

Wall effect in laminar flow of non-Newtonian fluid through a packed bed

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Abstract

Experiments were conducted on laminar flow of non-Newtonian fluid through a packed bed of low column to packing particle diameter ratio (3.8) to elucidate the wall effect on pressure drop and mass flux. Carboxy methyl cellulose (CMC) at different concentrations was passed through the packed bed and the pressure drop was measured at different CMC concentrations and flow rates. It was found that the pressure drop increases with the increase in CMC flow rate. The pressure drop also increases with the increase in CMC concentration for a given flow rate. The friction factor is plotted against Reynolds number and the data for different CMC concentration are found to be scattered around a line expressed as $f = 1.03/Re^{0.87}$. The tri-regional model of Cohen and Metzner [1] predicted correctly the mass flux in the packed bed at different pressure drop values and CMC concentrations with parameters K_0 (related to pore geometry) value of 1.5 and L_e/L (related to effective path length) value of 1.2, respectively.

Keywords: Wall effect; Voidage variation; Packed bed

1. Introduction

Wall effect in laminar flow of fluid through a packed bed with low column to packing particle diameter ratio gives rise to high mass flux in the transition and the wall region compare to bulk region. This occurs due to the channeling of liquid over a region of five particle diameter from the wall. The wall effect is more prominent in the case of flow of non-Newtonian fluid through a packed bed [1–3]. Thus wall effect should be considered in design of a packed bed since the rate of heat transfer and mass transfer and residence time distribution may alter. However, the wall effect become negligible for column to particle diameter ratio more than 50 in the case of a power law fluid. The wall effect is prominent in a packed bed with low column to packing particle diameter ratio. According to Cohen and Metzner [1], there are some instances where a packed bed with low column to particle diameter is used. In this study, flow of non-Newtonian fluid through a packed bed containing uniform spherical particle with low column to packing particle diameter ratio is studied.

Many investigators studied the laminar flow on non-Newtonian fluid in a packed bed where capillary tube–bundle theory is used to model the flow. Christopher and Middleman [4], Marshal and Metzner [5] and Gaintonde and

Middleman [6] modified the Blake–Kozeny equation for non-Newtonian and viscoelastic fluid to determine the correlation between friction factor and Reynolds number. Kembrowski and Mertl [7] modified Ergun equation for non-Newtonian fluid and tried to predict pressure drop in transition and turbulent region. Mishra et al. [8] and Brea et al. [9] studied the packed and fluidized bed considering high Reynolds number flow, non-power law pseudo plastic fluid and the effect of non-Newtonian properties. All these studies furnish friction factor f , and Reynolds number Re , relationship as, $f = C/Re$, where C is a constant and it varies between 1 and 1.3. The value of C varied in different studies because Re is defined differently. Kembrowski and Michniewicz [10] tried to resolve the difference in the value of C by defining f and Re in a different fashion by incorporating Rabinowitsch–Mooney correlation factor. While developing these correlations, the wall effect was not considered in the definition of pore geometry and effective path length. Further, these studies were conducted in high column to packing particle diameter ratio bed. The influence of wall effect will be high in low column to packing particle diameter bed. In the present study, a correlation between f and Re is determined following Kembrowski and Michniewicz [10] for laminar flow of non-Newtonian fluid in a packed bed with low column to packing particle diameter ratio. And it is compared with different f and Re relationship available in the literature. Srinivas and Chhabra

Nomenclature

A	cross-sectional area of the packed column (m^2)
A_b	cross-sectional area of the bulk region (m^2)
A_t	cross-sectional area of the transition region (m^2)
A_w	cross-sectional area of the wall region (m^2)
$B(n)$	parameter defined in Eq. (12) (dimensionless)
D_o	packed column diameter (m)
D_p	packing particle diameter (m)
f	friction factor defined in Eq. (4) (dimensionless)
G	mass flux in the packed column ($\text{kg}/(\text{m}^2 \text{ s})$)
G_b	mass flux in the bulk region ($\text{kg}/(\text{m}^2 \text{ s})$)
G_t	mass flux in the transition region ($\text{kg}/(\text{m}^2 \text{ s})$)
G_w	mass flux in the wall region ($\text{kg}/(\text{m}^2 \text{ s})$)
K	consistency index (Pa s^n)
K_0	constant, related to non-circular nature of the pore geometry (dimensionless)
K_1	constant, related to effective path length L_e , (dimensionless)
L	length of the packed region under consideration (m)
L_e	effective path length ($= K_1 L$) (m)
n	power law index (dimensionless)
m	aspect ration ($= D_o/D_p$) (dimensionless)
P	pressure (Pa)
ΔP	pressure drop (Pa)
Re	Reynolds number defined in Eq. (5) (dimensionless)
R_h	hydraulic radius in the wall region defined in Eq. (11) (m)
r_h	hydraulic radius ($= D_p \varepsilon_b / 6(1 - \varepsilon_b)$) (m)
V_e	mean linear velocity (m/s)
V_0	superficial velocity (m/s)
x	non-dimensional distance from the wall w.r.t. D_p
x_t	boundary of the transition region (dimensionless distance)
θ_{fl}	relaxation time of fluid
<i>Greek letters</i>	
ε	local porosity (dimensionless)
ε_b	bulk porosity (dimensionless)
ε_{wav}	average porosity in wall region (dimensionless)
ρ	fluid density (kg/m^3)

[11] attempted to determine the dependence of f and Re on column to packing particle diameter ratio of a packed bed. However, they have not studied for very low value of column to packing particle diameter ratio.

Cohen and Metzner [1] proposed tri-regional model to accommodate wall effect in predicting mass flow rate through a packed bed. Their model is tested for flow of

non-Newtonian fluid in a packed bed with low column to packing particle diameter ratio. The model accommodates porosity variation near the wall of the packed bed. The parameters involved in the model equation, i.e. the constant term related to the non-circular nature of the pore geometry K_0 , and the ratio of the effective path length followed by the fluid to the length of the column L_e/L , were determined. These two constant are lumped together and it is expressed as, $B(n) = K_0(L_e/L)^{n+1/n}$, where n is the flow index property of the non-Newtonian fluid. According to Cohen and Metzner [1], it is important to determine the value of $B(n)$ for non-Newtonian fluid and in the presence of wall effect. In this study, the value of $B(n)$ is determined for low value of D_o/D_p ratio, where D_o is the column diameter and D_p is the packing particle diameter.

2. Experimental

2.1. Material

Carboxy methyl cellulose (CMC) solution in distilled water was used as non-Newtonian fluid. Uniform sized spherical particle of 0.0135 m diameter was used as packing material. The particles were made of ceramic material. The CMC solution with appropriate concentration was used as the nomenclature fluid. The rheological equation for non-Newtonian purely viscous fluid (Sabiri and Comiti [12]) is given by, $T = K_i \gamma_i^{n_i}$, where T is the shear stress value for the corresponding shear rate γ , and K_i and n_i are the consistency and the behaviour index, respectively. The rheological properties of CMC solution at different concentrations are given in Table 1 [13].

2.2. Experimental setup

The experimental setup used is shown in Fig. 1. A glass column (A) of inner diameter 0.05 m and of length 1 m packed randomly with uniform sized spherical ceramic ball of diameter 0.0135 m was used. The packed bed is connected to a storage tank (E) through a pipe of 0.0127 m diameter. The pipe is connected to the packed bed through a nozzle just above the packing support. The nozzle is connected to a distributor in side the packed bed. The bottom of the packed bed is connected to a pipe (G) to drain liquid from the packed bed after the experiment is over. A centrifugal pump (C) and globe valve were connected to the line as shown in Fig. 1. An orifice meter (B) is also connected in the line to measure volumetric flow rate. There is a bypassing arrangement after the pump, which returns back to storage tank. The top of the packed bed is open to atmosphere and attached to a wide diameter funnel shape structure from where an outlet pipe (F) is connected to the storage. A U-tube manometer (D) is connected to column just below the packing support by inserting 2 mm glass tube through a rubber cork. A flexible rubber tube joins U-tube manometer and the glass tube.

Table 1
Physical properties (298 K) of carboxy methyl cellulose (CMC) solution

Carboxy methyl cellulose concentrations (wt.%)	Density (kg/m ³)	Flow index property (<i>n</i>)	Consistency (<i>K</i> , Pa s ^{<i>n</i>})
0.3	1005.0	0.9787	0.0614
0.5	1007.0	0.914	0.1358
0.7	1007.2	0.835	0.26

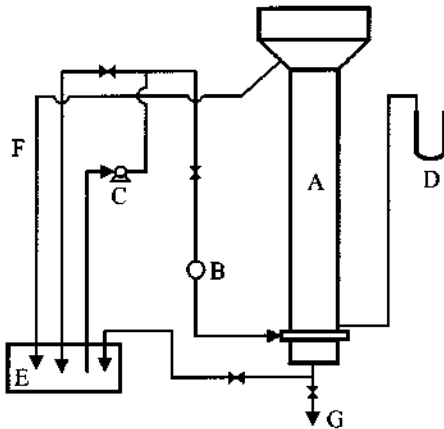


Fig. 1. A schematic diagram of packed bed ($D_c = 0.05$ m, $D_p = 0.0135$ m, $\epsilon_b = 0.41$) apparatus.

The manometer is attached to the structure of the column such that it can be moved upward and downward according to the range of pressure drop to be measured in the packed bed. The other end of the manometer is opened to the atmosphere since the top end of the packed bed is open to the atmosphere.

2.3. Experimental method

CMC of different concentrations by weight (0.3, 0.5 and 0.7 wt.%) was prepared in distilled water. The storage vessel was filled with CMC solution. The CMC solution is fed into the packed bed by using the centrifugal pump. The volumetric flow rate was measured by the calibrated orifice meter present in the pipe line. The CMC solution moved up through the packed bed and the overflowed solution returned to the storage vessel through a pipe. The U-tube manometer connected to the packed bed was used to measure the pressure drop across the bed at different flow rates and CMC concentrations. The manometer was shifted upward or downward accordingly to measure different ranges of pressure drop.

3. Model

3.1. Capillary bundle theory

Christopher and Middleman [4] used the Hagen–Poiseuille equation as the starting point to derive the Blake–Zozeny equation for power law fluid. However, Kembrowski and Michniewicz [10] used more general form of the equation

for flow of power law fluid through arbitrary cross-section. They incorporated Rabinowitsch–Mooney correlation factor, $4n/(3n + 1)$ in the expression for mean linear velocity. The mean linear velocity v_e , is given by

$$V_e = \frac{4n}{3n + 1} \frac{r_h}{K_0} \left(\frac{\Delta P r_h}{KL_e} \right)^{1/n} \quad (1)$$

where n and K are power law index and consistency, respectively, ΔP is the pressure drop. Introducing, the definition of hydraulic radius, r_h ($r_h = D_p \epsilon_b / 6(1 - \epsilon_b)$, where ϵ_b is the bed porosity), effective path length, L_e ($L_e = K_1 L$, where K_1 is a constant), and mean linear velocity, v_e ($v_e = v_0 L_e / (\epsilon_b L)$, where v_0 is the superficial velocity) in Eq. (1), the superficial velocity is given by

$$V_0 = \frac{4n}{3n + 1} \frac{D_p \epsilon_b^2}{6(1 - \epsilon_b) K_0 (L_e/L)} \times \left[\frac{\Delta P D_p \epsilon_b}{6(1 - \epsilon_b) K (L_e/L) L} \right]^{1/n} \quad (2)$$

The Eq. (2) is rearranged and it is presented as follows:

$$f = \frac{1}{Re} \quad (3)$$

where friction factor is defined as

$$f = \frac{\Delta P D_p \epsilon_b^3}{L \rho V_0^2 (1 - \epsilon_b)} \quad (4)$$

and Reynolds number is defined as

$$Re = \frac{D_p^n V_0^{2-n} \rho}{K (1 - \epsilon_b)^n} \left(\frac{4n}{3n + 1} \right)^n \frac{\epsilon_b^{2n-2}}{K_0^{n+1} (L_e/L)^{n+1}} \quad (5)$$

The value of K_0 and L_e/L is determined from the experimental results of the packed bed. It is possible to guess L_e/L value by visualizing flow behaviour in the packed bed and K_0 value from the exact shape of the pore geometry. Most of the investigators [7] used K_0 and L_e/L value of 2.5 and $\sqrt{2}$, respectively. It is to be noted that the Eq. (5) does not yield the definition of Re for Newtonian fluid at $n = 1$.

3.2. Tri-regional model

Cohen and Metzner [1] considered wall effect in their tri-regional model to accommodate the discrepancy in theoretical prediction and experimental data for mass flux observed in packed bed. The wall effect in packed bed is

observed because of porosity variation near the wall. The porosity variation is mainly due to the phase shift in the porosity oscillation, whose magnitude is at the most 0.2 particle diameter. The porosity variation near the wall in a randomly packed column is described by curve fitting the data available in Roblee et al. [14], Ridgway and Tarback [15], Hanghey and Beveridge [16] and Zou and You [17]. They found that the porosity ε , varies for randomly packed column from the wall to packing particle diameter up to 10. The variation in porosity is described by

$$\frac{1 - \varepsilon}{1 - \varepsilon_b} = 4.5 \left(x - \frac{7}{9}x^2 \right), \quad x \leq 0.25 \quad (6)$$

$$\frac{\varepsilon - \varepsilon_b}{1 - \varepsilon_b} = 0.3463e^{-0.4273x} \cos(2.4509x - 2.2011)\pi \quad (7)$$

where $0.5 < x < 8$

$$\varepsilon = \varepsilon_b, \quad 8 \leq x \leq \infty$$

where x is the distance from the wall, non-dimensionalized with respect to the particle diameter. The above equations are valid for uniform sized spherical particle.

Cohen and Metzner [1] divided the cross-section of the packed bed into three regions to consider the porosity variation. The region extended from wall to one particle diameter is the wall region and from one particle diameter to five particle diameter region is the transition region. Beyond $x > 5$, is the bulk region where the porosity remains constant and it is equal to bed porosity ε_b , determined from the experimental measurement. The voidage variation between five and eight particles diameter is less than 5% and it can be neglected. Thus a packed bed with $D_c/D_p \leq 5$ only wall and transition region exists.

The porosity in the bulk region is constant and the expression for mass flux through the bulk region G_b is given by

$$G_b = \rho \left[\frac{\Delta}{2KL} \right]^{1/n} \left[\frac{4n}{3n+1} \right] \left[\frac{D_p}{3} \right]^{n+1/n} \left[\frac{\varepsilon_b}{1 - \varepsilon_b} \right]^{n+1/n} \times \left[\frac{\varepsilon_b}{2B(n)} \right] \quad (8)$$

In the transition region, porosity varies with x and the last two terms in Eq. (8) is a function of area of the transition region A_t . The mass flux in the transition region G_t is expressed as

$$G_t = \rho \left[\frac{\Delta P}{2KL} \right]^{1/n} \left[\frac{4n}{3n+1} \right] \left[\frac{D_p}{3} \right]^{n+1/n} \times \left\{ \frac{1}{A_t} \oint_{A_t} \left(\frac{\varepsilon}{1 - \varepsilon} \right)^{n+1/n} \frac{\varepsilon dA}{2B(n)} \right\} \quad (9)$$

where local porosity ε is given by Eq. (7). In the wall region, the mass flux G_w is given by

$$G_w = \rho \left[\frac{\Delta P}{2KL_e} \right] \left[\frac{4n}{3n+1} \right] \frac{(2R_h)^{n+1/n}}{2B(n)} \frac{1}{A_w} \oint_{A_w} \varepsilon dA \quad (10)$$

Here ε is calculated using Eqs. (6) and (7) and A_w is the area of the wall region. R_h , is the hydraulic radius in the wall region and it is expressed as

$$R_h = \frac{D_p(m - x_t)x_t\varepsilon_{wav}}{m + 6x_t(m - x_t)(1 - \varepsilon_{wav})} \quad (11)$$

where $m = D_c/D_p$ and $\varepsilon_{wav} = \oint_{A_w} \varepsilon dA$. The total mass flux in the packed bed is given by $\dot{G} = G_b + G_t + G_w$. The detail derivation of Eqs. (8)–(11) is worked out by Cohen and Metzner [1]. The $B(n)$ in Eqs. (8)–(10) is a constant, where two unknown parameters are lumped. $B(n)$ is expressed as

$$B(n) = K_0 \left(\frac{L_e}{L} \right)^{n+1/n} \quad (12)$$

where K_0 accounts for the inadequacy in the choice of proper effective diameter of the pores inside the packed bed. The L_e is effective path length traveled by a fluid element to move in the axial direction of the packed bed of the length L . These two parameters K_0 and L_e/L are determined from the experimental data. $B(n)$ is a function of power law index, n for non-Newtonian fluid. The porosity variation near the wall will bring a change in $B(n)$ value for low column to packing particle diameter ratio bed.

4. Results and discussion

4.1. Pressure drop

Fig. 2 shows the pressure drop ($dP/d\lambda$) against liquid flow rate for three different CMC concentrations. It is seen that the pressure drop increases with the increase in liquid flow rate. Further, the pressure drop increases with the increase in CMC concentration for a given flow rate. The increase of CMC concentration leads to the increase in consistency K , and thus the Reynolds number decreases (Eq. (5)). The friction factor increases with the decrease in Reynolds number and therefore pressure drop increases (Eq. (4)) in the packed bed. A similar behaviour was observed by the previous investigators [5,9,12,18].

4.2. f versus Re

In Fig. 3, friction factor f (Eq. (4)), is plotted against Reynolds number Re (Eq. (5)), for flow of non-Newtonian fluid in a packed bed of D_c/D_p value of 3.8. It is seen from the figure that the friction factor–Reynolds number plot for 0.3, 0.5 and 0.7 wt.% CMC concentrations coincides on a line. The data are shown by the symbols for different CMC concentrations and line is fitted to the data points. The equation for the line is given by $f = 1.03/Re^{0.87}$. K_0 and K_1 used in the correlation are 2.5 and $\sqrt{2}$, respectively which is given in Kemblowski and Michniewicz [10]. It is seen in Fig. 4 that data do not coincide with $f = 1/Re$ line. A different f – Re correlation is determined may be due to the

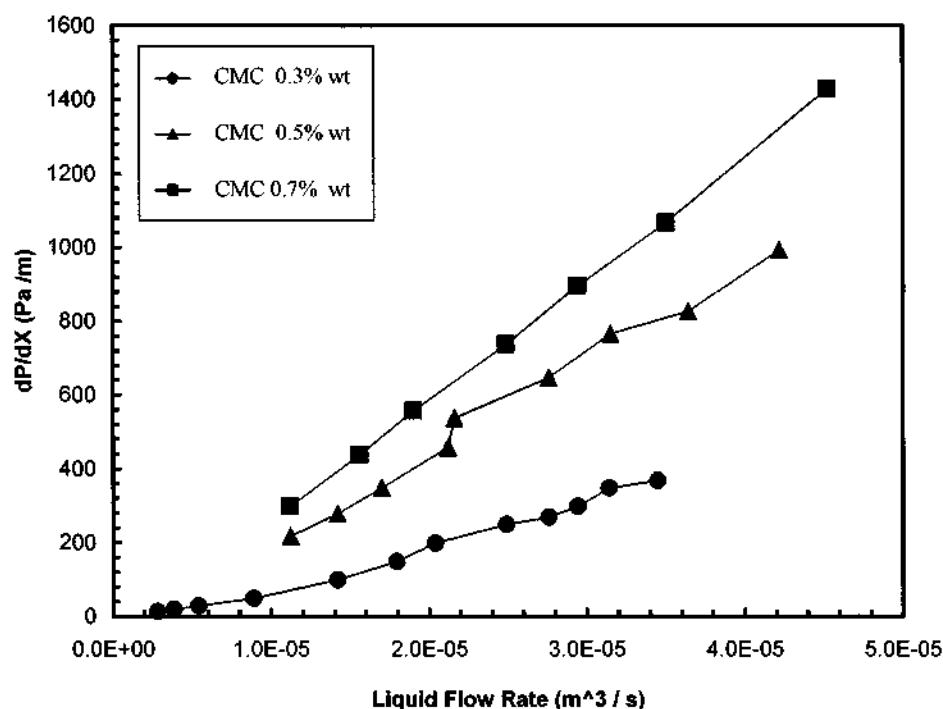


Fig. 2. Plot for pressure drop vs. liquid (CMC) flow rate at different CMC concentrations.

low column to packing particle diameter ratio used in the present study. It is to be noted that the variation in porosity near the wall is not considered in the definition of f and Re . In other words, the f and Re is not a function of D_c/D_p . The porosity variation near the wall is considered in tri-regional

model, which is discussed later on. The viscoelastic effect is checked by estimating the Deborah number (N_{Deb}) for the present system. Marshall and Metzner [5] pointed out that the viscoelastic effect is not important for very low value of N_{Deb} ($\ll 0.1$). The Deborah number is defined as, $N_{Deb} =$

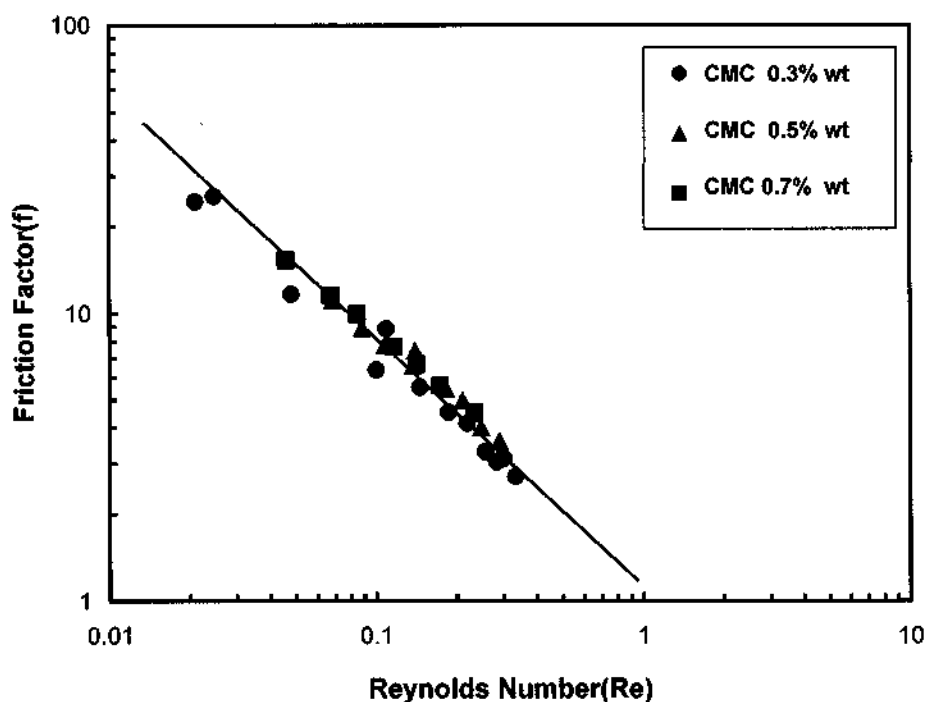


Fig. 3. Friction factor and Reynolds number plot at different CMC concentrations.

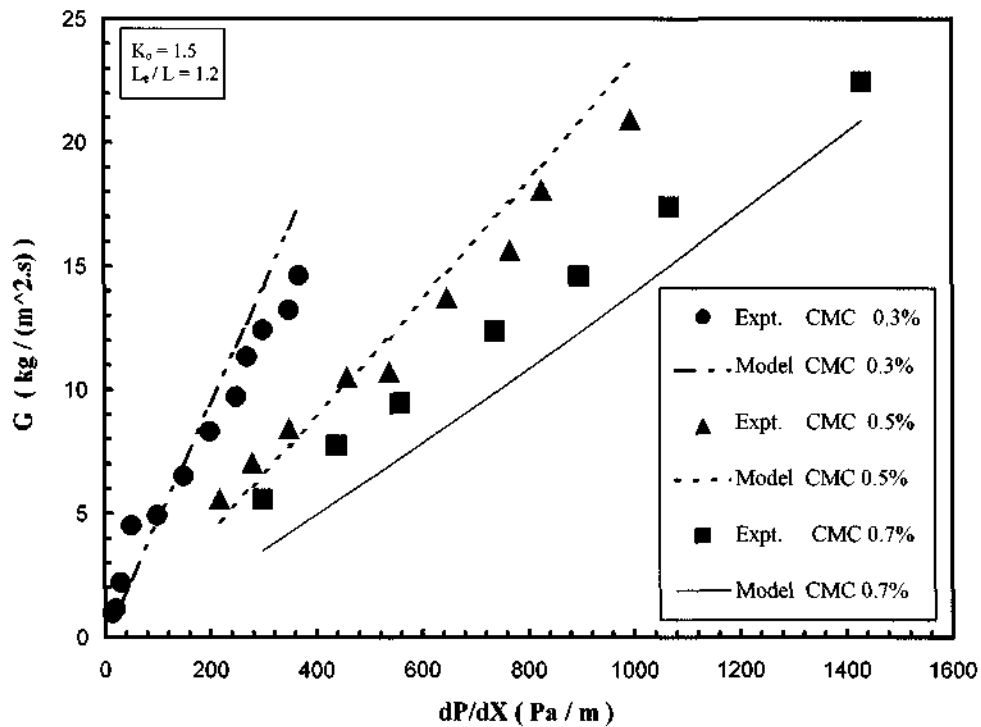


Fig. 4. Tri-regional model prediction for mass flux in the packed bed at different CMC concentrations.

$\theta_{fl} V_e/D_p$, where θ_{fl} is the relaxation time of the fluid. In the present study the flow of CMC is considered to be in the laminar region. In the laminar region of non-Newtonian fluid flow, the N_{Deb} is much smaller than 0.1 since V_e/D_p is of the order of 10^{-2} and the θ_{fl} is of the order of milliseconds.

4.3. Tri-regional model

The tri-regional model of Cohen and Metzner [1] was tested for the present experimental data. Since the $D_c/D_p = 3.8$, the packed bed is divided into wall and transition regions. The total mass flux in the bed is given by the sum of mass flux in the transition region Eq. (9) and the mass flux in the wall region Eq. (10). In Fig. 4, the mass flux is plotted against pressure drop for three different CMC concentrations. The symbols are experimental data whereas lines are the model predictions of Cohen and Metzner [1]. The unknown parameter B ($B = K_0[L_e/L]^{n+1/n}$), is fitted such that the model predicts the experimental data well.

The K_0 value of 1.5 is used in the present study. The K_0 value of 2 is used for pore geometry of circular in shape. The K_0 value of 2.5 is used for spherical packing particle in the packed bed with high column to particle diameter ratio (Cohen and Metzner [1]). In the present case, the K_0 value is small compare to high column to packing particle diameter ratio bed. It is to be noted that the shape of the pore near the wall is different from that in the bulk. In low column to packing particle diameter ratio bed, the walls are couple of particles away from each other and thus the pore geometry for the entire bed is affected by the presence of the wall.

Whereas, in high column to packing particle diameter ratio bed, the influence of wall diminishes within five particle diameter. Thus the K_0 value is different from that used by previous investigators for high column to packing particle diameter ratio bed.

Carman [19] conducted flow visualization experiment in a packed bed with $D_c/D_p = 4.0$. He observed that the average fluid path line is approximately 45° with the axis of the column. Based on this observation and on geometric consideration, L_e/L is equal to $\sqrt{2}$. This value slightly higher than (1.2) used in this work to predict the experimental data. L_e/L value of 1.2 corresponds to 55° angle between fluid path line and column axis. It is seen in Fig. 4 that the experimental data is under predicted by the model for 0.7 wt.% of CMC. This may be because of channeling occurring at high CMC concentrations and pressure drop.

5. Conclusion

Experiments were conducted on flow of non-Newtonian fluid through packed bed with low column to packing particle diameter ratio to study the wall effect. CMC at different concentrations was used as non-Newtonian fluid. The column to packing (spherical ball) particle diameter ratio used is 3.8. CMC is passed through the packed bed at different flow rates and concentrations and the pressure drop was measured. It was observed that the pressure drop increases with the increase in CMC flow rate. Further, the pressure drop increases with the increase in CMC concentrations for

a given flow rate. The friction factor–Reynolds number plot at different CMC concentrations coincide on a line and the correlation is given by, $f = 1.03/Re^{0.87}$. The tri-regional model of Cohen and Metzner [1] predicted the total mass flux in the packed bed at different pressure drop values with K_0 value of 1.5 and L_e/L value of 1.2.

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