Display of tilt information of vibrating object in time average mode using lateral shearing interferometry and interferometric grating

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Abstract

We have experimentally demonstrated vibration analysis of a reflecting object in time-average mode using shearing interferometry and interferometric grating. Experimental results show that time-average moiré fringes, formed between fringe pattern reflected from object and sinusoidal grating are modulated spatially by the amplitude of vibrating tilt. From the experimental results, information regarding tilt of vibrating objects can be determined.

Keywords: Vibration analysis; Shearing interferometry; Time-average mode

1. Introduction

Optical methods have advantage of being fast in response, non-contacting and full field. They are also very useful in mechanical engineering applications, especially when boundary conditions and material properties are complicated or even undetermined.

A number of optical methods have been investigated for vibration analysis. Prominent among them are classical interferometric techniques [1–5], Moiré techniques [6–8], holographic [9–13] and speckle [14–18]-based techniques, Talbot interferometric techniques [19, 20] and shearing interferometry [21]. Most of these techniques provide modal patterns of a vibrating object. In the present communication, we demonstrate the application of lateral shearing interferometry to measure the amplitude of vibrating tilt.

1.1. Theory

Munavadi [22] provided an overview of analytical description of lateral shearing interferometers. If a collimated beam of light with wavefront aberration \( \hat{W}(x, y) \) is assumed as an input to the interferometer, the shearing device divides the incoming beam into two outgoing beams with a lateral displacement \( S \), in the direction of shear. The interferogram indicates the resulting path difference \( \Delta W \). If \( S \) is assumed to be in the \( x \)-direction:

\[
\Delta W = [W(x, y) - W(x - S, y)].
\]

Usually \( S \) is small, and we can use the approximation \( \Delta W = (\partial W/\partial x)S \). This means that we measure the partial derivative of the wavefront in the direction of the shear. If the wavefront error due to defocusing of collimating lens is present, one observes straight-line fringes that are equally spaced and perpendicular to the direction of shear.

Schematic of the experimental set-up for vibration analysis using shearing interferometry is shown in Fig. 1. Straight-line fringes due to defocusing of the collimating lens are observed after the parallel plate. The fringe pattern from the shear plate can be treated as the grating like periodic structure; at the plane \( [g(x = x', z = 0)] \), it can be represented by Fourier series as:

\[
g(x') = \sum n C_n \exp \left( \frac{2\pi inx'}{d_1} \right),
\]

where \( d_1 = \) pitch of the fringe pattern.

The complex amplitude falling upon the grating \( G \) at a distance \( z \) along the optic axis is \( g_1(x, z) \), where \( g_1(x, z) \) is
Fig. 1. Schematic of experimental set-up for display of tilt information of vibrating object in time-average mode using lateral shearing interferometry and interferometric grating.

approximately the Fresnel transform of \( g(x') \)

\[
g_1(x, z) = \int g(x') \exp \left( \frac{i\pi(x - x')^2}{d_1} \right) \, dx'.
\] (1)

\[
= \sum C_n \exp \left( -\frac{i2\pi nx}{d_1} \right) \exp \left[ -\frac{i\pi L x^2}{d_1^2} \right].
\] (2)

The fringe pattern from the shear plate is incident on the object (reflecting glass plate). The local tilt of the object may be approximated by

\[
\phi(x, y, t) = \frac{\partial W(x, y, t)}{\partial x},
\] (3)

\[
= \left[ \frac{\partial W_0(x, y)}{\partial x} \right] \sin(\omega t + \phi_0).
\] (4)

\( W(x, y, t) = W_0(x, y) \sin(\omega t + \phi_0) \) is the deflection of the oscillating plate. The amplitude \( W_0(x, y) \) of the oscillation represent the maximum deflection of the plate, and \( \phi_0 \) the arbitrary initial phase. Depending on the tilt induced in the object because of the driving force, there is a shift in the fringe pattern. This shift can be mathematically represented in terms of grating function as; \( g_1(x) = g_1(x + \Delta x) \). The lateral shift \( \Delta x \) of the fringe pattern is proportional to the tilt of the plate

\[
\Delta x(x, y, t) = 3K \phi(x, y, t),
\] (5)

where \( K \) is the proportionality constant which depends on the separation between reflecting object and the grating G.

The fringe spacing \( (d_2) \) of the pattern reflected from the object is the same as that of the pitch of the grating G. The Moiré pattern is formed between the fringe pattern reflected from the object and the grating G, which is tilted about an axis perpendicular to the optic axis by an angle \( \theta \), so that the effective pitch \( (d_2) \) of grating G now varies and is given by the relation, \( d_2 = d_1 \cos \theta \). If \( g_2(x) \) is the amplitude transmittance of the sinusoidal grating, the light amplitude behind the sinusoidal grating can be represented as

\[
u(x) = g_1(x) g_2(x). \quad (6)
\]

Spatial filtering arrangement consisting of lenses L1, L2 and the spatial filter S helps to increase the visibility of the fringe pattern as it allows only the zeroth-order diffraction order or one of the first orders to pass. Zero-order filtering produces as an image the second-order derivative of the object, and first-order filtering the first-order derivative [23],

\[
\phi_0(x, y) = \frac{\partial^2 u(x, y)}{\partial x^2}, \quad (7)
\]

\[
\phi_1(x, y) = \frac{\partial u(x, y)}{\partial x}. \quad (8)
\]

If the filtering is performed in the first order and the recording time \( T \) is long compared to the vibration period one obtains time average intensity

\[
I_1(x, y) = \frac{1}{T} \int_0^T (u(x, y, t))^2 \, dt \quad (9)
\]

\[
= \frac{2}{J_0} \left( 1 + \cos \left[ \frac{2\pi}{J_0} \left( \frac{1}{d_1} - \frac{1}{d_2} \right) x \right] \right).
\] (10)

Here \( J_0 \) is the zeroth-order Bessel function of first kind. Eq (10) indicates that the time-averaged intensity is essentially the Moiré pattern of two gratings modulated by Bessel function \( J_0 \). The argument of the Bessel function is essentially \( (\partial W_0(x, y))/\partial x \), which is the maximum tilt of the vibrating plate at point \( (x, y) \).

2. Experimental

Schematic of experimental arrangement is shown in Fig. 1. The beam from a 15 mW He-Ne laser is expanded using a 40x microscope objective and pinhole of 5 μm diameter. The diverging beam is then collimated using a collimator of focal length 250 mm and diameter 50 mm, mounted on a precision translation stage. The collimated beam falls on the shear plate, which laterally shears the incident beam. If the collimating lens is then defocused, a typical straight-line fringe pattern results. This fringe pattern is recorded on a Slavich PFG-01 emulsion keeping the recording plate at G. PFG-01 plates were developed and fixed as per standard procedure given in the literature supplied by M/s Slavich International, Uab Geona, Lithuania. The pattern recorded
on the plate acts as a sinusoidal grating G. The pitch of the recorded pattern as measured under high-power microscope was 0.4 mm approximately.

The straight-line fringe pattern generated by shear plate is incident on reflecting glass plate (dim. 50 mm x 20 mm x 0.1 mm). The glass plate was excited into vibration at one end by a loudspeaker, which is driven by a frequency generator HP33120-A. On the other end, the glass plate was kept fixed to one corner of the loudspeaker. The light beam incident on the loudspeaker is reflected back by the glass plate and falls on the grating G, after being transmitted through the shear plate. The position of grating G with respect to collimating plate was optimized so as to achieve maximum contrast of moiré fringes. The position of the collimating lens is adjusted, so that the resulting fringe pattern has the same spacing as that of the grating G. The Moiré pattern was obtained by tilting the grating G by 23° approximately, about an axis perpendicular to the optic axis. The Moiré fringes inclined as a slight curvature is introduced in the glass plate while pasting it on the loudspeaker. However it will not effect the experimental results. The spatial frequency filtering arrangement is adjusted to filter out the desired order.

3. Results

Experimental results are shown in Figs. 2(a)–(d). Fig. 2(a) corresponds to the background Moiré pattern when no signal was applied to the loudspeaker. The contrast of the moiré pattern is homogenous over the observation plane, as long as the object does not vibrate. Figs. 2(b)–(d) are the photographs of experimental results when the loudspeaker is driven by a sinusoidal signal of amplitude 1 V and frequencies of 150, 300 and 580 Hz, respectively. Figs. 2(b)–(d) show the time-average intensity which is essentially the moiré pattern of the reflected fringe pattern from the vibrating object and grating G modulated by the zeroth-order Bessel function of first kind. The argument of Bessel function \( \frac{\partial W(x, y)}{\partial x} \), is the maximum tilt of the vibrating plate at point \((x, y)\). The negative parts of
Bessel function are indicated as a contrast reversal of moiré fringes.

4. Conclusions

We have experimentally demonstrated vibration analysis of a reflecting object in time-average mode using shearing interferometry and sinusoidal grating. In our experiment the reflecting object was vibrating and the tilt therefore varied as a function of time. The time-average moiré fringes are modulated spatially by the amplitude of vibrating tilt oscillations. From the experimental results, the information regarding tilt of vibrating glass plate can be deduced.

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