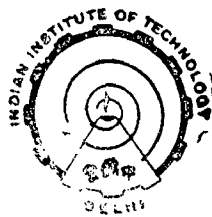


**Identification, Classification and Isomorphism  
of Kinematic Chains and Mechanisms  
via  
Identification Codes**

by  
**ASHOK G. AMBEKAR**

A Thesis submitted  
in fulfilment of the requirements of the degree  
of  
**DÓCTOR OF PHILOSOPHY**

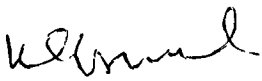


Department of Mechanical Engineering  
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MAY 1986

To My Wife  
SUSHAMA

C E R T I F I C A T E

This is to certify that the thesis entitled,  
"IDENTIFICATION, CLASSIFICATION AND ISOMORPHISM OF KINE-  
MATIC CHAINS AND MECHANISMS VIA IDENTIFICATION CODES"  
by Ashok G. Ambekar has been prepared under my supervision  
in conformity with the rules and regulations of the  
Indian Institute of Technology, Delhi. I further certify  
that the thesis has attained a standard required for  
a Ph.D. degree of the Institute. The results contained  
in this thesis have not been submitted, in part or full,  
to any other University for any degree or diploma.

  
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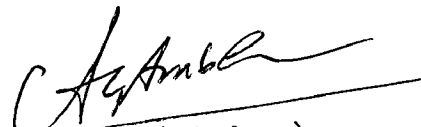
A C K N O W L E D G E M E N T

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The author would be failing in his duty if he did not express his gratitude to his wife and parents who have borne with him with patience during the course of investigation.

  
(Ashok G. Ambekar)

A B S T R A C T

Departing from the conventional approach of representing a kinematic chain through a set of identification numbers, an attempt has been made to represent a kinematic chain, with turning pairs, by a unique number, called identification code (IC). In the earlier part of this thesis, these ICs are established for kinematic chains by deriving coefficients of structural similarity of these chains with respect to corresponding complete chain. IC of mechanism is expressed as a set consisting of IC of parent chain and IC representative of frame link. Following the conventional approach, rooted graph method and an analytical method is also proposed for the identification of mechanisms, path generators and function generators. A new method of graph representation is shown to result in a more realistic characteristic polynomial and variable characteristic polynomial for a multiple jointed kinematic chain. ICs, derived in a certain way, are shown; (i) to produce a more refined number representation scheme and (ii) a more convincing classification scheme.

In the latter part of the thesis, an entirely new concept-namely, the concept of canonical numbers, is shown to produce unique and decodable identification codes

for kinematic chains with simple turning pairs. Testing two chains for isomorphism is, thus, equivalent to a comparison of their canonical numbers (max code or min code). Based on proposed algorithms, for max/min code, programs are written in Fortran IV and executed on ICL-2960 computer. All possible schemes of canonical numbering, obtained from these programs are shown to produce automorphism groups. A canonical number approach is shown to be quite useful in identifying kinematic chains with simple/multiple joints and also with revolute/non-revolute joints. A method of identifying mechanisms, driving link mechanisms, and paths is also proposed. Symmetry operations of point and line group, derived from automorphism groups, are shown to be useful in enumerating structurally distinct mechanisms, driving mechanisms, path generators and function generators.

C O N T E N T S

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