

ACTIVE VIBRATION CONTROL OF SMART PLATES AND SHELLS CONSIDERING PIEZOELECTRIC NONLINEARITY

by

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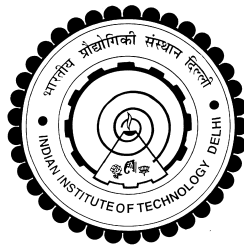
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Certificate

This is to certify that the thesis entitled “**Active Vibration Control of Smart Plates and Shells Considering Piezoelectric Nonlinearity**” being submitted by **Mohd. Yaqoob Yasin** to the Indian Institute of Technology Delhi, for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

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Abstract

Increasing demand for the development of lightweight structures particularly in aerospace, automobile and space applications has brought active vibration control of structures into the focus of research in recent times. Experiments have shown that piezoceramic materials exhibit constitutive nonlinearity, when the electric field exceeds the coercive limit. Analysis and design of smart structures made of composite and sandwich laminates integrated with distributed piezoelectric sensors and actuators for achieving vibration control require a computationally efficient and accurate model for the laminate mechanics, its appropriate numerical implementation and a suitable control algorithm, all incorporating the piezoelectric nonlinearity.

In this thesis, a new electromechanically coupled efficient layerwise finite element (FE) model is developed for the analysis of smart piezoelectric multilayered shallow shells considering the nonlinear behavior of piezoelectric materials under strong electric field. The nonlinearity is modeled using the rotationally invariant nonlinear constitutive equations, with the assumption of large electric field and small strains. For the laminate mechanics, an efficient coupled zigzag theory (ZIGT) is developed for the doubly curved shallow hybrid shells. The theory considers the normal deformation in the piezoelectric layers due to transverse piezoelectric coefficient d_{33} , without introducing additional deflection variables. A consistent quadratic variation of the electric potential across the piezoelectric layers with the provision of satisfying the equipotential condition of electroded surfaces is adopted. A quadrilateral shallow shell element with four kinetic nodes and one electric node is developed. The electric node conveniently enforces the equipotential condition over a number of elements under an electrode, and also leads to significant reduction in the number of electric DOF. In spite of the requirement of C^1 continuity of the ZIGT, interpolation functions very similar to the isoparametric shape functions in C^0 formulations are used for the rotation variables using the improved discrete Kirchhoff quadrilat-

eral technique, which makes it suitable for standard FE programming. The governing equations of motion are derived using the extended Hamilton's principle for piezoelectric solids. For the static analysis, the equations are solved using the direct iteration method. For active vibration control, the FE model is transformed to the reduced order modal space considering the first few modes, and is expressed in the state space format. The resulting nonlinear control problem is solved using the feedback linearization approach.

Since such an element did not exist even for elastic laminated shallow shells, the new element is first critically assessed for its accuracy for static and free vibration responses of elastic laminated shells. This is followed by its assessment for the linear and nonlinear static responses under electro-mechanical loading, free vibration and active vibration control study of smart plates and shallow shell structures. The element has only seven kinematic degrees of freedom per node, and does not need any discretization in the thickness direction. Yet, its predictions are very close to the analytical three-dimensional (3D) piezoelectricity solutions or a detailed 3D FE analysis, for elastic as well as hybrid shells made of not only single-material composite substrates, but also sandwich substrates with soft core for which the equivalent single layer (ESL) theories with the same or higher number of primary variables perform very badly. A separate efficient ZIGT FE model with two kinetic nodes and one electric node is also developed for active vibration control study of smart piezoelectric laminated beams.

The constant gain velocity feedback (CGVF) control, optimal LQR and LQG control as well as clipped optimal control strategies are studied under step, impulse as well as harmonic excitations. Case studies on CGVF control are performed to show the instability phenomena in closed-loop response with conventionally collocated actuator-sensor pairs, and a truly collocated arrangement to remove the instability is illustrated. The effectiveness of directional actuation and sensing capability of piezoelectric composites for vibration suppression is studied. The effect of prescribing a cutoff for actuation potential is studied, and the optimum cutoff voltage which yields the control performance similar to the case of no cutoff voltage is presented. The nonlinear model is validated in comparison with the experimental data available in the literature. The effect of the piezoelectric nonlinearity on the static response and deflection/stress control is studied. Finally, the effect of piezoelectric nonlinearity on the peak control voltage required to achieve a given settling time for the vibration amplitude is studied for smart plates and shells.

Contents

Certificate	i
Acknowledgements	ii
Abstract	iii
Contents	v
List of Figures	xii
List of Tables	xxii
1 Introduction	1
1.1 PREFACE	1
1.2 LITERATURE REVIEW	3
1.2.1 Modeling of Hybrid Beams for Vibration Control	3
1.2.2 Modeling of Hybrid Plates for Vibration Control	5
1.2.3 Modeling of Elastic Laminated Shells	7
1.2.4 Modeling of Hybrid Shells	10
1.2.5 Modeling of Hybrid Shells for Vibration Control	13

1.2.6	Control Algorithms for Vibration Control of Beams	15
1.2.7	Control Algorithms for Vibration Control of Plates	16
1.2.8	Control Algorithms for Vibration Control of Shells	16
1.2.9	Nonlinear Modeling of Piezoelectric Materials	17
1.2.10	Use of PFRC Actuators and Sensors in Smart Structures	20
1.3	OBJECTIVES OF THE PRESENT WORK	21
1.4	ORGANISATION OF THE THESIS	22
2	Active Vibration Control of Smart Beams	27
2.1	INTRODUCTION	27
2.2	EFFICIENT ZIGZAG THEORY FOR SMART COMPOSITE BEAMS	28
2.3	VARIATIONAL PRINCIPLE	33
2.4	BEAM ELEMENT WITH ELECTRIC NODE	35
2.5	ACTIVE VIBRATION CONTROL	38
2.5.1	State Space Representation	39
2.5.2	Feedback Control Laws	40
2.5.3	Optimal Control	41
2.5.4	Clipped Optimal Control	43
2.6	RESULTS AND DISCUSSION	43
2.6.1	Validation	43
2.6.2	Vibration Control of Hybrid Al Beams	45
2.6.3	Direct Feedback Control	47

2.6.4	Optimal Control	53
2.6.5	Vibration Control of Hybrid Sandwich Beams	54
2.6.6	Clipped Optimal Control	56
2.7	CONCLUSIONS	57
3	Nonlinear Efficient Layerwise Finite Element Model for Smart Shallow Shells under Strong Applied Electric Field	59
3.1	INTRODUCTION	59
3.2	NONLINEAR CONSTITUTIVE EQUATIONS FOR PIEZOELECTRIC LAMINA	60
3.3	COUPLED ZIGZAG THEORY APPROXIMATIONS	63
3.4	EXTENDED HAMILTON'S PRINCIPLE	67
3.5	FINITE ELEMENT FORMULATION	72
3.6	SOLUTION OF GOVERNING EQUATIONS	78
3.6.1	Nonlinear Static Response	78
3.6.2	Dynamic Response	78
3.6.3	Nonlinear Active Vibration Control	79
4	Linear Static and Free Vibration Responses of Smart Shallow Shells	82
4.1	INTRODUCTION	82
4.2	STANDARD TEST FOR ELEMENT ACCURACY	82
4.3	STATIC RESPONSE OF ELASTIC SHELLS	83
4.3.1	Composite Cylindrical Shells	84
4.3.2	Composite Spherical Shells	89

4.3.3	Sandwich Cylindrical Shells	89
4.3.4	Shells with Non-Simply Supported Boundary Conditions	93
4.4	FREE VIBRATION RESPONSE OF ELASTIC SHELLS	95
4.4.1	Composite Cylindrical Shells	95
4.4.2	Sandwich Cylindrical Shells	96
4.4.3	Shells with Non-Simply Supported Boundary Conditions	98
4.5	STATIC RESPONSE OF HYBRID SHELLS	99
4.5.1	Validation	101
4.5.2	Hybrid Composite Cylindrical Shell	105
4.5.3	Hybrid Sandwich Cylindrical Shell	109
4.5.4	Hybrid Shells with Non-Simply Supported Boundary Conditions	113
4.6	FREE VIBRATION RESPONSE OF HYBRID SHELLS	116
4.6.1	Validation	116
4.6.2	Hybrid Composite Cylindrical Shell	117
4.6.3	Hybrid Sandwich Cylindrical Shell	118
4.6.4	Hybrid Doubly Curved Shells with Non-Simply Supported Boundary Con- ditions	118
4.7	CONCLUSIONS	121
5	Linear Active Vibration Control of Smart Plates and Shallow Shells	123
5.1	INTRODUCTION	123
5.2	MICROMECHANICS OF PFRC MATERIALS	124
5.3	ACTIVE VIBRATION CONTROL OF SMART PLATES	125

CONTENTS

5.3.1	Validation	125
5.3.2	Control of Hybrid Composite Plate	128
5.3.3	Control of FML Plate Integrated with PFRC Layers	137
5.4	DIRECTIONAL ACTUATION AND SENSING IN SKEW PLATES USING PFRC	141
5.4.1	Closed-Loop Response	142
5.4.2	Optimum PFRC Fiber Orientation	144
5.5	CLIPPED OPTIMAL CONTROL OF SMART PLATES	150
5.6	ACTIVE VIBRATION CONTROL OF SMART SHELLS	153
5.6.1	Validation	153
5.6.2	True Collocation for Stability in CGVF Control	154
5.7	LQG CONTROL OF SHALLOW SHELLS	160
5.7.1	Comparison of Shell Theories for Control Response	163
5.7.2	Effect of a/h and R/a	165
5.7.3	Effect of Fiber Orientation and Fiber Volume Fraction of PFRC	167
5.7.4	Control under Harmonic Excitation	168
5.8	CONCLUSIONS	171
6	Nonlinear Static Response of Smart Plates and Shallow Shells under Strong Electric Field	173
6.1	INTRODUCTION	173
6.2	VALIDATION	174
6.3	NONLINEAR STATIC RESPONSE FOR SMART LAMINATED PLATES	176
6.3.1	Static Shape Control	180

6.3.2	Sensory Response	182
6.4	NONLINEAR STATIC RESPONSE FOR SMART LAMINATED SHALLOW SHELLS	183
6.4.1	Static Shape Control	186
6.4.2	Sensory Response	186
6.5	CONCLUSIONS	189
7	Nonlinear Active Vibration Control of Smart Plates and Shallow Shells under Strong Field Actuation	191
7.1	INTRODUCTION	191
7.2	VIBRATION CONTROL OF SMART PLATES	192
7.3	VIBRATION CONTROL OF SMART SHALLOW SHELLS	197
7.3.1	Control of Bending and Twisting in Smart Angle-ply Composite Shell . . .	197
7.3.2	Control of Sandwich Cylindrical Shell	203
7.4	CONCLUSIONS	206
8	Conclusions	208
8.1	CONCLUSIONS FROM THE PRESENT WORK	208
8.2	FUTURE SCOPE OF WORK	213
	References	215
Appendix A		235
A.1	LINEAR 3D CONSTITUTIVE EQUATIONS	235
A.2	CONSTITUTIVE EQUATIONS FOR 1D MODELS OF BEAMS AND PANELS	237

CONTENTS

Appendix B	239
B.1 DERIVATION OF $\mathbf{R}^k(z)$ MATRIX FOR ZIGT	239
Appendix C	242
C.1 IDKQ INTERPOLATION OF DEFLECTION	242
Appendix D	248
D.1 EFFECTIVE PROPERTIES OF PFRC	248
D.1.1 Model A:	248
D.1.2 Model B:	249
D.1.3 Combined Model	250
Brief Biodata of the Author	251
List of Publications from the Thesis	252