SELF FOCUSING AND DEMODULATION OF ELECTROMAGNETIC BEAMS

by

Awdhesh Kumar

Thesis submitted to the Indian Institute of Technology, Delhi
for the award of the Degree of
DOCTOR OF PHILOSOPHY

Department of Physics
Indian Institute of Technology, Delhi
November, 1974

I. I. T. DELHI LIBRARY
PROCESSED

I. I. T. DELHI
LIBRARY
Acc. No. TH-243

538.56
AWD-S

ACKNOWLEDGEMENTS

The author expresses his deep gratitude to Prof. M.S.Sodha, Dr. D.P.Tewari and Dr. V.K.Tripathi for their valuable guidance, kind encouragement and inspiration without which the present work would not have been possible.

The author also wishes to express his sincere thanks to Shri S.N.Mitra, Shri S.C.Mazumdar and Shri C.S.R.Rao, All India Radio for their kind encouragement and useful suggestions. Thanks are due to Dr. A.P.Mitra, National Physical Laboratory for his continuous interest and valuable suggestions.

The author expresses his sincere thanks to his colleagues and staff members of the Physics Departments and the Computer Centre of IIT Delhi. Thanks are due to Shri T.N.Gupta for the efficient typing of the thesis.

Financial support of IIT Delhi and CSIR India is gratefully acknowledged.

Akagarwal
22.11.74
(AWDHESH KUMAR)

PREFACE

Self focusing of electromagnetic beams in nonlinear media has attracted wide attention during the past decade[1-10] on account of its implications in harmonic generation, parametric power conversion and other nonlinear processes. The earlier investigations of this phenomenon were confined to dielectrics. The self focusing of laser beams in plasmas has assumed tremendous importance on account of its application to thermonuclear fusion[11-17]. The realization of fusion involves two major problems, viz., (1) the excitation of instabilities in the plasma in the presence of a nonuniform laser beam which may be exploited for heating the plasma and (2) the inertial confinement of the plasma. The earlier problem necessitates an understanding of the mechanism of nonlinearity in the plasma. Since the laser powers used in these experiments are very high ($\sim 10^{12}$ watts) as compared to the powers at which nonlinear effects starts occuring, the self focusing i.e. propagation of a laser beam in selfmade waveguide is important in this context.

A study of self focusing of electromagnetic beams in plasmas is also very useful to the physical understanding of the nonlinear processes; experiments on plasmas should be easy to undertake because the critical power for self focusing in plasmas is several orders of magnitude smaller than that in dielectrics[18,19].

Litvak[18], Prasad and Tripathi[19], Sodha et al.[20] and others[21-27] have recently discussed the self focusing of laser beams in plasmas in which the nonlinearity arises due to the inhomogeneous heating of electrons by the nonuniform beam and their subsequent redistribution in the plasma. These treatments are applicable for moderate powers where the rise in electron temperature due to the beam may be treated as a small perturbation. Hora[28], Sodha et al.[29] and others[30] have considered another mechanism of nonlinearity, which leads to self focusing viz. the ponderomotive force on electrons and the subsequent redistribution of electron density, caused by it. In the steady state this nonlinearity is much less important than the one discussed earlier, because the critical power for self focusing in this case is much larger than that in the previous case.

The nonlinearity in the effective dielectric constant of a plasma arises through th heating and modulation of collision frequency of ectrons also. This mechanism of nonlinearity has been invtigated in great detail in literature[31]. However, it inot relevant in causing the self focusing of the beam[2(The nonlinearity arising through the breakdown of the pha[32] is also not effective in the self focusing of the b because the nonlinear dielectric constant of the plasma, by this mechanism, is a decreasing function of intensity of the beam.

An additional mechanism of nonlinearity has been recently investigated by Stenflo[33] and Sodha et al.[34] in the gaseous plasmas showing Ramsaur effect i.e. in which according to Harp model[35] we may assume the collision frequency to be very low below some critical speed of electrons $v = v_0$ and above which the collision frequency may be taken as infinite. Thus only those electrons contribute to the current density which have speed below v_0. When a heating electric field (of the wave) is applied to a plasma the electrons below $v = v_0$ (on assuming ν to be small but finite) are heated i.e. transferred to high velocity range $v > v_0$. Hence the effective number density of electrons responsible for current density is reduced. This nonlinearity gives rise to self focusing of a Gaussian beam at accessible powers. However, this is valid in very rare cases and is not a subject of present interest.

The self focusing of electromagnetic beams in solid state plasmas has also been studied in considerable detail in recent years[36-40]. These studies lead to a better understanding of the free carrier nonlinearity in semi-conductors and aids in their characterization. The nonlinearity in semiconductor (and semimetals)[41,42] is important at much lower powers as compared to those in dielectrics and the density of electrons is higher than that of usually encountered plasmas; hence the studies on self focusing in semiconductors may be made for modelling the phenomenon in dielectrics and plasmas.

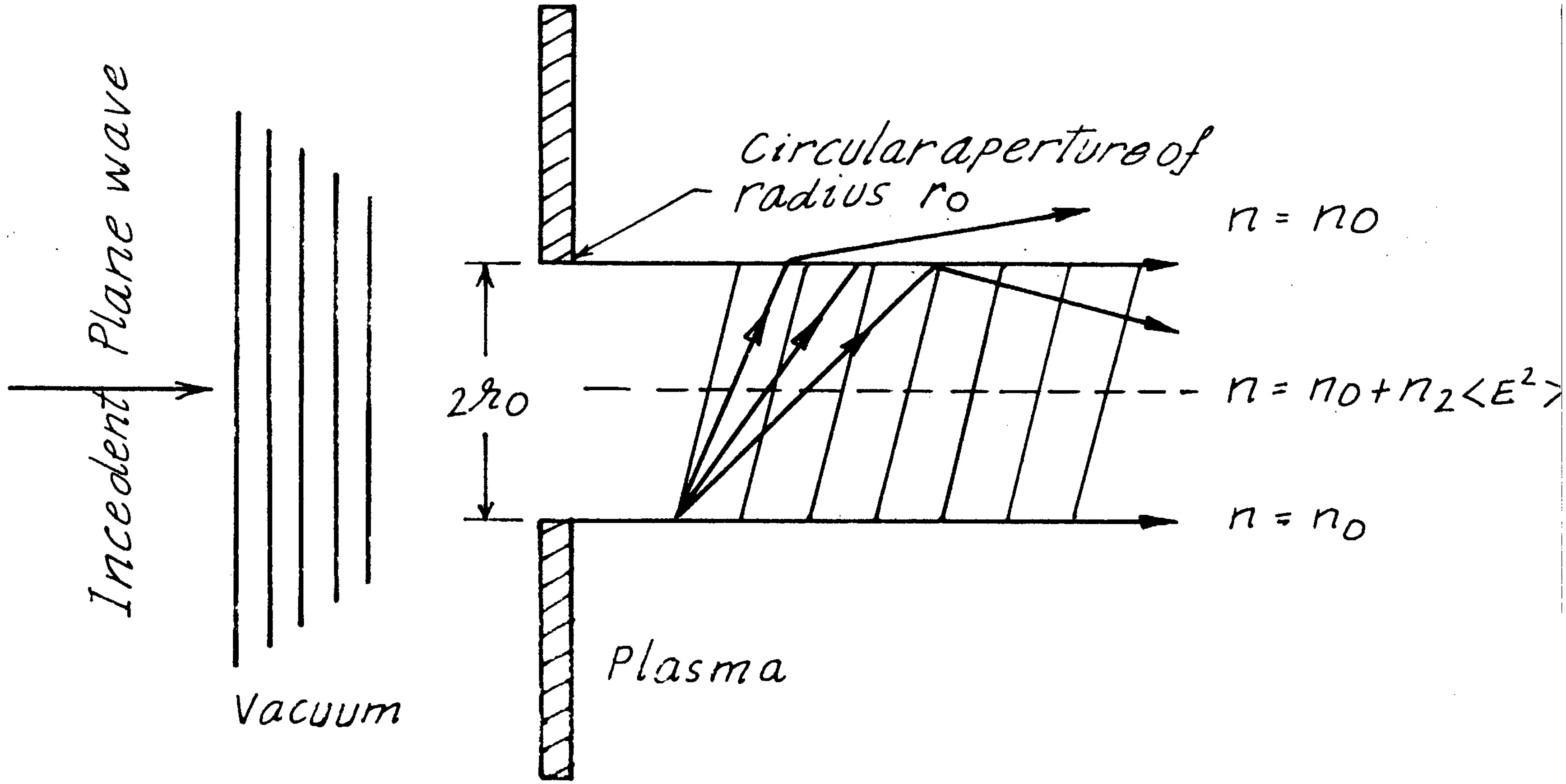
The nonlinear dependence of effective dielectric constant of a semiconductor on the intensity of the beam may be understood as follows: the free carriers absorb energy from the field and, in the steady state, attain a temperature higher than the equilibrium value such that the rate of power absorption from the field becomes equal to the rate of power lost by the carriers in collisions with heavy particles (cf. Conwell (1967))[43]. Such carriers whose temperature is modified appreciably by the field are called hot carriers and may give rise to the following important effects:

(i) The mobility of carriers changes on account of the energy dependence of the relaxation time and the energy dependence of the carrier mass; the latter holds for nonparabolic semiconductors (cf. Conwell (1967)) while the former is not relevant for self focusing[20].

(ii) The local concentration of carriers may change on account of nonuniform heating by the beam. It is interesting to mention here that, owing to the space charge effects the positive and negative charge carriers are redistributed through ambipolar diffusion[18,19].

In the present thesis, the author has investigated the mechanism of nonlinearity in plasmas and semiconductors in the steady state. In the case of plasmas and parabolic semiconductors, a rigorous kinetic treatment for the derivation of nonlinear dielectric constant has been given when the power of the beam has been taken to be arbitrary below the limit of breakdown of the plasma. In the case of nonparabolic semiconductors and semimetal (e.g. Bismuth) a phenomenological treatment has been followed. The nonlinearity is then used to investigate the self focusing of the beam in the plasma. A problem of nonlinear self distortion of an amplitude modulated electromagnetic wave in a magnetoplasma has also been investigated as it is much relevant to high frequency communication.

It would be useful to visualize physically the phenomenon of self focusing in a medium where the refractive index is an increasing function of the intensity EE^* of the beam (say $n = n_0 + n_2 EE^*$ where n is the refractive index of the medium, n_0 and $n_2 EE^*$ are the field independent and field dependent components of n respectively). We consider a plane uniform wavefront incident on a circular aperture (of radius r_0) in a nonlinear medium (see Fig.1). The portion of the medium



Incedent Plane wave
Vacuum
2r0
Circular aperture of radius r0
Plasma
n = n0
n = n0 + n2 <E2>
n = n0

FIG. 1

illuminated by the beam has a refractive index $(\eta = \eta_0 + \eta_2 EE^*)$ higher than that of the non-illuminated portion. Therefore, the secondary wavelets diverging at an angle θ from the wavenormal suffer total internal reflection at the boundary of the fictitious cylinder of radius r_0 when

$$\theta < \theta_c \quad ,$$

where

$$\theta_c = \cos^{-1} \frac{\eta_0}{\eta_0 + \eta_2 EE^*}$$

corresponds to the critical angle. It is also known from the diffraction theory that a very large fraction of the power will be carried by rays making an angle less than θ_D with the axis;

$$\theta_D \simeq 0.61 \lambda_0 / 2 r_0 \eta_0 \quad ,$$

where λ_0 is the wavelength of radiation in free space. We may now consider three obvious possibilities.

(i) When $\theta_D > \theta_c$, the beam will diverge due to the predominence of diffraction effects.

(ii) When $\theta_D = \theta_c$, the beam should propagate in the selfmade waveguide. The corresponding power of the beam can easily be shown to be given by

$$P \equiv P_{cr} = \frac{(1.22)^2 \lambda_0^2 c}{128 \eta_2} \quad ,$$

where P_{cr} is the critical power of the beam and c is the velocity of light in vacuum.

(iii) When $\theta_D < \theta_c$ one expects the beam to focus. The above speculations are supported by rigorous analyses carried out by Akhmanov et al. and Sodha et al.[20] based on the solution of wave equation on WKB and paraxial ray approximations. Putting $\theta_D = \frac{r_0}{R_d}$ we obtain

$$R_d = \frac{r_0}{\theta_D} = r_0 \cdot \frac{2 r_0 n_0}{0.61 \lambda_0} \simeq \frac{k r_0^2}{2}$$

where

$$k = 2\pi n_0/\lambda_0$$

The condition expressed by $\theta_D = \theta_c$ is equivalent to

$$R_d \simeq R_n$$

where

$$R_n = r_0 \left(n_0/n_2 E_0^2 \right)^{1/2} .$$

The conditions for convergence and divergence are likewise $R_d > R_n$ and $R_d < R_n$ respectively. From the above analysis it can be seen that the diffraction acts as a divergent lens of focal length R_d. The condition of non convergence and nondivergence for $R_d = R_n$ implies that the nonlinear medium acts as a converging lens of focal length R_n.

It is instructive to realize the avalanche nature of the self focusing. Consider a beam propagating in a nonlinear medium with refractive index defined by $n = n_0 + n_2 E E^*$. As the beam gets focused due to nonlinearity, the intensity increases causing enhancement of the nonlinearity and hence in the extent of nonlinear focusing.

In case the beam is stronger near the axis than at the edges, i.e. there is an intensity distribution along the wavefront (which is indeed true for laser beams), the refractive index at the edges will be less than the refractive index at the central portion of the beam and hence the rays will tend to bend towards the axis. In case this tendency is stronger than the tendency to diverge by diffraction, focusing will occur and vice versa.

The present thesis has been divided into seven chapters. A chapterwise summary may be given as follows:-

In the first chapter the self focusing of the electromagnetic beam has been studied; the analysis is general and not limited to the perturbation approximation. The nonlinearity in the medium arises due to the heating and redistribution of carriers. The isotropic part of the electron velocity distribution function is found to be Maxwellian at an effective local temperature[44] T_e (T_e is identical to the one obtained by phenomenological theory) when $\nu^2 << \omega^2$ where ν is the collision frequency of electrons with heavy particles and ω is the angular frequency of the wave. The redistribution of carriers is, however, very sensitive to the velocity dependence of collision frequency. The redistribution is maximum for an electron-ion collision dominated plasma while there is no redistribution in a plasma where collision cross section is proportional to electron velocity. In a hypothetical case, when the collision frequency depends on the velocity as ϑ^{S} (S is an integer greater than 2), the redistribution is such that the carriers are more where the intensity of the beam is high. The real part of nonlinear dielectric constant shows a saturating behaviour with respect to the field. The imaginary part of dielectric constant (though it is small) tends to zero at high fields in a Coulomb collision dominated plasma; while in a plasma having constant collision cross section, this tends to a constant value. The self focusing of the beam in such media showing saturating nonlinearity rules out the possibility of point focusing. The axial intensity of the beam increases when the self focusing effect dominates over the diffraction divergence effect. It then attains a maximum (but finite) value at some distance Z_{opt}; for $Z > Z_{opt}$ the diffraction effects dominate over the self focusing effects so that the divergence of the beam occurs and it goes on till the divergence and self focusing effects become equal. With sufficient broadening of the beam, diffraction effects become less dominant so that the beam starts focusing. In this way the beam propagates in a selfmade oscillatory waveguide.

In the second chapter of the thesis the self focusing of a Gaussian beam has been studied in a magnetoplasma. A rigorous kinetic treatment has been given for the derivation of effective dielectric constant of the medium when the magnetic field is applied in the direction of propagation without putting any limit on the wave intensity and accounting properly for the space charge field. The nonlinearity appears through the heating and redistribution of carriers. The self focusing of the beam shows that the minimum spot size increases with the increase in field intensity of the wave and it occurs at greater distances. The effect of static magnetic field is found to enhance the tendency of self focusing at low intensities and to suppress it at higher intensities. As the cyclotron frequency ω_c approaches the frequency of the wave, the minimum spot size and the distance of focusing increase. However, in a typical case with the wave intensity not too large, the minimum spot size attains a maximum value for some magnetic field beyond which it goes on decreasing with the magnetic field although 'f' always grows. The extraordinary mode of the wave propagates in an oscillatory waveguide (in the absence of absorption) in such a medium.

The third chapter of the thesis deals with the self focusing of a right handed circularly polarized beam in a nonparabolic semiconductor (InSb) in the presence of a static magnetic field. The interaction of beam with the medium increases the carrier temperature and makes the medium nonlinear through two mechanisms (i) mass modulation of carriers due to the increase in their random energy and (ii) redistribution of carriers. The two mechanisms have been studied explicitly. In the case of a n-InSb sample the redistribution of carriers is limited due to the space charge field so that the mass modulation of carriers is the source of nonlinearity. On the other hand, in a compensated sample the electrons and holes diffuse simultaneously and hence the nonlinearity appears mainly due to the redistribution of carriers. It is found that all frequency waves can be focused in a n-type InSb. It is shown that the above two mechanisms are dominent over all other mechanisms e.g. the non-linearity due to the mass modulation of carriers owing to their drift energy given by Tzoar et al.[45] and the energy dependent relaxation type nonlinearity given by Dubey et al.[46].

In the fourth chapter, the self focusing of a Gaussian electromagnetic beam has been studied in a semimetal namely Bismuth. The semimetal is chosen because of low impurities and hence less attenuation even at large carrier concentrations. A rigorous treatment has been given for the evaluation of nonlinear dielectric tensor of the medium, under perturbation limit, when the static magnetic field is applied along the trigonal axis of the sample. Under this condition the normal modes of wave propagation are circularly polarized and the heating of carriers are identical in all valleys. This is not true in the case of a linearly polarized wave (even in the absence of static magnetic field) unless the electric vector is aligned along the trigonal axis and the propagation of wave is normal to it. The interaction of wave with the carriers increases their temperature and results in their redistribution which makes the medium nonlinear. The analysis shows that the magnetic field has appreciable contribution in reducing the focusing length and the critical power of the beam. However, self focusing is not possible in the case of Alfven/Helicon waves because the nonlinear dielectric tensor in these cases increases with the increasing carrier concentration and hence the nonlinear divergence of the wave takes place. The self focusing length is found to be more in the case of a plane polarized wave propagating along z direction than a circularly polarized wave in the x-y plane for the same power of the beam.

In the fifth chapter we have investigated the stability of a plasma/semiconductor for small fluctuations in the intensity of a high amplitude plane electromagnetic wave. The nonlinearity in the medium appears through the redistribution of carriers on account of their nonuniform heating (due to fluctuations in the intensity of the beam). It is found that the hot carrier nonlinearity in the dielectric constant of a plasma/semiconductor is very effective in making the medium unstable for small fluctuations in the intensity of an electromagnetic beam. The perturbation grows with the advancement of the beam. Fluctuations of optimum size and long duration (greater than the energy relaxation time of the medium) have been found to grow at moderate powers. The effect of absorption is, however, to suppress this effect.

In the sixth chapter of the thesis we have investigated the problem of demodulation of an electromagnetic wave in the ionosphere. A general case has been chosen in which the magnetic field is applied at an arbitrary direction to the direction of wave propagation and no limit has been put on the modulation frequency of the wave. It is found that the modulation of the wave is sensitive to the angle between the phase velocity and static magnetic field, and gyro resonance.

In the seventh chapter some miscellaneous work connected with electromagnetic wave propagation and diffraction phenomena has been presented. The results obtained are quite interesting and informative.

This work has resulted in the following publications:-

1. The saturating nonlinear dielectric constant and self focusing of electromagnetic waves in plasmas: Kinetic approach, M.S.Sodha, D.P.Tewari, Awdhesh Kumar and V.K.Tripathi, J.Phys.D: Appl.Phys. 7, 345 (1974).

2. Hot carrier diffusion induced instabilities of electromagnetic waves in plasmas and semiconductors, M.S.Sodha, Awdhesh Kumar, V.K.Tripathi and P.K.Kaw, Opto Electronics 5, 509 (1973).

3. Self focusing of electromagnetic beams in Bismuth, D.P.Tewari and Awdhesh Kumar, J.Appl.Phys. (In Press 1974).

4. Periodic focusing of gaussian electromagnetic beam in a magnetoplasma, D.P.Tewari and Awdhesh Kumar, Plasma Physics (In Press 1974).

5. Self focusing of laser beams in InSb in presence of a magnetic field, Awdhesh Kumar, V.K.Tripathi and D.P.Tewari, Communicated (1974).

6. Demodulation of electromagnetic waves in ionosphere, M.S.Sodha, D.P.Tewari, Awdhesh Kumar and V.K.Tripathi Communicated (1974).

7. Enhancement of diffracted microwave signal by a metallic knife edge mounted over a cylindrical hill, D.C.Dube, Awdhesh Kumar, S.Prasad and V.K.Tripathi, Communicated (1974).

8. Self focusing of an intense laser beam in a dense plasma, Awdhesh Kumar, V.K.Tripathi and D.P.Tewari, Communicated (1974).

9. On the self reflection of intense electromagnetic waves in plasmas, D.P.Tewari and Awdhesh Kumar, Communicated (1974).

The author has also worked on the phenomenon of cross modulation in the ionosphere with Shri C.S.R. Rao of All India Radio. The report of this study which indicates good agreement of theory with experiments published by All India Radio (See Appendix 'B').

I. I. T. DELHI.
LIBRARY
Acc. No TH-243.

CONTENTS