Reactive power compensation and load balancing in electric power distribution systems

Bhim Singh[a], Ambrish Chandra[b], Kamal Al-Haddad[b], Anuradha[c] and D.P. Kothari[c,*]
[a]Electrical Engineering Department, IIT, Hauz Khas, New Delhi, 110 011 India
[b]GREPCI, Electrical Engineering Department, Ecole de Technologie Superieure, 1100 rue Notredame Oust, Montreal, Quebec, Canada H3C 1K3
[c]Centre for Energy Studies, IIT, Hauz Khas, New Delhi, 110 011 India

This article presents a new method for reactive power compensation and load balancing in a four-wire, three-phase distribution system. An IGBT-based PWM voltage source inverter with a dc bus capacitor is used as a compensator. The hysteresis-based carrierless PWM current control is employed to derive switching signals to the devices of the compensator. A detailed dynamic model of the complete scheme is developed to study the steady-state and transient behaviour of the proposed compensator. Simulated results for the cases of single-phase, two-phase and three-phase lagging power-factor loads are presented and discussed in detail to demonstrate the reactive power compensation and load balancing capabilities of the compensator.

Keywords: reactive power compensation, load balancing, compensator

I. Introduction

The majority of electrical loads in distribution systems are non-unity power-factor reactive loads which draw a reactive power component of current along with the active power component. These reactive power components of load current cause low power factor, low efficiency and poor utilization of the distribution system. Moreover, sometimes there is an excessive load unbalancing caused by loads such as those from arc furnaces, traction, lighting, welding, heating, commercial building, air-conditioning and domestic industries. These loads sometimes draw large single-phase currents, two-phase currents and three-phase unbalanced currents, which cause large neutral conductor current, voltage unbalance and voltage fluctuations. These problems affect other consumers near by and cause increased losses and poor utilization of the electric distribution system. The severity of reactive power and load unbalancing was recognized long ago and many attempts [1–17] have been made to define, study and derive methods for load balancing and reactive power compensation. Gyugyi *et al.* [1,2,7], Miller [3], Kneschke [4], Sadek [5] and Kern and Schroder [15] have reported various concepts for load balancing by using lossless reactive elements and reactive power compensation, but they used thyristor-controlled inductors and capacitors which introduce harmonics and switching surges. Akagi *et al.* [8] have given a novel concept of instantaneous reactive power compensation without energy storage elements by using mainly new, self-commutated switching devices. However, their attempt was confined to balanced loads. Cox and Mirbod [9] have developed a single-phase static VAR compensator for an arc furnace confined to single-phase only. Lin *et al.* [10] have reported a real-time calculation method for optimal reactive power compensator. Moran *et al.* [11] have given an analysis and design of a three-phase, synchronous, solid-state VAR compensator for a balanced system. The concepts of instantaneous reactive power theories [12,13] were developed by Akagi *et al.* and used by Furuhashi *et al.* for harmonic and reactive power compensation. Kojori *et al.* [14] have developed a large-scale PWM solid-state synchronous condenser for balanced loads. Bhavaraju and Enjeti [16] used negative sequence theory and inductive storage components to balance unbalanced loads. Recently, Dixon *et al.* [17] reported a control system for a three-phase active filter which simultaneously compensates power-factor and unbalanced loads, but it is confined to a three-wire system. However, the majority of unbalanced poor power-factor loads occur in three-phase, four-wire distribution systems. Hence, a need is felt to design and analyse a compensator for load balancing and reactive power compensation for four-wire systems typically found in supply utilities, process industries, traction, furnaces, commercial buildings and captive power generation.

The objective of this article is to compensate reactive power and to balance the load of the three-phase, four-wire distribution system. An IGBT-based PWM voltage source inverter (VSI) with dc bus capacitor is employed as a compensator. Reference currents of the compensator are estimated by using reference source currents and load currents. Three-phase balanced reference source currents are computed using load currents, dc bus voltage and source voltages. A hysteresis-based carrierless PWM current control technique is used over reference currents of the compensator to derive gating signals to its devices. A mathematical model of the whole scheme is developed and simulated to demonstrate reactive power compensation and load balancing of single-phase, two-phase and three-phase loading.

II. System configuration and control scheme

Figure 1 shows the basic circuit of the compensator, including a four-wire, three-phase, unbalanced lagging power-factor load. This load may be single-phase, two-phase, three-phase unbalanced or balanced leading, lagging or unity power-factor loads. An IGBT-based voltage source inverter [13] having an energy storage capacitor on a dc bus is realized as a compensator. It consists of three single-phase bridge VSI with a common dc bus to facilitate independent control of all three phases and a return path through a neutral conductor. A hysteresis-rule-based carrierless PWM current control is employed to give fast response of the compensator. The main function of the compensator is to compensate for reactive power of the load and to balance out the load (if any unbalance exists in the load) equal to all three phases of the ac source, such that the ac supply feeds only unity power-factor balanced currents.

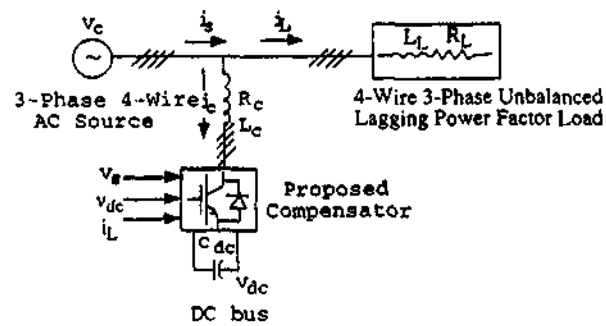

Figure 1. Fundamental building block of the proposed compensator

Figure 2 shows the control scheme of the compensator. An ac source feeds the active power component of load currents and an active power component of a current to maintain the average voltage across the dc bus capacitor at a constant value. The latter component of active power of source current is to supply losses in the VSI such as switching losses, capacitor leakage current, etc. under steady-state conditions and to recover stored energy on the dc bus during transient conditions such as a sudden change in loads, etc. This component of peak source currents (I^*_{smd}) is estimated from the capacitor value, average voltage on the dc bus and desired reference value of the dc bus voltage. The other active power component of peak source currents corresponding to load (I^*_{smp}) is estimated by using sensed load currents and source voltages through an averaged value of instantaneous load power. The total peak of reference source currents (I^*_{sm}) is computed by adding both components (I^*_{smd} and I^*_{smp}) of it. Three-phase instantaneous reference source currents (i^*_{sa}, i^*_{sb} and i^*_{sc}) are estimated by multiplying their peak values (I^*_{sm}) and unit current vectors (u_{sa}, u_{sb} and u_{sc}) in phase with the respective source voltages. Three-phase reference compensator currents (i^*_{ca}, i^*_{cb} and i^*_{cc}) are estimated by subtracting load currents (i_{La}, i_{Lb} and i_{Lc}) from instantaneous source currents (i^*_{sa}, i^*_{sb} and i^*_{sc}). The hysteresis-rule-based carrierless PWM current controller is used over the current errors among three-phase reference currents (i^*_{ca}, i^*_{cb} and i^*_{cc}) and sensed currents (i_{ca}, i_{cb} and i_{cc}) of the compensator to result in gating signals to IGBTs of the compensator. The compensator draws the desired currents from the system, meets the requirement of the reactive power

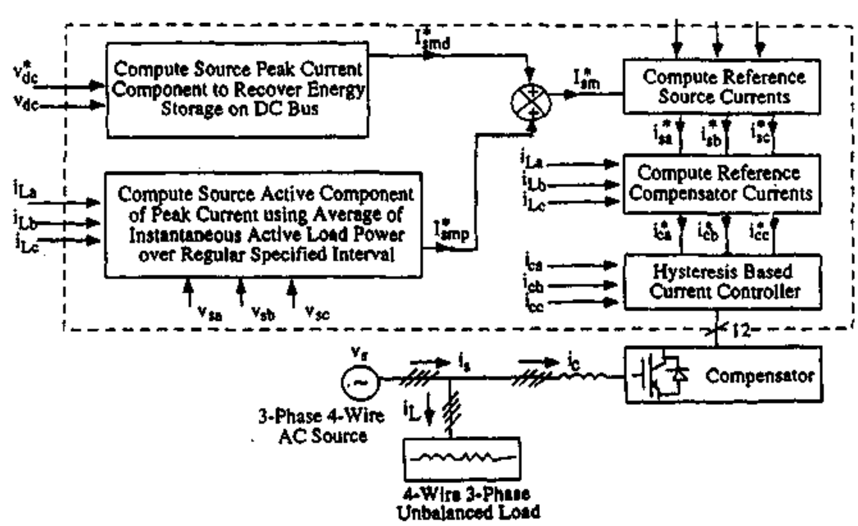

Figure 2. Control scheme of the compensator

of the load and balances the load currents equally to the three phases of the ac source locally, resulting in unity power-factor balanced three-phase source currents in all operating conditions.

III. Mathematical modelling of the compensator system

The complete system consists of ac source, unbalanced load and compensator. Different components of the system are analysed individually and then integrated together to develop a detailed mathematical model to simulate its transient and steady-state behaviour.

III.1 *Control scheme*

The functional details of the control scheme are described in the previous section. Here only mathematical model equations are given. It consists of many segments, so all are modelled in sequence.

III.1.1 *Computation of peak current of the source*

The peak current of the ac source (I^*_{sm}) has two components which are computed as follows. The first active component of peak source current corresponding to load currents is estimated from the instantaneous load power (p_L) as:

$$p_L = v_{sa}i_{La} + v_{sb}i_{Lb} + v_{sc}i_{Lc} = p_s + p_c \quad (1)$$

where i_{La}, i_{Lb} and i_{Lc} are three-phase load currents which may have different values because of unbalanced loads. v_{sa}, v_{sb} and v_{sc} are the three-phase line to neutral ac source voltages and, under ideal conditions, may be expressed as:

$$v_{sa} = V_{sm} \sin \omega t \quad (2)$$

$$v_{sb} = V_{sm} \sin(\omega t - 2\pi/3)$$

$$v_{sc} = V_{sm} \sin(\omega t + 2\pi/3)$$

where V_{sm} is the peak value of source voltage and ω is its frequency in electrical rad s^{-1}.

In equation (1), p_s is the real power of the load to be supplied by the ac source and p_c is the reactive power of the load to be supplied by the compensator. p_s may be estimated by taking the averaged value of p_L over the period of ripple in its (p_L) instantaneous variation, which may be represented as:

$$p_s = (3/2)V_{sm}I_m \cos \phi \quad (3)$$

where I_m is the peak value of source current flowing to the load without compensator and $\cos \phi$ is the equivalent power factor.

From equation (3) the source peak current (I^*_{smp}) corresponding to active power supplied to the load may be computed as:

$$I^*_{smp} = P_s/[(3/2)V_{sm}] = I_m \cos \phi \quad (4)$$

The second component of peak source current (I^*_{smd}) is to feed active power to the compensator for maintaining the average voltage across the dc bus to a constant value. This component of source current feeds switching losses, leakage current, etc. of the dc capacitor and regulates stored energy on the dc bus. It is estimated from the sensed dc voltage, and takes care of all these losses in the practical system. In this model switching losses and leakage current are neglected, so it is derived by balancing the stored energy on the dc bus. If the reference dc bus voltage is v^*_{dc}, then the stored energy corresponding to this reference voltage is:

$$e^*_{dc} = C_{dc}(v^*_{dc})^2/2 \quad (5)$$

However, the average stored energy in the dc capacitor is:

$$e_{dc} = C_{dc}(v^*_{dca})^2/2 \quad (6)$$

where v_{dca} is the average voltage across the dc capacitor over the one periodic variation. Therefore, the energy loss of the dc capacitor over a period is:

$$\Delta e_{dc} = e^*_{dc} - e_{dc} = C_{dc}\left[(v^*_{dc})^2 - (v_{dca})^2\right]/2$$
$$= C_{dc}(v^*_{dc} - v_{dca})(v^*_{dc} + v_{dca})/2$$

Considering the small variation in the average voltage within one period, the term $v^*_{dc} + v_{dca}$ may be approximated to $2v^*_{dc}$. Hence energy losses are approximated to:

$$\Delta e_{dc} = C_{dc}v^*_{dc}(v^*_{dc} - v_{dca}) \quad (7)$$

This energy loss must be supplied by the three-phase ac source to regulate the dc bus voltage. The corresponding peak value of charging/discharging equivalent source current (I^*_{smd}) may be estimated as:

$$\int_0^{T_x} (V_{sm} \sin \omega t)(I^*_{smd} \sin \omega t)\, dt = \Delta e_{dc}/3$$

Hence

$$I^*_{smd} = (2/3)\Delta e_{dc}/(T_x V_{sm})$$
$$= (2/3)C_{dc}v^*_{dc}(v^*_{dc} - v_{dca})/(T_x V_{sm}) \quad (8)$$

where T_x is the period of symmetrical periodic variation of the capacitor dc bus voltage.

The total peak source current from equations (4) and (8) is:

$$I^*_{sm} = I^*_{smp} + I^*_{smd} \quad (9)$$

III.1.2 *Estimation of instantaneous source reference currents*

The unity power-factor, three-phase ac source currents may be computed by using unit current vectors in phase with source voltages and their peak values. The unit current vectors in phase with ac source voltages may be derived from equation (2) as:

$$u_{sa} = V_{sa}/V_{sm}, \quad u_{sb} = V_{sb}/V_{sm}, \quad u_{sc} = V_{sc}/V_{sm} \quad (10)$$

The three-phase reference source currents may be estimated as:

$$i^*_{sa} = I^*_{sm}u_{sa}, \quad i^*_{sb} = I^*_{sm}u_{sb}, \quad i^*_{sc} = I^*_{sm}u_{sc} \quad (11)$$

III.1.3 *Estimation of instantaneous three-phase compensator currents*

The three-phase compensator currents may be estimated from the reference source currents given by equation (11) and sensed load currents as:

$$i^*_{ca} = i^*_{sa} - i_{La}, \quad i^*_{cb} = i^*_{sb} - i_{Lb}, \quad i^*_{cc} = i^*_{sc} - i_{Lc} \quad (12)$$

III.1.4 *Hysteresis-rule-based current controller*

The compensator consists of three single-phase bridge VSI connected to a common dc bus. The current controllers for all three phases are designed to act independently. Each current controller contributes the switching pattern to the compensator devices. The switching logic is formulated as:

If $i_{ca} < (i^*_{ca} - h_b)$, the upper switch is OFF and the lower switch is ON for the phase "a" leg of the "a" phase compensator.
If $i_{ca} > (i^*_{ca} + h_b)$, the upper switch is ON and the lower switch is OFF for the phase "a" leg of the "a" phase VSI.h_b is the width of the hysteresis band around respective reference currents. Moreover, the neutral leg devices are switched in complementary mode with the respective phase leg devices in each individual phase VSI.

The switching of the devices of the other two phases (b and c) are derived similarly.

III.2 *Compensator*

A three-phase ac source with neutral through inductances (L_c, R_c) is the input to the compensator (three single-phase VSI) with a common dc bus having a capacitor (C_{dc}). The compensator is operating in current-controlled mode and modelled by the following state space equation:

$$pi_{ca} = -(R_c/L_c)i_{ca} + (v_{sa} - v_{ca})/L_c \quad (13)$$

$$pi_{cb} = -(R_c/L_c)i_{cb} + (v_{sb} - v_{cb})/L_c \quad (14)$$

$$pi_{cc} = -(R_c/L_c)i_{cc} + (v_{sc} - v_{cc})/L_c \quad (15)$$

$$pv_{dc} = (i_{ca1} + i_{cb1} + i_{cc1})/C_{dc} \quad (16)$$

where p is the differential operator (d/dt). v_{ca}, v_{cb} and v_{cc} are three-phase PWM voltages reflected on the ac input side of the compensator. They may be expressed in terms of instantaneous dc bus voltage (v_{dc}) and switching functions of the compensator as:

$$v_{ca} = v_{dc}(SA1 - SA2) \quad (17)$$

$$v_{cb} = v_{dc}(SB1 - SB2)$$

$$v_{cc} = v_{dc}(SC1 - SC2)$$

where SA1, SA2, SB1, SB2, SC1 and SC2 are the switching functions stating ON/OFF positions of the three single-phase VSIs of the compensator.

Moreover, i_{ca1}, i_{cb1} and i_{cc1} are the respective charging/discharging currents to the dc bus corresponding to three-phase currents of the compensator and, similar to the three-phase PWM ac voltages of equation (17), these currents may also be expressed in terms of switching functions and three-phase compensator currents.

III.3 *Load on the compensator system*

Three single-phase loads (each phase to neutral) lagging power factors are taken which are a very common load on ac systems. These inductive–resistive three-phase loads may be modelled in the form of the following state space equations:

$$pi_{La} = -(R_L/L_L)i_{La} + v_{sa}/L_L \quad (18)$$

$$pi_{Lb} = -(R_L/L_L)i_{Lb} + v_{sb}/L_L \quad (19)$$

$$pi_{Lc} = -(R_L/L_L)i_{Lc} + v_{sc}/L_L \quad (20)$$

These three-phase load parameters (R_L, L_L) are taken to be equal but these may be taken as different in the case of unbalanced loads. In the case of single-phase or two-phase loading on the ac system, only one or two differential equations of equations (18)–(20) are required, the remaining ones being omitted.

The set of first-order differential equations, equations (13)–(16) and equations (18)–(20), along with other expressions, defines the mathematical model of the compensator system. These first-order nonlinear differential equations are solved by using the fourth-order Runge–Kutta method to analyse transient and steady-state behaviour of the compensator system. A generalised software package has been developed for the above algorithm in FORTRAN language and simulated on Pentium PC to study the behaviour of the proposed compensator under varying operating conditions.

IV. Performance of the compensator system

Performance characteristics of the compensator are given in Figures 3–6 to illustrate its steady-state and transient behaviour. The necessary parameters of the system are

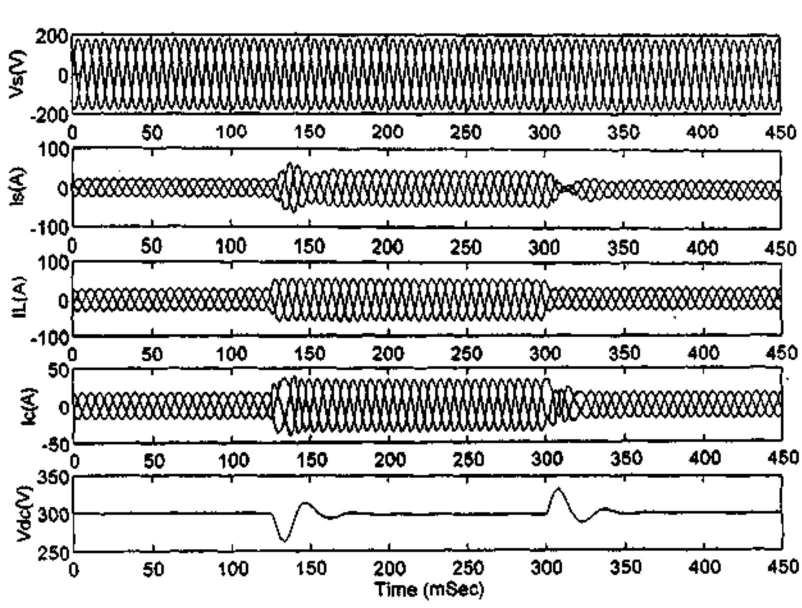

Figure 3. Performance of the compensator system under three-phase lagging power factor (0.8): load change from 6 kW to 12 kW and to 6 kW

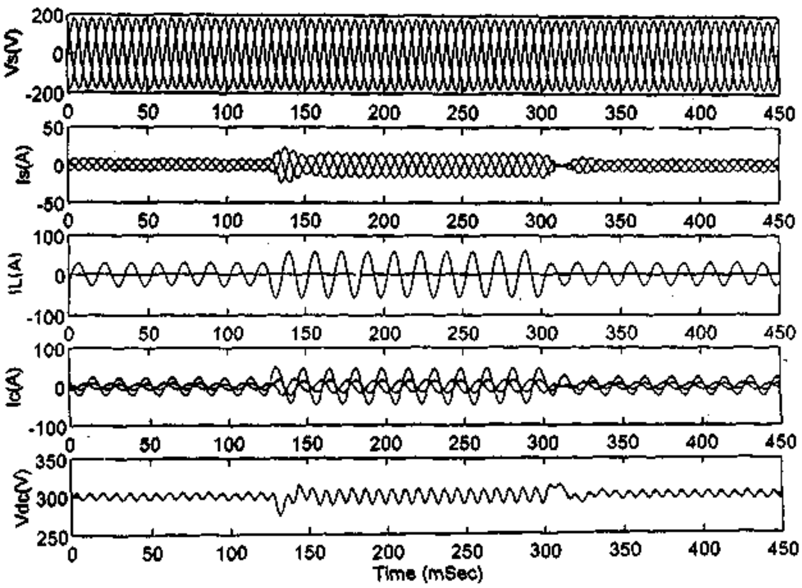

Figure 4. Performance of the compensator system under single-phase lagging power factor (0.8): load change from 2 kW to 4 kW and to 2 kW

given in Appendix A. From these results the following observations are made.

Figure 3 shows three-phase source voltages (v_s), source currents (i_s), load currents, compensator currents and dc bus voltage under three-phase balanced load of lagging power factor (0.8) for load changes from 6 kW to 12 kW to 6 kW. The compensator improves the power factor of the ac source to unity in all load conditions, thus demonstrating the reactive power compensation of varying loads. The source currents respond very quickly and settle to a steady-state value within one cycle. The compensator currents change instantaneously at the load change and exchange the energy quickly from the dc bus, resulting in a dip and rise of dc bus capacitor voltage.

Figure 4 shows similar variation of various quantities as in Figure 3, except under a single-phase load of lagging power factor (0.8) for load change from 2 kW to 4 kW to 2 kW. The compensator balances the single-phase load to a three-phase supply, resulting in three-phase balanced unity power-factor currents of the source and less than one-third in magnitude of the load current. Source currents, compensator currents and dc bus voltage settle to steady-state values within less than a cycle.

Figure 5 shows similar variation of different quantities as Figure 3 under the two-phase load currents drawing from the supply at lagging power factor (0.8) for the load change from 4 kW to 8 kW to 4 kW. The compensator perfectly balances two-phase loads equally to all three phase of the supply, resulting in three-phase balanced unity power-factor source currents and less than two-thirds in magnitude of the load currents. All the of quantities such as source currents, compensator currents and dc bus voltage settle to

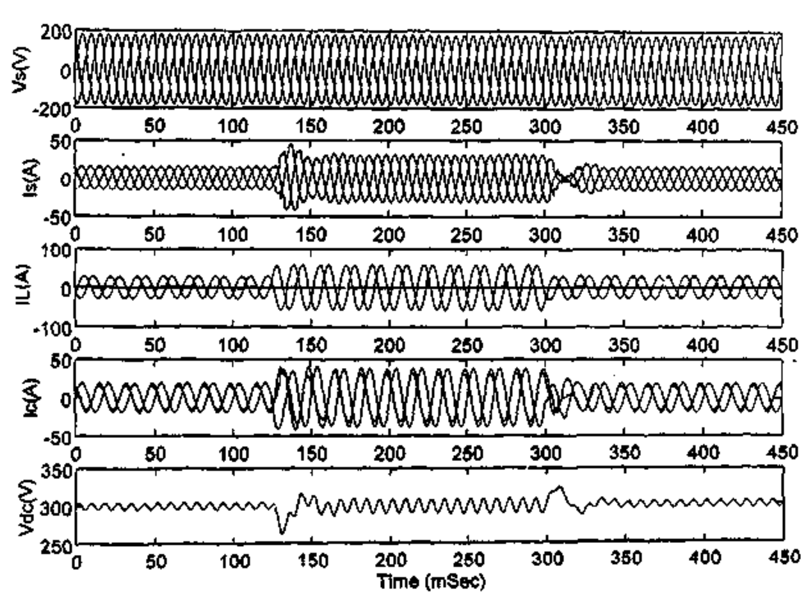

Figure 5. Performance of the compensator system under two-phase lagging power factor (0.8): load change from 4 kW to 8 kW and to 4 kW

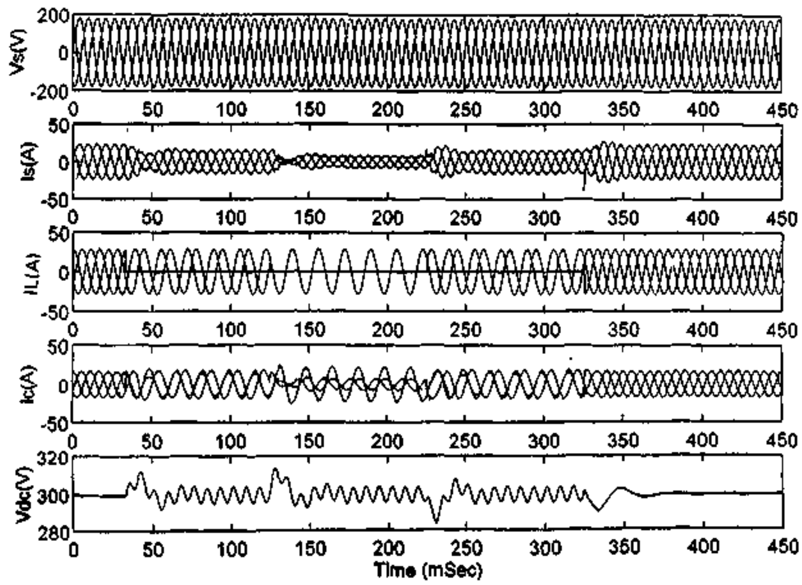

Figure 6. Performance of the compensator system under three-phase to two-phase to single-phase to two-phase and to three-phase load change from 6 kW to 4 kW to 2 kW to 4 kW and to 6 kW

steady-state values within a cycle, demonstrating excellent dynamic performance of the compensator.

Figure 6 shows three-phase source voltages, source currents, load currents, compensator currents and dc bus voltage under the load change from three-phase (6 kW) to two-phase (4 kW) to single-phase (2 kW) to two-phase (4 kW) to three-phase (6 kW), all at 0.8 lagging power factor. The compensator balances unbalanced loads either of single-phase or two-phase and improves the power factor of the ac source to unity under all load conditions. Source currents remain balanced and at unity power factor under balanced and unbalanced load conditions. Source currents, compensator currents and dc bus voltage settle to steady-state values within a cycle for any type of change in the load. The average voltage across dc bus remains constant under steady-state conditions. However, the ripple in dc bus voltage changes with load magnitude under unbalanced loading. There are no ripples in dc voltage at balanced load conditions.

V. Conclusions

The proposed compensator demonstrates the validation of a new approach for reactive power compensation and load balancing and may be used effectively in electric distribution systems. The performance of the compensator is observed to be excellent: it balances unbalanced loads and improves the power factor of the source to unity under transient and steady-state conditions. The proposed compensator enhances the system efficiency because it avoids the reactive power and unbalanced loading of distribution equipment and ac source. This type of compensator can be implemented easily by using IGBT-based VSI and single chip micro controllers; solid-state power devices, namely IGBT, with high switching frequency will result in a low-cost, lightweight and harmonics-free compensator. It is hoped that this compensator will find a number of applications in supply utilities, traction, furnaces, commercial buildings, process industries and captive power generation systems.

VI. References

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Appendix A System parameters

V_s (rms/phase) = 127 V, $F = 60$ Hz, $R_c = 0.1\ \Omega$, $L_c = 3$ mH, $C_{dc} = 3000\ \mu$F, $R_L = 5$ and 2.5 Ω, $L_L = 9.95$ and 4.98 mH.