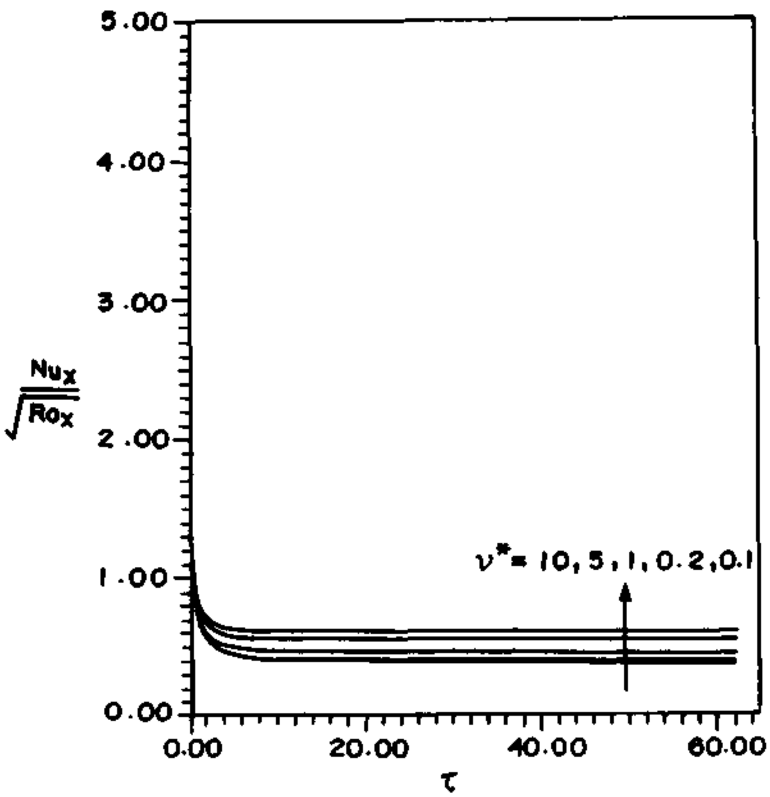

FIG. 2
Effect of Viscosity Ratio Parameter ν^* on Heat Transfer Rate.

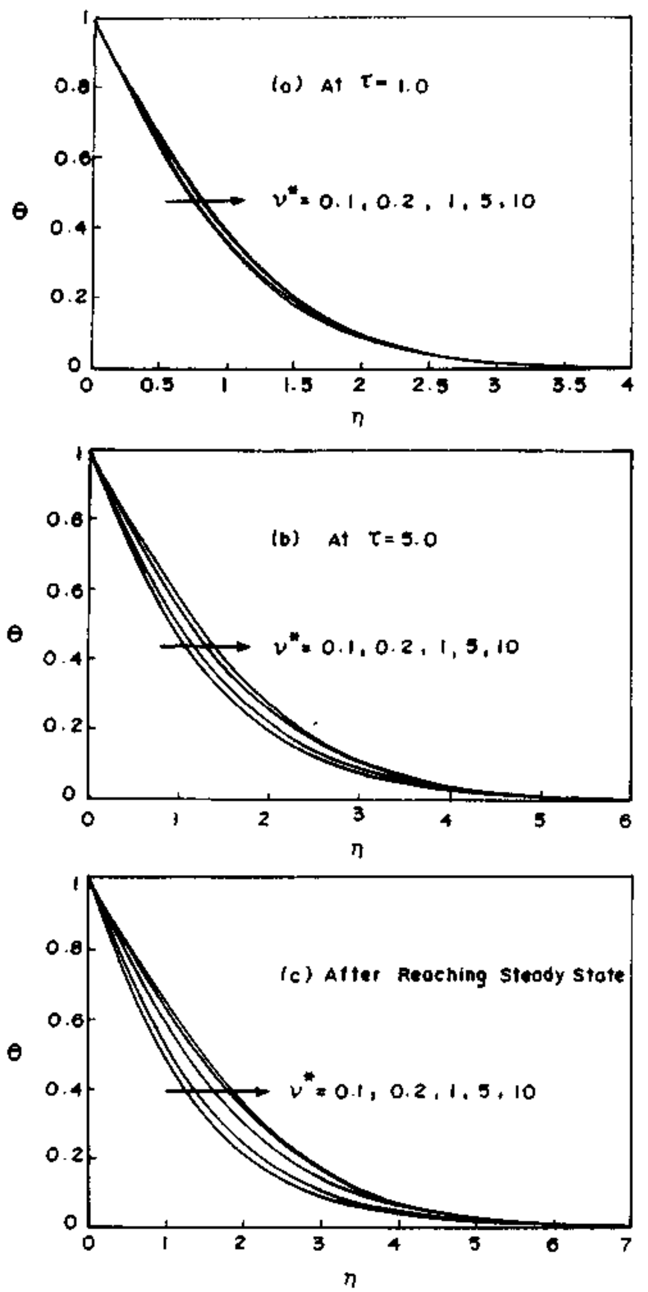

FIG. 3
Temperature Profiles

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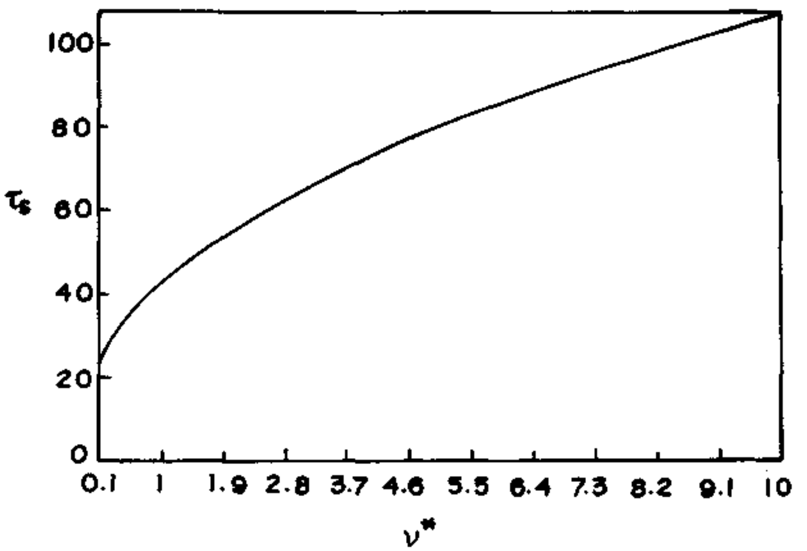

FIG. 4

Time Taken to Reach Steady State with ν^*

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