FILAMENTATION INSTABILITY IN PLASMAS

by

Jai Kishan Sharma

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Jai Kishan Sharma

(Jai Kishan Sharma)
Recently there has been a surge of interest in the interaction of intense electromagnetic radiations with plasmas on account of its close relevance to controlled fusion (both magnetically confined and laser induced) and some exotic ionospheric phenomena. In laser induced fusion, the most important problem is the efficient coupling of the energy of the laser beam to plasma to heat the latter. In this coupling process many nonlinear phenomena such as self-focusing, filamentation instabilities, parametric instabilities, harmonic generation and stimulated scattering etc. play crucial role. The parametric instabilities provide a mechanism for efficient transfer of electromagnetic energy to plasmas. An incident (pump) wave may excite two new waves in a plasma. If both the excited modes are purely electrostatic, they are eventually absorbed in the plasma and this decay process thus leads to enhanced (or anomalous) absorption of the incident electromagnetic wave. If one of the excited mode is electromagnetic, it can escape from the plasma and appears as enhanced (or stimulated) scattering of the incident electromagnetic wave. Further, if the electrostatic mode is of high frequency such that ion response may be neglected the scattering is stimulated Raman scattering and if the electrostatic mode is of low frequency such that ion response cannot be neglected, then stimulated Brillouin scattering takes place (Brueckner and Jorna, 1974; Drake, Kaw, Lee, Schmidt, Liu and Rosenbluth,
1974 and others). Thus the scattering process in which large amount of beam energy is wasted, is detrimental to laser induced fusion.

When the density perturbations (on account of the deviation from the quasi-neutrality of the plasma because of modification in electron density) propagate in a direction perpendicular to the direction of incoming radiation, the modulational instability is known as 'Filamentation Instability'. Physically this phenomenon can be understood as follows:

If we consider that plasma is irradiated by a pump wave

$$\mathbf{E}_0 = E_0 \hat{\mathbf{e}}_z \exp \left[ (\omega_c t - \mathbf{k}_0 \cdot \mathbf{r}) \right]$$

then in steady state electrons oscillate with high velocity in the incident electric field $\mathbf{E}_0$ with the ions form a stationary background. If this equilibrium is perturbed by a propagating density perturbation $(\omega, \mathbf{k})$ associated with an electrostatic wave, the electron density fluctuations are imposed on the oscillating field $\mathbf{E}_0$ and lead to currents at frequencies $\omega \pm l \omega_0, \mathbf{k} \pm l \mathbf{k}_0$ where $l$ is an integer; the lowest order coupling corresponds to $l = 1$. These currents will generate mixed electromagnetic-electrostatic side band modes at $\omega \pm l \omega_0, \mathbf{k} \pm l \mathbf{k}_0$. The side band modes in turn interact with the pump wave field producing a ponderomotive force $\sim \nabla E^2$ which
amplifies the original density perturbation. It should be mentioned here that for the modulational instability both of the side bands contribute to the density fluctuations. Thus there is a positive feedback system, which leads to the instability of the original density perturbation. When the perturbation is perpendicular to the direction of propagation of the incident beam \((\mathbf{K} \cdot \mathbf{v} = 0)\), the modulational instability is called as filamentation instability. This instability may also arise on account of small scale fluctuations in the intensity distribution of the pump wave. When such a beam propagates in the plasma, the effective dielectric constant of the plasma for the high intensity portion is higher than that for the lower intensity portion. On account of the inherent characteristics of a nonlinear medium, where the refractive index increases with the intensity of the beam, the electromagnetic energy is concentrated around the point of intensity maximum. Thus the perturbation could grow inside the plasma, leading to the phenomenon of filamentation. On account of this filament formation, the plasma is rendered inhomogeneous and the condition for the growth of absorptive instabilities become localized; thus the heating of the plasma is significantly affected.

These filaments have already been studied in dielectrics by Chiao, Garmire and Townes (1964) and observed by Garmire, Chiao and Townes (1966). Bespalov and Talanov (1966) have presented a theory for filament formation. They have concluded
that in a nonlinear dielectric liquid, the amplitude and phase perturbations of a plane electromagnetic wave bring about its decay into individual beams.

Kaw, Schmidt and Wilcox (1973) and Sodha, Kumar, Tripathi and Kaw (1973) have studied the growth of perturbation in the intensity distribution of plane electromagnetic beams in a plasma, where the nonlinearity arises on account of the redistribution of the electron density caused by the ponderomotive force or non uniform heating of the carriers by the main beam or by the perturbations in the beam intensity. For a uniform plane beam, the redistribution of the carriers does not occur, because the ponderomotive force due to a uniform beam is zero. However, on account of the perturbation in the intensity distribution along the wave front, the electrons do get redistributed. Sodha, Sharma and Tripathi (1974) have also studied the spatial growth of filamentation instability for a Gaussian laser beam in a plasma. Drake et al. (1974) have derived a general dispersion relation and used it to investigate the parametric excitation of plasma wave, parametric excitation of electrostatic kinetic instabilities, stimulated Raman and Brillouin scattering off natural or strongly modified plasma modes and modulational instabilities like filamentation instabilities. The nonlinear coupling terms responsible for the interaction has been taken to arise from (a) the nonlinear current density due to density fluctuations in one wave interacting with the oscillating electron velocities
due to the pump wave and (b) the ponderomotive force \((\nabla E^2)\) produced by the pump wave and one of the excited waves.

Bingham and Lashmore-Davies (1976) have also discussed the self-modulation and filamentation of electromagnetic waves in a plasma. Lee, Liu, Chen and Nishikawa (1974) have obtained expression for the growth of filamentation instability in an inhomogeneous plasma. Manheimer and Ott (1974) have discussed the filamentation instability in the presence of a weak magnetic field for a uniform beam. Perkins and Valeo (1974) have reported the self-focusing in highly collisional plasmas, where filamentation is due to thermal effects. Yu and Shukla (1976) have discussed the filamentation instabilities of ordinary electromagnetic waves in collisional plasmas.

Bujarbarua, Sen and Kaw (1975) have investigated the filamentation instability of electromagnetic waves in a finite \(B\) \((B\) is the ratio of particle kinetic pressure to magnetic pressure) inhomogeneous plasma. They have derived a general dispersion relation for stimulated scattering of high frequency EM waves off drift-alfven waves. The dispersion relation has been solved for the case of filamentation instabilities and estimates of growth rate, threshold power have been obtained. Mio et al. (1976) have also investigated the modulatory instability of alfven waves propagating along the magnetic field in a plasma.

Thome and Perkins (1974) have experimentally observed ionospheric striations, while Langdon and Lasinski (1975) and
Eidmann et al. (1975) have observed the filamentation instability in computer simulation and in laser target experiments. Stenzel (1976) has observed the filamentation of a high power whistler wave in a laboratory plasma. The temporal and spatial evolution of duct formation and wave propagation has been shown. In an attempt to explain the experimental results, Sodha and Tripathi (1977A) have discussed the steady state self-focusing and filamentation of whistlers in a plasma. Donaldson and Spalding (1976) have also observed experimentally the density cavitons and X-ray filamentation in CO₂ laser produced plasmas (nearly collisionless). More recently Mori et al. (1977) have experimentally observed the interaction of a high power pulsed microwave incident on the plasma. The spatial and temporal perturbations of plasma density and internal field perpendicular to the direction of propagation have been experimentally investigated.

The earlier theoretical analyses on the filamentation instability are confined to homogeneous and isotropic plasmas and for uniform beams. Since Gaussian beams are generally used in laser plasma interaction experiments, the self-focusing of the beam occurs, which leads to the enhancement in the intensity of the beam at the axis, thus all the nonlinear phenomenon such as filamentation instability, scattering, harmonic generation etc. get drastically modified. Furthermore the nonlinearity in a plasma is mode sensitive; the process of self-focusing and filamentation are expected to be much affected
by the application of a static magnetic field.

This thesis reports the author's investigations on filamentation instability in more realistic situations, taking into account inhomogeneity in the plasma, inhomogeneity in the beam and the presence of a magnetic field. The author has also discussed the growth of a Gaussian ripple on a plane wave front. Excitation of the plasma waves by an intense Gaussian electromagnetic beam and the associated second harmonic generation has also been analyzed.

It is appropriate to mention here that the self-focusing is a process in which an electromagnetic beam of finite spatial transverse width focuses itself in transverse extent due to nonlinear effects and produces a single hot spot, while the filamentation is a uniform plane wave analogue of the self-focusing phenomenon and results in multiple hot spots.

It would be useful here to summarise the various nonlinear effects in the plasma. Physically the nonlinear behaviour of the plasma may be understood as follows:

The free carriers acquire momentum and energy from the field which are lost in collisions with ions and neutral particles. In an idealized collision momentum is almost completely randomized. In equilibrium (i.e. in the absence of an electric field), the temperature of the electrons is the same as that of the heavy particles, so that the net energy exchange between electrons and heavy particles is zero. When an electric field is applied, the electrons gain energy from
the field and in the steady state attain a temperature higher
than that of the heavy particles such that the power gained by
the electrons from the field equals the power lost by the
electrons in collisions (Ginzburg 1970).

When the applied laser beam has a nonuniform distribu-
tion of intensity along the wavefront, the carrier temperature
is also nonuniform and a corresponding variation in electronic
concentration takes place. The space charge field generated
in this process makes ions to follow the electrons. Due to
this redistribution of the carriers, a transverse gradient of
effective dielectric constant is established which leads to
the self-focusing of the beam.

In collisionless plasmas this mechanism of nonlinearity
is not operative and the nonlinearity arises on account of the
ponderomotive force (Hora, 1969). Ponderomotive force arises
because of the interaction of the electron drift velocity with
the magnetic field of the wave and the motion of the electrons
in the inhomogeneous electric field. When the intensity
distribution of the beam is nonuniform, the force on account
of this interaction has a finite time independent component
in a direction perpendicular to that of propagation. On
account of this force a redistribution of electronic concen-
tration takes place. The ions are also dragged by the electro-
in view of the strong Coulomb interaction between them. This
effect makes the effective dielectric constant nonlinear and
self-focusing of beams may occur. This nonlinearity is dominan
in plasmas where the duration of the electromagnetic beam is much smaller than the energy relaxation time \( t \ll \tau_c \) of the electrons; such situations occur in the case of fast pulse laser interaction with strongly ionized collisionless plasmas. In the case when \( t \geq \tau_c \), the nonlinearity appearing through the heating of the carriers is much more important than the ponderomotive force effect. The nonlinearity because of ponderomotive force is set up in a time of the order of \( r_o/v_s \) where \( r_o \) is the dimension of the beam and \( v_s \) is the ion sound speed. This time is much smaller than the time required by the hot carrier nonlinearity to set up.

The nonlinearity in the effective dielectric constant of a plasma may also arise through the heating and modulation of collision frequency of electrons. This mechanism of nonlinearity has been investigated in detail. However, it does not cause appreciable self-focusing (Sodha, Ghatak and Tripathi, 1974). The nonlinearity arising through the breakdown of the plasma is also not effective in the self-focusing of the beam because the nonlinear dielectric constant of the plasma (on account of this mechanism) is a decreasing function of the intensity of the beam.

In relativistic plasmas (Kaw and Dawson 1970) where the velocity of the electron is comparable to the velocity of light (i.e. \( v \sim c \)) the nonlinearity due to the relativistic variation of the mass of the electron may arise but this nonlinearity is important only at very high powers of the
laser beams ($\approx 10^{16}$ watts/cm$^2$).

It is instructive here to understand the phenomenon of self-focusing in a medium where the refractive index is an increasing function of the intensity ($E^*E$) of the beam (say $n = n_o + n_2 E^*E$, where $n$ is the refractive index of the medium, $n_o$ and $n_2 E^*E$ are the field independent and field dependent components of $n$ respectively). We consider a plane uniform wavefront incident on a circular aperture (of radius $r_o$) in a nonlinear medium (see Fig.1). The portion of the medium illuminated by the beam has a refractive index ($n = n_o + n_2 E^*E$) higher than that of the non-illuminated portion. Therefore, the secondary wavelets diverging at an angle $\Theta$ from the wave normal suffer total internal reflection at the boundary of the fictitious cylinder of radius $r_o$ when

$$\Theta < \Theta_c$$

where

$$\Theta_c = \cos^{-1}\left[\frac{n_o}{(n_o + n_2 E^*E)}\right]$$

corresponds to the critical angle. It is also known from diffraction theory that a very large fraction of the power will be carried by rays making an angle less than $\Theta_D$ with the axis

$$\Theta_D = 0.61 \frac{\lambda_0}{2 n_o n_o}$$

where $\lambda_0$ is the wavelength of the radiation in free space.
Circular aperture of radius $n_0$

$n = n_0$

$n = n_0 + n_2 \langle E^2 \rangle$

$n = n_0$

Fig. 1
Now three possibilities may arise

(i) When $\Theta_D > \Theta_c$, the beam will diverge due to predominance of diffraction effects.

(ii) When $\Theta_D = \Theta_c$, the beam should propagate in the self-made wave guide. The corresponding power of the beam can easily be shown to be given by

$$P = P_{cr} = \frac{(1.22)^2 \lambda_0^2 c}{12 \theta \kappa \alpha}$$

where $P_{cr}$ is the critical power of the beam and $c$ is the velocity of light in vacuum and

(iii) When $\Theta_D < \Theta_c$ one may expect the beam to focus.

The above speculations are supported by rigorous analysis carried out by Akhmanov, Sukhorukov, and Khokhlov (1966) and Sodha, Ghatak and Tripathi (1974) based on the solution of wave equation on using WKB and paraxial ray approximations. Putting $\Theta_D = \frac{2 \pi n_0}{R_d}$ one may write

$$R_d = \frac{2 \pi n_0}{\Theta_D} = n_0 \left( \frac{2 \pi n_0}{0.6 \lambda_0} \right) = \frac{k r_0^2}{2}$$

where

$$k_0 = \frac{2 \pi n_0}{\lambda_0}$$

The condition expressed by $\Theta_D = \Theta_c$ is equivalent to

$$R_d < R_n$$

where

$$R_n = n_0 \left( \frac{n_0}{n_2 EE^*} \right)^{1/2}$$
The conditions for convergence or divergence are likewise $R_n$ and $R_d < R_n$ respectively. From the above analysis it can be seen that the diffraction acts as a divergent lens of focal length $R_d$. The condition of nonconvergence and nondivergence for $R_d = R_n$ implies that the nonlinear medium acts as a converging lens of focal length $R_n$.

It is instructive to realize the avalanche nature of the self-focusing. Consider a beam propagating in a nonlinear medium with refractive index defined by $n = n_0 + n_2 E E^*$. As the beam gets focused due to nonlinearity, the intensity increases causing enhancement of the nonlinearity and hence in the extent of nonlinear focusing.

In case the beam is stronger near the axis than at the edges i.e. there is an intensity distribution along the wave front (which is indeed true for laser beams; Yariv, 1971), the refractive index at the edges will be less than the refractive index at the central portion of the beam and hence the rays will tend to bend towards the axis. In case this tendency is stronger than the tendency to diverge by diffraction, focusing will occur and vice versa. On the other hand, a beam in which the intensity increases with increasing radius, will defocus in a medium, because the refractive index will be higher at the boundary of the beam than at the axis.

In the proposed thesis the author has investigated the effect of self-focusing on the growth rate of filamentation
instability, excitation of plasma waves and second harmonic generation. It is found that the above mentioned phenomena are modified significantly on account of self-focusing effects.

The steady state self-focusing of intense electromagnetic waves in plasmas has been discussed by Askaryan (1962), Talanov (1964), Litvak (1970), Sodha, Ghatak and Tripathi (1976), Kaw et al. (1973) and others. In these investigations the conditions for self-trapping of waves (i.e. uniform wave guide propagation) in plasmas were investigated. Litvak (1970) and Prasad and Tripathi (1973) investigated the self-focusing of microwaves in a weakly ionised plasma; they considered the nonuniform heating and consequent redistribution of electrons. These treatments were phenomenological in nature and applicable only in the perturbation approximations. Sodha, Tewari, Kumar and Tripathi (1974) studied the same problem with a rigorous kinetic treatment. The velocity dependence of collision frequency governs the redistribution of the carriers. It was shown that at high powers the nonlinear dielectric constant of the plasma is a saturating function of the wave intensity, which in turn, results in the periodic focusing of the beam i.e. the wave propagates in a self made oscillatory wave guide. Sodha, Khanna and Tripathi (1973) studied the self focusing of laser beams in a strongly ionised plasma, where the thermal conduction plays a dominant role in the loss of excess electron energy; the analysis predicted periodic focusing of the beam
even when the dielectric constant is not in the saturating region.

Lindl and Kaw (1970), Hora (1969) and Sodha, Mittal, Virmani and Tripathi (1974) have investigated the self-focusing in a collisionless plasma by the ponderomotive force nonlinearity, which is important on short time scale (i.e. duration of pulse $t \ll T_e$, the energy relaxation time). Sodha, Kaushik and Kumar (1975) have also studied the self-focusing in a moving plasma. The nonstationary self-focusing of E.M. waves in collisional and collisionless plasmas have also been studied by Sodha, Sharma and Tripathi (1974A) and Sodha, Prasad and Tripathi (1975) where the finite relaxation time of the nonlinearity is found to affect the self-focusing considerably.

Moreover the self-focusing of electromagnetic beams in collisional and collisionless magnetoplasma has also been investigated by Litvak (1970), Sodha, Mittal, Kumar and Tripathi (1974). In the presence of a magnetic field two modes of propagation exist. The heating of electrons and the ponderomotive force on electrons on account of the extra ordinary mode display marked cyclotron resonance and consequently the self-focusing of the beam occurs at much lower powers. In collisional magnetoplasmas the self-focusing of ordinary mode is possible in the frequency range $\omega \gg \gamma$, where $\omega$ is the frequency of the wave and $\gamma$ is the collision frequency. But the extra ordinary mode is focused only in the
range of $\omega > \omega_c$ where $\omega_c$ is the electron cyclotron frequency. For $\omega < \omega_c$ defocusing of extra ordinary mode occurs. In collisionless magnetoplasmas extra ordinary mode shows a peculiar behaviour on reversing the direction of ponderomotive force on electrons in going from $\omega < \frac{\omega_c}{2}$ to $\omega > \frac{\omega_c}{2}$ range. Therefore, the self-focusing of extra ordinary mode is possible in all frequency range except $\frac{\omega_c}{2} < \omega < \omega_c$. Self-focusing of ordinary mode is retarded on account of magnetic field but that of extra ordinary mode is enhanced. Tewari and Kumar (1975) have studied the self-focusing of electromagnetic beam in magnetoplasma, following a rigorous kinetic approach; dielectric constant of the magnetoplasma is saturated and periodic focusing of the beam occurs. Sodha, Khanna and Tripathi (1974) have also studied the self-focusing in a magnetoplasma, when the collisions are few and the energy dissipation is caused by thermal conduction. The important result was that even at low powers of the beam, when the dielectric constant is not saturated, periodic focusing of the beam occurs (For details see Sodha, Ghatak and Tripathi 1976). Recently Sodha and Tripathi (1976, 1977B) have concluded that the self-focusing of electromagnetic beams for a given set of parameters, occurs only when the power of the beam lies between two critical values $P_{cY1}$ and $P_{cY2}$. For $P < P_{cY1}$ the beam suffers monotonic divergence; for $P_{cY1} < P < P_{cY2}$ the beam propagates in a self made oscillatory wave guide, with the intensity on the axis being always greater than the
initial value; for \( P > R y^2 \) the propagation of the beam takes place in oscillating wave guide, with the intensity on the axis being always less than the initial value.

The nonlinear phenomena discussed so far are very much affected by the application of an external magnetic field. The current density in a magnetoplasma due to an electric field is not aligned along the electric field; i.e. the conductivity and effective dielectric constant of the plasma are tensors. Thus the plasma becomes anisotropic and the effective dielectric tensor of the magnetoplasma is not symmetrical. Two independent modes (in general elliptically polarized) of propagation exist in the limit of linear theory in such a medium (Ginzburg 1970). When the direction of the phase velocity coincides with the external magnetic field the two modes are circularly polarized. The right handed circularly polarized mode is sensitive to electron cyclotron resonance because both the electric and the external magnetic field rotate the electrons in the same sense. The author has studied the spatial growth of perturbation in a magnetoplasma in Chapter I. The investigations are limited by considering only one mode at a time. The cross focusing of the two modes (extra ordinary mode and ordinary mode) has been neglected on account of weak coupling between them.

Besides the filamentation instability of Gaussian laser beam in a plasma, the author has also discussed the dynamics of the growth of a ripple on a uniform plane wavefront. The
nonuniformity in the intensity of the beam along its wave front considered here is on account of perturbation field (i.e. Gaussian ripple). For a ripple of low power in comparison to that of the main beam, the nonuniform field responsible for the redistribution of carriers arises on account of the nonlinear coupling of the field of the main beam and that of the ripple. Consequently the redistribution of carriers (and hence nonlinear part of the effective dielectric constant of the plasma) is highly dependent on the phase difference $\phi_p$ between the electric vector of ripple and main beam, power of the main beam, the nonlinear absorption if present.

In addition to it the generation of plasma wave and second harmonic generation has also been analysed. In recent experiments [Lee et al. (1974), Yamabe et al. (1977), Sigel et al. (1976), Jackel et al. (1976)] on irradiation of targets by intense laser beam, the scattering at $\frac{3\omega}{2}$, $\frac{3\omega}{2}$ and $2\omega$ have been observed. Liu and Rosenbluth (1976) has given an explanation for the scattered loss at $\frac{3\omega}{2}$ by considering the parametric decay of an incident pump wave into two plasmas. Sodha, Sharma and Kaushik (1976) also investigated the effect of the self-focusing of the pump wave on the excitation of plasma wave and predicted that scattered power at $\frac{3\omega}{2}$ gets enhanced. Sodha, Sharma and Kaushik (1976) have also explained the observed scattered loss at $3\omega$ by considering the excitation of a plasma wave at $2\omega$ frequency on account of the $\vec{U} \times \vec{B}$ force (where $\vec{U}$ is the drift velocity of
the electron and \( \mathbf{B} \) is the magnetic field of the wave). This excited plasma wave interacts with the incoming pump wave and gives the scattered power at \( 3\omega_0 \). In the present thesis the author has also given a possible explanation of the second harmonic \((2\omega_0)\) observed in laser plasma interaction experiments. The physical mechanism can be briefly understood as follows:

In the presence of a Gaussian beam the carriers get redistributed from high field region to low field region, on account of ponderomotive force and a transverse density gradient is established in the plasma. When the electric vector of the main beam is parallel to the density gradient, the plasma wave at the pump wave frequency is generated. In addition to this the transverse intensity gradient of the electromagnetic wave also contributes significantly to the plasma wave generation. The generated plasma wave interacts with the electromagnetic wave and leads to the generation of second harmonic. If the initial power of the pump wave is more than the critical power for self-focusing, the main beam gets self-focused and hence the generated plasma wave and second harmonic which depend on the background electron concentration and power of the main beam also get significantly modified.

The thesis has been divided into six chapters; a chapterwise summary is given below:
Chapter-I: Spatial growth of filamentation

**Instability in a magnetoplasma**

This chapter presents an investigation of the effect of saturating nonlinearity on the spatial growth of filamentation instability of a Gaussian electromagnetic beam in a magnetoplasma. The nonlinearity arises on account of the nonuniform heating and consequent redistribution of plasma in a collisional plasma and because of ponderomotive force in a collisionless plasma. The growth rate is a sensitive function of the distance of propagation on account of self-focusing effects and is enhanced with the increase of static magnetic field for extraordinary waves; the reverse is true for ordinary waves.

Chapter-II: Growth of filamentation instability in a strongly ionized plasma

This chapter presents an investigation of the growth of perturbation in the intensity distribution of a Gaussian electromagnetic beam propagating in a strongly ionized plasma. The loss of energy is assumed to be due to electronic thermal conduction. For beams converging, on account of self-focusing, the amplitude of perturbation changes much faster with distance than the axial amplitude in the absence of perturbations and hence the growth rate becomes a sensitive function of the distance of propagation.
Chapter-III: Temporal growth of Filamentation Instability in plasmas

In this chapter author has derived an expression for the growth rate of filamentation instability, valid for arbitrary intensity of the electromagnetic wave and up to plasma resonance. \((\omega_p = \omega_o)\) here \(\omega_p\) and \(\omega_o\) are the field free plasma frequency and EM wave frequency respectively. On the short time scale \((t \ll T_e, T_e\) being the energy relaxation time) i.e. in a collisionless plasma, the nonlinearity arises on account of ponderomotive force and consequent depletion of electrons in regions of large wave amplitude. On the time scale of energy relaxation time i.e. in a collisional plasma, the dominant nonlinearity arises on account of preferential heating of electrons. It is found that (i) maximum growth rate attains its maximum for an optimum intensity of the beam; (ii) filamentation is predicted even in the over dense plasmas \((\omega_p > \omega_o)\).

Chapter-IV: Filamentation instability of electromagnetic beams in an inhomogeneous plasma

This chapter presents an investigation of the spatial and temporal growth rates of filamentation instability due to the propagation of a Gaussian laser beam in an axially inhomogeneous plasma \((N_e = N_0 (1 + z/L))\), \(L\) being the characteristic length of inhomogeneity). On the time scale of energy relaxation time \((t \gg T_e, T_e\) being the energy relaxation time) the nonlinearity arises due to preferential heating and consequent depletion of carriers. While on the
short time scale ($t \ll T_e$) ponderomotive force is taken to be the source of nonlinearity. The effect of self-induced inhomogeneity (introduced due to self-focusing of the main beam) and that of background, on the temporal and spatial growth of filamentation instability is found to be drastic. The theory is valid even around plasma resonance and into the over-dense plasmas within the WKB and paraxial ray approximations.

Chapter-V: Growth of a Gaussian ripple on a uniform plane wavefront in plasmas

This chapter presents an investigation of the growth of a Gaussian ripple on a uniform plane wavefront of an EM wave in plasmas. In collisionless plasmas and in short time scale ($t \ll T_e$, $T_e$ being the energy relaxation time) in a collisional plasma, ponderomotive force is the dominant source of nonlinearity. In the time scale of energy relaxation time ($t \gg T_e$) in a collisional plasma, preferential heating of electrons in the regions of large intensity and consequent redistribution of plasma is the sole mechanism of nonlinearity. The theory reveals that the ripple gets self-focused when the initial power of the ripple ($P$) is between two critical powers $P_{c1}$ and $P_{c2}$. When $P > P_{c2}$ or $P < P_{c1}$ respectively, oscillatory and monotonous defocusing of the ripple occurs; oscillatory self-focusing of the ripple occurs when $P_{c1} < P < P_{c2}$. The critical power for self-focusing of the ripple and the nature of self-
focusing is highly dependent on the power of the main beam, the phase difference between the electric vector of the ripple and the main beam and also the nonlinear absorption. The theory is also valid for a weakly inhomogeneous plasma within the WKB approximation.

Chapter VI: Generation of plasma wave and second harmonic generation

This chapter presents an investigation of the generation of a plasma wave at pump wave frequency and second harmonic generation on account of self induced transverse inhomogeneity introduced by a Gaussian EM beams in a hot collisionless plasma. In the presence of a Gaussian beam the carriers get redistributed from high field region to low field region on account of ponderomotive force and a transverse density gradient is established in the plasma. When the electric vector of the main beam is parallel to this density gradient, plasma wave at the pump wave frequency is generated. In addition to this the transverse intensity gradient of the EM wave also contributes significantly to the plasma wave generation. The power of the plasma wave exhibits maximum and minimum with the power of the pump wave (at z=0). The generated plasma wave interacts with the EM wave and leads to the generation of second harmonic. Further, if the initial power of the pump wave is more than the critical power for self-focusing, the beam gets self-focused and hence the generated plasma wave and second harmonic which depend on the background electron concentration and power of the main beam also get accordingly modified.
The thesis is partly based on the following publications:


5. Filamentation instability of electromagnetic beams in an inhomogeneous plasma (Communicated, 1977).


In addition to the above mentioned publications the author has also been associated with the following publications which have not been included in the thesis:


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## CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PREFACE</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>SPATIAL GROWTH OF FILAMENTATION INSTABILITY IN A MAGNETOPLASMA</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>Introduction</td>
<td>27</td>
</tr>
<tr>
<td>1.2</td>
<td>Nonlinear Dielectric Tensor of Magnetoplasma</td>
<td>31</td>
</tr>
<tr>
<td>1.3</td>
<td>Growth Rate of Perturbation</td>
<td>34</td>
</tr>
<tr>
<td>A. Uniform plane wave propagation</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>B. Uniform self-made waveguide propagation</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>C. Oscillatory waveguide propagation</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td>Discussion and Conclusions</td>
<td>44</td>
</tr>
<tr>
<td>II</td>
<td>GROWTH OF FILAMENTATION INSTABILITY IN A STRONGLY IONIZED PLASMA</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>49</td>
</tr>
<tr>
<td>2.2</td>
<td>Nonlinear Dielectric Constant</td>
<td>51</td>
</tr>
<tr>
<td>2.3</td>
<td>Growth Rate of Perturbation</td>
<td>57</td>
</tr>
<tr>
<td>2.4</td>
<td>Discussion of the Results</td>
<td>60</td>
</tr>
<tr>
<td>III</td>
<td>TEMPORAL GROWTH OF FILAMENTATION INSTABILITY IN A PLASMA</td>
<td></td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>64</td>
</tr>
<tr>
<td>3.2</td>
<td>Dielectric Constant of Plasma</td>
<td>65</td>
</tr>
<tr>
<td>3.3</td>
<td>Growth Rate of Perturbation</td>
<td>67</td>
</tr>
<tr>
<td>3.4</td>
<td>Discussion</td>
<td>71</td>
</tr>
<tr>
<td>IV</td>
<td>FILAMENTATION INSTABILITY OF EM BEAMS IN AN INHOMOGENEOUS PLASMA</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>76</td>
</tr>
<tr>
<td>4.2</td>
<td>Nonlinear Dielectric Constant of an Inhomogeneous Plasma</td>
<td>78</td>
</tr>
<tr>
<td>4.3</td>
<td>Dispersion Relation and Growth Rate</td>
<td>81</td>
</tr>
<tr>
<td>4.4</td>
<td>Discussion</td>
<td>86</td>
</tr>
</tbody>
</table>
V  GROWTH OF A GAUSSIAN RIPPLE ON A UNIFORM PLANE WAVE FRONT IN PLASMA

5.1 Introduction 89
5.2 Nonlinear Dielectric Constant 91
5.3 Self-focusing of the Ripple 95
5.4 Discussion 100

VI  GENERATION OF PLASMA WAVE AND SECOND HARMONIC GENERATION

6.1 Introduction 104
6.2 Generation of Plasma Wave 106
6.3 Second Harmonic Generation 112
6.4 Discussion 117