Performance of friction materials based on variation in nature of organic fibres
Part II. Optimisation by balancing and ranking using multiple criteria decision model (MCDM)

B.K. Satapathy, J. Bijwe*

Industrial Tribology, Machine Dynamics and Maintenance Engineering Centre (ITMMEC), Indian Institute of Technology, Hauz Khas, Delhi 110016, India

Received 25 June 2003; received in revised form 10 February 2004; accepted 22 March 2004

Abstract

Multiple criteria decision model (MCDM) is used for optimisation of several conflicting criteria dependent systems. A multiple criteria decision model (MCDM) approach taking into account the performance defining attributes (PDAs) such as fi-fade, fi-recovery, performance-fi, wear and temperature rise in the rotor disc, was adopted to determine the performance ranking of five non-asbestos fibre reinforced organic friction materials. It is a three-stepped procedure to derive an overall complete final order of the options. The out-ranking matrix is derived, indicating the frequency of the relative superiority of options with respect to each other based on each criterion. The out-ranking matrix is triangularised to obtain an implicit ordering or provisional order of options, based on sequential application of a balancing principle supported by the pair wise comparison of the options with the help of advantages-disadvantages table. The method has been used to rank a series of friction materials (FMs) based on the combinatorial variation of the fibres, in particular, the organic fibres. The carbon fibre based composite (C) was found to be functioning optimally for its practical selection and implementation in similar evaluating conditions. Cellulose fibre based composite (S) was found to be the poorest in this regard.

Keywords: Friction material; Optimisation; Performance ranking; MCDM

1. Introduction

The performance of friction materials is controlled by the selection of the constituents, their relative volume fractions, shape, orientation and distribution. The property defining attributes (PDAs) are predicted on the basis of micro-mechanics analysis by explicitly taking into account of the design variables. By identifying an objective function, which is the weighted sum of PDAs, the overall performance in an n-dimensional performance space can be traced [1]. Composite material selections have been reported to be a combinatorial optimisation problem, which could be addressed by simulated annealing technique [2]. Performance assessment and evaluation of mechanical and tribological systems using ranking techniques is a widely accepted practice among the decision-makers and design optimisation engineers. Several methods have been reported in this regard. Among them the wear-digraph, Hasse diagram, MADM and various other operation research techniques have been reported to be efficient [3-7]. MCDM refers to a model enabling decision making in the presence of multiple and usually conflicting criteria. MCDM problems are commonly categorised as continuous or discrete, depending on the domain of the alternatives. The conceptual superiority of MCDM over single criteria methods for the attainment of an optional objective, rests on the facilitation of collaborative decision making, performance-benefit analysis and the validity of performance defining attributes which are mostly conflicting in nature. Further they provide more realistic empirical results without the weight assignment and by taking into account the realistic values. A common MCDM procedure is to incorporate criteria weights into a simple additive weighted function [5]. This type of utility function is popular because of its simplicity and its relevance to real world problems [8,9].

The objective of comparison of advantages and disadvantages of friction materials, which needs the actual distribution of stress, strain, temperature and sensitivity to other operating variables within the brake pad, is a complex task. Hence performance assessment and optimisation there of from a set of available options is a conflicting decision making task. In this scenario the present paper focuses on the use of multiple criteria optimisation by ranking and balancing method for the selection and design of an optimal composite, for friction applications. The method adopted in the present paper overcomes some of the deficiencies of other MCDM methods, such as subjective evaluation of criteria, scoring of options, statistical estimation of weights and specification of utility function for criteria [5,10].

2. The MCDM approach

2.1. Data table and out-ranking matrix

A detailed description of the five friction material systems (FMs) and the five criteria (Ci; i = 1, 2 … , 5) are given in Tables 1 and 2, respectively. Table 3 elaborates the criteria values for the five friction material systems (FMs) evaluated and discussed in Part I of this paper [11].

The transitive overall final order of a finite set of options is derived on the basis of a stepwise ordering procedure following the balancing and ranking as the variants of MCDM. The model heeds to the assumptions of pair wise comparison of options among all the selected friction materials and the principle of balancing [10]. The method overcomes the conventional MCDM approaches based on the empirical relative weight assignments. The MCDM problem is solved on the basis of a three-stepped method called the balancing and ranking method. The ranking of the friction materials based on the scores of performance defining attributes (PDAs) such as fade, recovery, friction coefficient, temperature rise and wear are taken for the definition of the superiority sequence of the options, on the basis of the data table as given in Table 3. Based on Table 3 and taking into account the pair wise comparison an advantages-disadvantages table is constructed (Table 4). The data table is used to derive an out-ranking matrix (Table 5). The latter indicates the frequency with which an option is superior to other options based on each criterion. Secondly the triangularisation of the out-ranking matrix is carried out to obtain an implicit pre-ordering of options, which is essentially a provisional ordering [12,13]. Thirdly the provisional ordering is further evaluated using screening and balancing operations based on an advantages-disadvantages table (Table 4), where the

Table 1
Description of the five friction materials

Friction material system	Specification based on net composition (wt.%)
FM1 (NL)	28% $BaSO_4$ + 0% organic fibre
FM2 (AL)	25% $BaSO_4$ + 3% aramid fibre
FM3 (PL)	25% $BaSO_4$ + 3% PAN fibre
FM4 (SL)	25% $BaSO_4$ + 3% cellulose fibre
FM5 (CL)	25% $BaSO_4$ + 3% carbon fibre

Table 2
Description of the criteria for evaluating friction materials (FM)

Criterion	Description of criterion
Performance friction fade (PFF)	It is calculated on the basis of the difference between the performance friction coefficient and the friction coefficient at the maximum disc temperature rise for every fade cycle runs which is normalised against the number of fade cycle runs (5 in this case). The higher the fade the poorer the performance. Therefore the options are ranked from the lowest to the highest values
Performance recovery (PR)	It is the revival of the braking efficiency in terms of attaining the same performance after the friction material is cooled down (by air blowing in this case) to a lower temperature. Therefore the options are ranked from the highest to the lowest
Performance friction (PF)	It is the average friction coefficient of all the 35 braking operations irrespective of the nature of the run, viz. cold cycle, fade cycle and recovery cycle. The options are ranked from the highest to the lowest
Disc temperature rise (DTR)	It is the frictional temperature rise of the rotor disc due to the friction braking irrespective of all the runs. The lower the temperatures rise the better the performance as the thermal distortions and frictional undulations will be minimum. Hence the options are ranked from the lowest to the highest
Wear volume (WV)	It is the progressive removal of the material from the surface due to thermo-mechanical stresses caused by the frictional interactions. The lower the wear the higher the operational life expectancy. Hence the options are ranked from the lowest to the highest

Table 3
Details of the performances of selected friction materials (FMs)

	Criteria		Options				
	Feature	Measure	FM1	FM2	FM3	FM4	FM5
C1	Performance-[x	–	0.316	0.329	0.295	0.438	0.297
C2	[x-Fade	%	13.92	13.37	18.2	18.26	2.357
C3	[x-Recovery	%	106.32	106.99	109.83	111.87	108.00
C4	Temperature rise	°C	347	373	331	455	302
C5	Wear volume	cm^3	3.31	1.347	1.953	5.218	1.841

Table 4
Advantages-disadvantages table for 10 pairs of options and five criteria

	FM1/FM2	FM1/FM3	FM1/FM4	FM1/FM5	FM1/FM3	FM1/FM4	FM1/FM5	FM1/FM4	FM1/FM5	FM1/FM5
C1	1/2D1	/3A11	/4D11	1/5 Al2	2/3A1	2/4D1	2/5A1	3/4D1	3/5D1	4/5 A1
C2	1/2D2	/3D21	/4A21	1/5 D_2	2/3D2	2/4A2	2/5D2	3/4A2	3/5 D_2	4/5 D_2
C3	1/2D3	/3D31	/4D31	1/5 D3	2/3D3	2/4D3	2/5D3	3/4D3	3/5 A3	4/5 A3
C4	1/2 A4	/3D41	/4A41	1/5 D_4	2/3D4	2/4A4	2/5D4	3/4A4	3/5 D_4	4/5 D4
C5	1/2D5	/3D51	/4A51	1/5 D5	2/3A5	2/4A5	2/5A5	3/4A5	3/5 D5	4/5 D5
EA;	1	1	3	1	2	3	2	3	1	2
EA	4	4	2	4	3	2	3	2	4	3

Table 5
Outranking matrix *(R)*

	FM1	FM2	FM3	FM4	FM5
FM1	-	1	1	3	1
FM2	4	-	2	3	2
FM3	4	3	-	3	1
FM4	2	2	2	-	2
FM5	4	3	4	3	-

advantages and disadvantages are represented asAi and D *(i* = 1, 2,... , 5), respectively. This helps in establishing strict superiority relations forthe ordering of pairs of options.

2.2. Triangularisation of the out-ranking matrix

Triangularisation of the out-ranking matrices is conducted to obtain a new order of options. This resulting triangular out-ranking matrix is denoted as RT shown in Table 6. The triangular matrix systematically reorders thej· options such that, out of a set of $P = j!$ orders (in this case $P = 5! = 120$), the sum of the values above the main diagonal is a maximum in the matrix of the final order. The triangularisation method is generally applicable to quadratic matrices, such as input-output matrix or a voting matrix [13].

In a completely triangular matrix, there are only zeros below the main diagonal, a situation, which is referred to as the total order structure. The occurrence of a total order structure implicates the transitive overall final order of options. Normally the order of options implied by the out-ranking matrix is not the final overall order of options. Therefore "Triangularisation" can be understood as a method for both, to test and to display the degree of achievement of (strong) transitive overall order of options.

Table 6
Triangular outranking matrix (R^T)

	FM5	FM3	FM2	FM1	FM4
FM5	-	4	3	4	3
FM3	1	-	3	4	3
FM2	2	2	-	4	3
FM1	1	1	1	-	3
FM4	2	2	2	2	-

The degree of linearity of a triangularised matrix is measured by λ, where

$$k = \frac{\sum_{j<k} r_{jk}}{\sum_{j\neq k} r_{jk}}, \quad 0.5 \leq \lambda \leq 1$$

The degree of linearity of the matrix given in the table is 0.68. λ indicates how much an order of options deviates from the ideal case of $\lambda = 1$, which implies a strong linear order, say, A-C, for which the transitivity condition applies [8].

The performance orders of the friction materials with respect to each criterion on the basis of Table 3 are given below

C1 : FM4 > FM2 > FM1 > FM5 > FM3

C2 : FM5 > FM3 > FM2 > FM1 > FM4

C3 : FM4 > FM3 > FM5 > FM2 > FM1

C4 : FM5 > FM3 > FM1 > FM2 > FM4

C5 : FM2 > FM5 > FM3 > FM1 > FM4

2.3. Advantages-disadvantages table

The advantages-disadvantages table is developed, which combines the criteria with the pair wise comparison of options. The head row contains all possible pairs of options. If there are 'm' options, the maximum number of pairs is $z = m(m - 1)/2$. In this case, $z = 10$. The pair wise comparisons are made in terms of quantities, i.e., on a cardinal scale. For example: FM1 has comparative disadvantage relative to FM2 since FM2 was superior to FM1 with respect to the first criteria (C1). Hence it is denominated as 1/2D1. In contrast, with respect to the fourth criterion (C4), the FM1 has a comparative advantage as compared to FM2, as FM1 was superior to FM2. This is denominated as 1/2A4. This table containing the votes of out-ranking matrix shows how the quasi-votes are split by criteria or equivalently, the criterion dependent advantages and disadvantages.

2.4. Balancing of the problems

The balancing problem involves the comparison of two options with respect to a set of advantages and disadvantages, which are separate binary decision problems. For example, in the first column of Table 5 of FM1/FM2 represents separate binary decision problem consisting of one advantage and four disadvantages. This implies that FM1 is at a disadvantage as compared to FM2. The binary problem is then solved with respect to the advantages and disadvantages of options and they are further reordered. The triangular out-ranking matrix given in Table 6 indicates the following provisional ordering of options: FM5 > FM3 > FM2 > FM1 > FM4. It is the starting matrix used in the stepwise procedure. This matrix is at the helm of the procedure and the goal of the same is to convert as many pairs of the entries above the diagonal to 5:0 pairs as warranted by the judgements of the decision-maker. A final solution in terms of the overall ordering of the options is reached when this conversion is completed. The provisional ordering of the triangular out-ranking matrix is attempted sequentially so as to attain a maximised value of the difference of *J2'jk* below the diagonals from that of the above the diagonal elements [13].

The logical implication of the transitivity condition is used while going for the stepwise procedure. For example, if the *(m* - 1) pair wise comparisons above and along side the diagonal is given in Table 6 and the remaining pair wise comparisons in the upper triangle are implied by transitivity. To elicit a maximum number of transitivity implications triangularisation is the prime objective, when the *(m* - 1) pairs of options above and along side the diagonal are given.

For example, if the pair wise comparisons above and along side the diagonal, FM5/FM3, FM3/FM2, FM2/FM1 and FM1/FM4, are given, the remaining six pair wise comparisons FM5/FM2, FM5/FM1, FM5/FM4, FM3/FM1, FM3/FM4 and FM2/FM4 are implied. Such implicative comparisons are determined as follows:

FM5 > FM3 and FM3 > FM2 -> FM5 > FM2

FM5 > FM2 and FM2 > FM1 -> FM5 > FM1

FM5 > FM1 and FM1 > FM4 -> FM5 > FM4

FM3 > FM2 and FM2 > FM1 -> FM3 > FM1

FM3 > FM1 and FM1 > FM4 -> FM3 > FM4

FM2 > FM1 and FM1 > FM4 -> FM2 > FM4

These implicative comparisons simplify the balancing problems. In the best case, shown above, where all pair wise comparisons above and along side the diagonal (FM5/FM3, FM3/FM2, FM2/FM1 and FM1/FM4) are confirmed which leaves out four balancing problems that are solved as illustrated in Table 7.

To reach the final order of options through the final triangular out-ranking matrix a multi-stepped approach has been adopted. For example suppose that the first decision, FM3 is superior to FM5, instead of FM5 being superior to FM3. Then the entries 3 versus 5 have to be inversed. This change requires a new "Triangularisation", which results in another provisional order of options and another first provisional triangular out-ranking matrix. Consequently, the pair wise comparisons above and alongside the diagonal become different and then subsequently another second balancing problem will be chosen, and so on, till all the entries above the diagonal are maximised (in the present case it is 5, since the number of criteria are 5), as elaborated in Table 7.

Table 7
Illustration of stepwise procedure to the final triangular outranking matrix and the final order of options (final ranking of the materials)

	FM5	FM3	FM2	FM1	FM4
Triangular' outranking matrix					
FM5	-	4	3	4	3
FM3	1	-	3	4	3
FM2	2	2	-	4	3
FM1	1	1	1	-	3
FM4	2	2	2	2	-
First provisional triangular outranking matrix					
FM5	-	5	3	4	3
FM3	0	-	3	4	3
FM2	2	2	-	4	3
FM1	1	1	1	-	3
FM4	2	2	2	2	-
Second provisional triangular outranking matrix					
FM5		5	5	4	3
FM3	0		5	4	3
FM2	0	0		4	3
FM1	1	1	1		3
FM4	2	2	2	2	
Third triangular outranking matrix					
FM5	-	5	5	5	3
FM3	0	-	5	5	3
FM2	0	0	-	5	3
FM1	0	0	0	-	3
FM4	2	2	2	2	-
Final triangular outranking matrix					
FM5	-	5	5	5	5
FM3	0	-	5	5	5
FM2	0	0	-	5	5
FM1	0	0	0	-	5
FM4	0	0	0	0	-

2.5. The significance of judgements

The balancing approach differs from the traditional MCDM method of assignment of prior weights to the criteria. Further it also allows integration of both the balancing of the relative advantages and disadvantages of pairs of options while simultaneously taking into account the different importance of the criteria. The advantages-disadvantages table operates at the factual level because other than the factual relations between the alternatives comprising each pair no other qualitative relations are taken into account. The final order of options for decision making is determined by solving the 10 balancing problems stated in Table 4.

2.6. Final ordering of options

The sequential elimination from the complete enumeration of possible orders, which are inconsistent with the superiority relations, leads to the final ordering of the options. The number of possible orders in our example is $p = n! = 5! = 120$. Of these 120 orders, 60 orders having FM2 before FM5 are eliminated as FM5 shows a strict superiority over FM2. If the pair wise comparisons above and along side the diagonal, FM5/FM3, FM3/FM2, FM2/FM1 and FM1/FM4 are as assumed, then a stepwise reduction of the remaining 60 orders becomes possible. Specifically another 20 orders are eliminated from the decision FM5/FM3, another 20 orders are eliminated from the decision FM3/FM2, another 15 orders are eliminated from the decision FM2/FM1 and another 4 orders are eliminated from the decision FM1/FM4 leaving only one order of option, which is the optimised option elicited from the decision model in terms of the selected set of performance criteria. The ensuing final overall order of options (of the five selected friction materials) in terms of performance is FM5 > FM3 > FM2 > FM1 > FM4. Thus the carbon fibre based friction material is adjudged to be the optimised option with maximisation of the overall performance that is immediately followed by PAN based one. The aramid fibre based composite performed moderately. The cellulose fibre based friction material is adjudged to be the least performing.

3. Conclusion

The application of a new balancing and ranking method based on the MCDM technique has proved to be useful in performance ranking of the friction materials, especially to avoid the confusion of selection of a particular friction material out of the available lots. Since the performance of the friction material is based on diversified magnitudes of various performance indicators, hence to approach an optimum level of safety and reliability through proper selection and meeting the desired operational variations, by the use of the MCDM method was proved to be successful. The method entails three steps involving the definition of out-ranking matrix based on the criteria values for all the options, while taking into account the frequency of superiority of one option, with respect to each option. Based on the information from the advantages-disadvantages table the provisional ordering of options through the triangularisation of the out-ranking matrix and the final balancing operations of the provisional ordering of the options has been performed. The balancing of the ordering is supplemented by the construction of auxiliary tables until attainment of the partial or complete strict ordering of the options. Thus, this method is effectual in establishing an analytic preview of the options available for a particular application in terms of its efficiency before the decision-maker. The current illustrations have indicated that the presence or absence or the variation of the nature of the organic fibre in a friction material formulation alters the performance order. Hence the balancing and ranking method based on the MCDM approach was proved to be a powerful technique for a rapid and comparative assessment of the friction materials.

Acknowledgement

Authors gratefully acknowledge the financial support by Department of Science and Technology, New Delhi, India to carry out this research work.

References

[1] D.M. Elzey, R. Vancheeswaran, S. Myers, R. McLellan, Multi-criteria optimization in the design of composites for friction applications, in: Proceedings of the International Conference on Brakes 2000 Automotive Braking-Technologies for the 21st Century, Leeds, UK, July 11-12, 2000, pp. 197-205.

[2] D.M. Elzey, R. Vancheeswaran, S.W. Myers, R.G. Mc Lellan, Intelligent selection of materials for brake linings, SAE Proceedings, 2000-01-2779, 2000, pp. 181-192.

[3] O.P. Gandhi, V.P. Agarwal, A digraph approach to system wear evaluation and analysis, ASME Trans. J. Tribol. 116 (1994) 268-274.

[4] R. Sehgal, O.P. Gandhi, S. Angra, Wear evaluation and ranking of tribomaterials using a Hasse diagram approach, in: Proceedings of the STLE/ASME Tribol. Conf., Seattle, WA, October *1-4*, 2000, pp. 1-8.

[5] P. Sen, J.B. Yang, Multiple Criteria Decision Support in Engineering Design, Springer-Verlag, London, 1998.

[6] T.L. Saaty, The Analytic Hierarchy Process, University of Pittsburgh, USA, 1988.

[7] C.L. Hwang, K. Yoon, Multiple Attribute Decision Making Methods and Applications: A State-of-the-Art Survey, Springer-Verlag, Berlin, 1981.

[8] J. Zak, The methodology of multiple criteria decision making in the optimization of an urban transportation system: case study of Poznan city in Poland, Int. Trans. Op. Res. 6 (1997) 571-590.

[9] O.L. Chau, C. Parkan, Selection of a manufacturing process with multiple attributes: a case study, J. Eng. Technol. Manage. 12 (1995) 219-237.

[10] G. Strassert, T. Prato, Selecting farming systems using a new multiple criteria decision model: the balancing and ranking method, Ecol. Econ. 40 (2002) 269-277.

[11] B.K. Satapathy, J. Bijwe, Performance of friction materials based on variation in nature of organic fibres. Part I. Fade and recovery behaviour, Wear (in press).

[12] Z.F. Lansdowne, Outranking methods for multi-criterion decision making Arrow's and Raynaud's conjecture, Social Choice and Welfare 14 (1997) 125-128.

[13] J. Bartnick, Optimal triangulation of a matrix and a measure of interdependence for a linear econometric equation system, in: J. Gruber (Ed.), Econometric Decision Models, Springer, Berlin, 1991, pp. 487^95.