Parametric study of a greenhouse by using Runge–Kutta methods

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Abstract

This communication presents the analysis for a greenhouse using Runge–Kutta methods. The energy balance equations have been written considering the effects of evaporation from the plant, conduction through the ground, ventilation, etc. The parametric study has also been performed to find the effects of various parameters, namely number of air changes/h, transmissivity (α), heat capacity of plants and movable insulation on the plant and enclosure room air temperature. Numerical computations have been performed for a typical day in the month of March for Delhi climatic conditions. It is observed that there is significant reduction of plant and room air temperatures due to an increase in number of air changes, the heat capacity of plants and the movable insulation.

Keywords: Greenhouse; Summer; Off-season cucumber production

Nomenclature

\( A \) area (m\(^2\))
\( A_c \) area of cooling pad (m\(^2\))
\( A_d \) area of door (m\(^2\))
\( A_g \) greenhouse floor area (m\(^2\))
\( A_p \) area of foliage (m\(^2\))
\( A_r \) area of roof (m\(^2\))
\( F \) greenhouse efficiency factor
\( h_0 \) heat transfer coefficient between room air and ambient air through walls (W/m\(^2\)°C)
$h_p$  heat transfer coefficient between plant and enclosure air (W/m²·°C)
$h(t)$  overall total heat transfer coefficient from inside room to ambient through walls, floor and canopy cover (W/m²·°C)
$M_p$  heat capacity of plants (mass of plants $\times$ specific heat of plants) (J/°C)
$M_a$  heat capacity of enclosed air (mass of air $\times$ specific heat) (J/°C)
$N$  number of air changes/h
$p(T)$  partial vapour pressure at temperature $T$ (N/m²)
$S(t)$  intensity of solar radiation (hourly average) at time $t$ (W/m²)
$T_{x=0}$  temperature of ground at $x = 0$ (floor) (°C)
$T_a$  ambient temperature (°C)
$T_p$  plant temperature at time $t$ (°C)
$T_{p0}$  plant temperature at time $t = 0$ (°C)
$T_R$  greenhouse enclosure room air temperature (°C)
i  time (s)
$V_i$  rate of exchange due to ventilation and infiltration (W)
$x$  position coordinate along depth inside ground (m)

Greek letters
$\alpha_g$  absorptivity of greenhouse cover (canopy cover)
$\alpha_p$  absorptivity of plant
$\tau$  transmissivity of canopy cover
$\gamma$  relative humidity

Suffix
D  door
E  east
G  floor
N  north
p  plant
r  room
S  south
W  west

1. Introduction

The greenhouse environment is represented by a group of spatial average values of climatic factors, such as radiation, temperature, humidity and CO$_2$ concentration, which affect plant growth and development. The environment thus described or controlled is referred to as the greenhouse microclimate [1]. Studies on greenhouse microclimate have been conducted by several researchers [2-4]. Efforts have been made to predict the greenhouse thermal environment under both steady state and transient conditions [5-8]. A dynamic thermal performance simulation model for a greenhouse was also developed numerically [9]. In this, the system equations were solved numerically by using a Runge-Kutta predictor-corrector...
technique for the differential equations and a Newton–Raphson iteration technique for the algebraic equations to determine the various dependent variables.

A mathematical model was further developed to predict the plant and the room air temperatures besides other parameters, such as instantaneous thermal efficiency, greenhouse efficiency factor, etc., considering the effect of evaporation from the plants, conduction through the ground, ventilation, etc. [10]. In this model, to simplify the analysis, the heat capacity of the room air was neglected. However, in the present analysis, the heat capacity of the room air has also been considered, and the analysis has been done by using the Runge–Kutta method to predict different parameters for a greenhouse.

2. Thermal analysis

Energy balance equations for the different components of the proposed greenhouse have been written with the following assumptions: (i) the properties of the plant mass are considered equivalent to water mass for all thermal analysis purposes due to the high content of water in the plant; (ii) the relative humidity inside the greenhouse does not vary with height due to the wetted floor/watering channel; (iii) the analysis is based on quasi-steady-state conditions inside the greenhouse due to transient behaviour for short time intervals (Δt); (iv) no stratification in temperature of plant, greenhouse enclosure, covers, etc., due to the low operating temperature range; (v) the ground heat loss from the floor to the ground has been considered in a steady-state mode; and (vi) the heat capacity of air remains constant. The resulting energy balances for the different components of the greenhouse [Fig. 1(a)] are given below.

For greenhouse plants
\[
\alpha_p(t) = \frac{d T_p}{d t} + h_p A_p (T_p - T_R) + h_0 A_p \left[ p(T_p) - \gamma p(T_R) \right].
\] (1)

For greenhouse floor
\[
\alpha_e(1 - \alpha_p(t)) = -k \frac{\partial T}{\partial x} \bigg|_{x=0} A_G + h_G A_G (T_{h=0} - T_R).
\] (2)

For greenhouse enclosed air
\[
(1 - \alpha_e)(1 - \alpha_p(t)) = M_a \frac{d T_R}{d t} + \left[ h(\theta)(T_R - T_a) + h_d A_d (T_R - T_a) \right]
+ V_1 A_e (T_R - T_a) - h_p A_p (T_p - T_R)
+ h_0 A_p \left[ p(T_p) - \gamma p(T_R) \right] - A_G h_G (T_{h=0} - T_R).
\] (3)

where \( h_0 = 0.016 \), \( S = A_E S_E + A_W S_W + A_R S_R + A_S S_S + A_R S_R \), and \( V_1 = NV/3 \).

Here, in Eqs. (1) and (3), the partial vapour pressures of the plant and the room air temperatures have been linearized as \( p(T_e) = R_1 T_e + R_2 \) and \( p(T_i) = R_1 T_i + R_2 \), which are the equations of straight lines where \( R_1 \) is the slope and \( R_2 \) is the intercept of the straight lines, both of which can be obtained by linear regression analysis using steam tables.
The above-mentioned Eqs. (1) to (3), after simplifying, may be reduced to the following two first-order differential equations of the form:

\[
\frac{dT_p}{dt} + a(1, 1)T_p + a(1, 2)T_e = b_1
\] (4)

and

\[
\frac{dT_e}{dt} + a(2, 1)T_p + a(2, 2)T_e = b_2
\] (5)

where

\[a(1, 1) = \frac{h_pA_p + h_0A_pR_1}{M_p}, \quad a(1, 2) = -\frac{h_pA_p + h_0A_p\gamma R_1}{M_p},\]

\[a(2, 1) = -\frac{h_pA_p + h_0A_pR_1}{M_a}, \quad a(2, 2) = \frac{h(t) + h_dA_d + V_1A_c + h_pA_p + h_0A_p\gamma R_1 + U_bA_g}{M_a}.\]
Fig. 1(b). Continued.

\[ b_1 = \frac{z_p (tS) - h_0 A_p R_c (1 - \gamma)}{M_p} \]

and

\[ b_2 = \frac{h_{\text{eff}} + \tau_{\text{eff}} S + \left[ h(t) + h_d A_d + V_1 A_e + U_0 A_d \right] T_a + h_0 A_p R_c (1 - \gamma)}{M_a} \]

Eqs. (4) and (5) have been solved numerically by using the fourth order Runge–Kutta method.
Table 1
Constants used for the experimental study

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>( \alpha_G )</td>
<td>24.0</td>
<td>( \tau )</td>
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</tr>
<tr>
<td>( \alpha_B )</td>
<td>26.4</td>
<td>( \alpha_T )</td>
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<td>( \alpha_D )</td>
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<td>( \alpha_G )</td>
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<td>( \alpha_T )</td>
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<td>( \gamma )</td>
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<td>( M_p )</td>
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<td>( h_p )</td>
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<tr>
<td>( h_G )</td>
<td>5.7</td>
<td>( h_T )</td>
<td>3.99</td>
</tr>
<tr>
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<tr>
<td>( \tau )</td>
<td>3600</td>
<td>( M_a )</td>
<td>60 \times 1006.0</td>
</tr>
</tbody>
</table>

to obtain the values of \( T_p \) and \( T_r \) at the first four steps. Further, the values of \( T_p \) and \( T_r \) at the subsequent steps are obtained by the Adam–Moulton’s predictor-corrector method. The constants used in the study are given in Table 1.

For numerical analysis, a computer program has been written in FORTRAN 77, the flow chart of which is shown in Fig. 1(b). Computer solutions were obtained for the greenhouse dependent variables for the plant and inside room temperatures. At the start of the computer run, for calculating values of the temperatures, the input constants and the solar radiations on each wall and roof of the greenhouse are read in. The iteration process then starts to determine new values of the temperatures from the initial values assigned. For the next step, this value becomes the new value and so on.

3. Numerical results and discussion

Fig. 2(a) shows the hourly variation of the solar intensity (total, beam and diffuse) and the ambient temperature \( (T_a) \) for a typical day in March. These data of solar intensity have been used to calculate the solar intensity on each wall and roof of the greenhouse by the Liu and Jordan formula [11]. These data, along with the relative humidity inside the greenhouse, have been used as the input parameters in the model. Fig. 2(b) shows the hourly variation of the plant and room air temperatures. The results show that there is a fair agreement between the two. The plant temperatures obtained are less than the room air temperatures due to evaporation from the plant, as expected [12].

The effect of transmissivity of the canopy cover \( (\tau) \) on the plant and room air temperatures has been shown in Fig. 3. As indicated in this figure, these temperatures increase with transmissivity \( (\tau) \) during sunshine hours, as expected, due to more availability of solar radiations to the plant and the room air.

The effect of heat capacity of the plants on the plant and the room air temperatures has been shown in Fig. 4. The increase of heat capacity helps in reduction of the plant temperatures during sunshine hours and increasing plant temperature during off sunshine hours due to the increase in storage capacity of the plant, as expected. This also helps in phase
Fig. 2. Hourly variations of (a) solar intensity and ambient temperature and (b) plant and room air temperatures.

shifting of the maximum plant temperature. There is insignificant effect on the room air temperature.

The effect of movable insulation provided on the north and south walls and roof during sunshine hours (10 a.m. to 4 p.m.) on the plant and room air temperatures has been shown in Fig. 5. This figure shows that the movable insulation (jute cloth) during sunshine hours help in reduction of both the plant and the room air temperatures due to less transmittance of solar radiations inside the greenhouse. From the figure it is clear that there is a reduction of about
Fig. 3. Effect of transmissivity of cover on (a) plant and (b) room air temperatures.

1.5°C and 4°C in the plant and room air temperatures, respectively, during the March period. This reduction is sufficient to achieve a desirable temperature. For further reduction in temperature, other cooling methods, namely free and forced convection, can be used, particularly during April to June. This effect has also been shown in Fig. 6.

Fig. 6 shows the effect of the number of air changes/hour on the plant and room air temperatures. The plant and room air temperatures decrease with the increase in the number of
Fig. 4. Effect of the heat capacity of the plants on (a) plant and (b) room air temperatures.

air changes/h due to more withdrawal of the room air, as expected. However, the effect of a number of air changes beyond 60 is insignificant.

4. Summary and conclusions

The mathematical model presented in this paper is quite simple and yet very comprehensive to predict different parameters of a greenhouse. It can be used for a greenhouse of any shape
and size and in any location. The software is very simple and can be used on any computer with a minimal time.

On the basis of this study, it may be concluded that there is a significant reduction of plant and room air temperatures due to an increase in the number of air changes, the heat capacity of the plants and the movable insulation. However, these temperatures are found to increase with an increase in the transmissivity of the canopy cover.
Fig. 6. Effect of number of air changes per hour on (a) plant and (b) room air temperatures.

References