A THESIS ON

SOME FLOW PROBLEMS IN HYDROMAGNETICS AND HYDRODYNAMICS

By

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C E R T I F I C A T E

This is to certify that the thesis entitled "Some Flow Problems in Hydromagnetics and Hydrodynamics" which is being submitted by Shri Harbans Lal Sathi to the Indian Institute of Technology, Delhi for the award of the Degree of Philosophy (Mathematics) is a record of bonafide research work. He has worked under my guidance and supervision for the last three years and three months.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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S Y N O P S I S

The importance of the effects of rotation, suction, slip and magnetic field on the fluid flow and heat transfer problems is well known.

Rotation plays a significant role in several important phenomenon in cosmical fluid dynamics. Similarly, a great deal of meteorology depends upon the dynamics of a revolving fluid. The large scale and moderate motions of the atmosphere are greatly affected by the vorticity of earth's rotation. In the case of an infinite fluid rotating as a rigid body about an axis, the amount of energy possessed by the liquid is infinite and it is of great interest to see how small disturbances propagate in such a liquid.

The effects of suction are important in controlling the boundary layer and thus avoiding separation which further results in the reduction of skin friction on the body. It also causes delay in transition from the laminar to the turbulent flow.

The develoment of rockets whether for ballistic missiles or designed to probe into the outer space, has given a new impetus to the study of rarefied gas dynamics. It has been seen that for large Knudsen number, the gas is considered to be in rarefied state and kinetic theory approach is more appropriate. But for small Knudsen number, the flow of gas is in the so-called 'slip' region and the use of continuum theory with modified boundary conditions, allowing for a slip at the surface gives good approximate results.

Recently, it has been recognised that the electromagnetic interaction on the fluid flow leads to a much more varied phenomenon. Thus it has been the usual practice to extend the problems of hydrodynamics to hydromagnetics, a new but fast developing science as well as technology. This latter discipline has made itself felt because of its own merits and has more than proved its worth in the problems of missile and space dynamics, propulsions and communication

The purpose of this thesis is to investigate some problems in hydromagnetics and hydrodynamics regarding flow patterns, viscous resistance at the walls and heat propagation in the region under consideration. In all, it comprises eight chapters. Chapter I is introduction to the thesis, followed by remaining seven chapters which are the result of theoretical investigations in the 'slip' as well as no-slip regions. An exact solution of a rotating flow is obtained in chapter II. The results of electromagnetic interactio on the fluid flow are reported in the remaining chapters III to VIII. Suction effects are studied in chapters III to VII.

In chapter I which is of introductory nature, a critical survey of the relevant literature is presented. The fundamental equations governing the fluid flow and heat transfer phenomenon are also included.

In chapter II, a study is made of unsteadyflow engendered in a viscous, incompressible, rotating fluid by an infinite flat plate set suddenly in motion in its own plane. The exact solution of the problem is obtained by the method of Laplace Transforms. It is shown

that the rotation induces lateral motion and it also generates vortic
It is further found that it tends to oppose the penetration of longitudinal flow. The effect of rotation on drag is obtained and several limiting cases are deduced from the solution.

Chapter III deals with the effects of Coriolis forces on Rayleigh problem in hydromagnetics. As in chapter II, it is seen that the rotation induces lateral motion. For small rotation, it is found that the longitudinal components of velocity, induced magnetic field and shear stress remain the same as in the non-rotating case. The lateral stress, however, is seen to develop linearly with time which gives an erroneous impression about the non-existence of steady solution. But this is misleading and can be explained from the fact that the expression obtained for lateral shear stress is merely the leading term of the limiting solution obtained by considering the angular velocity negligibly small in the solution for general rotation. Another effect of rotat is the generation of vorticity as in the non-magnetic case.

The effects of uniform suction on Rayleigh problem in hydromagnetics are studied in chapter IV. The problem is solved by the method of Laplace Transforms. For a finitely conducting plate, solutions valid for small as well as large time are obtained for magnetic Prandtl number unity or close to unity. It is found that the shearing stress on the plate decreases from infinity and tends asymptotically to a finite value for large time. This limiting value comes out to be greater than the corresponding limiting value obtained for a perfectly conducting plate in the absence of suction. Further, an exact solution is obtained for a non-conducting plate in case of magnetic prandtl number unity and close to unity. Results for the shear stress are also analyzed.

In chapter V, a study is made of the hydromagnetic fluctuating flow past a flat plate with variable suction. It is assumed that the suction velocity normal to the plate as well as the free stream velocity fluctuate about a mean constant in magnitude (but not in direction) and that the Alfven velocity is less than the mean suction velocity. All the quantities of aerodynamical interest such as velocity distribution, skin-friction, temperature field and heat flux upto two harmonic fluctuations are evaluated. It turns out that the amplitude of first harmonic fluctuations in the skin-friction increases with the increase in the frequency of fluctuations, while that of second harmonic fluctuations tends to a finite limit. Further, for small frequencies and large values of suction parameter, the phase of the first harmonic fluctuations may be negative but it ultimately tends to $\pi/4$ at very high frequencies. Also, the phase of the second harmonic fluctuations drops to zero when frequency is large. It is also seen that for the case of no heat transfer, there is no effect on the wall temperature at high frequencies, i.e., it remains the same as in the non-magnetic case or in the absence of variable suction. But in case of heat transfer, the amplitude of the first harmonic fluctuations in heat flux tends to a finite limit and the phase drops to zero. The amplitude of second harmonic fluctuations rises with frequency and for very high frequencies, the phase lead tends to $\pi/4$.

In chapterVI, using the continuum theory approach, the effects of 'slip' are considered on the hydromagnetic fluctuating flow past a porous flat plate with oscillatory suction. As in the earlier investigation in chapter V, distributions for velocity, skin-friction and temperature fields are calculated. The case of flow past a flat plate in the no-slip region is deduced as a particular case and the results in the two regions compared. It turns out that in both 'slip' as well as no-slip regions, the periodic suction velocity induces more than one harmonic fluctuations in the flow and temperature fields. Also for both the regions, the amplitude of first harmonic fluctuation in skin-friction increases with the increase in the value of either of the magnetic or suction parameters, though it cannot exceed a finite limit in the 'slip' flow regime. Further, it is seen that the amplitude of second harmonic fluctuations in skin-friction decreases with increase in the magnetic parameter in the 'slip' flow region while it increases in the no-slip region. For very large values of frequency in the 'slip' flow region the phase of first harmonic fluctuations in skin-friction drops to zero --a result contrary to the one for the no-slip region. Further for both regions, second harmonic fluctuatio in the skin-friction always have a phase lag whatever be the frequency. It also turns out that because of the fluctuating suction even for large values of the rarefaction parameter, the fluctuations in the temperature field persist which result is quite contrary to the one for the case of uniform suction.

Chapter VII concens with a more genearl problem of hydromagnetic flow past a porous flat plate when a time dependent suction is applied at the wall. Two cases of this time dependence are considered : (i) the suction may vary periodically, (ii) the suction as well as the free stream velocity may vary slowly but are otherwise arbitrary functions of time. It turns out that the suction, in general, induces an unsteady flow parallel to the wall. Further, the periodic suction changes the shape of the mean profile. The effect magnetic field is that the mean value of the shear stress on the plate is altered - a result contrary to the one obtained for the non-magnetic case. But when the free stream velocity also has a periodic component which oscillates with the suction frequency, the mean flow is further distorted. For slowly varying but otherwise arbitrary suction and free stream velocity, a solution is obtained in terms of the derivatives of the suction and free stream velocities.

The thesis ends with chapter VIII which deals with hydromagnetic, fluctuating, axis-symmetric flow near the stagnation point region. It is assumed that the main-stream outside the boundary layer region fluctuates in magnitude (but not in direction) about a steady mean. Using Runge-Kutta method, a numerically exact solution for the mean flow as well as for the fluctuating part of the flow at small frequencies is obtained. For large frequencies, an approximate solution is developed for the oscillatory part. The 'critical frequenc at which the two solutions overlap is also calculated for different values of the magnetic parameter. It is seen that the presence of magnetic field affects the transition from one type to the other type of flow. Further, the mean value of the skin-friction decreases as the magnetic parameter increases. It is also found that an increase in the magnetic parameter results in a decrease in the value of the 'mean displacement thickness'.

C O N T E N T S