Free Electron Laser Operation in the Whistler Mode

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***Abstract*—The introduction of a strongly magnetized plasma in the inner region of a Free Electron Laser opens up the possibility of generating coherent radiation in the slow whistler mode using mildly relativistic electron beams. The frequency of emission, however, is limited to below the electron cyclotron frequency. The efficiency of the device can be enhanced by tapering the guide magnetic field.**

I. INTRODUCTION

FREE electron laser (FEL) is a fascinating device producing high power coherent radiation at millimeter to optical wavelengths. The frequency of radiation w_L depends upon three parameters [1], [2]: the wiggler period A™, beam velocity v_b and the radiation phase velocity v_{ph} =_c//x where /J, is the refractive index.

$$\omega_L = \frac{2nv_b/X_w}{(l-v_b/v_{ph})'} \quad (1)$$

In a plasma-free FEL, $w_{ph} \simeq c$; hence, $u>_L \simeq 4wy\%c/\backslash_w$ where $-y_o = (1 - i\wedge/c^2)^{"1}/^2$. If one introduces a strongly magnetized plasma in the interaction region, the phase velocity of the right circularly polarized radiation is reduced by a factor /x.

$$M = \left[1 + \frac{\omega_p^2}{\omega_L(J_C - w_L)}\right]^{1/2} \quad (2)$$

where UJ_P and iv_c are the plasma and cyclotron frequencies respectively of the electrons [3]. This significantly diminishes the required beam voltage to generate a particular frequency $U>L$ as long as $CUL < u_c$. One may recall here that a whistler wave can also be excited by an electron beam via Cerenkov interaction, even in the absence of a wiggler, when: (i) the whistler wave has a component of electric vector along the direction of beam propagation (often, such a component is small), (ẁ)t;ph $\simeq$ Þ&. For a mildly relativistic electron beam($i>_b/c \leq 0.5$), the second condition is satisfied when LOL is close to $u>_c$. Such waves may suffer strong cyclotron damping at the throat of the interaction region where the magnetic field is weak [4]. In the presence of a wiggler, the beam is not in phase synchronism with the whistler but with the ponderomotive or space charge wave of frequency $U>L$ and wave vector $(k_L + k_w)$. Hence, $ui_L \approx (k_L + k_w)vb$ where k_L is the wave number of the radiation and k_w is the wiggler wave number. This condition can be satisfied at frequencies well below the cyclotron frequency; hence, the problem of cyclotron damping by the plasma electrons could be less significant. Further, in the FEL process, one does not require a longitudinal field component and the beam-wave coupling is expected to be stronger.

The presence of a plasma in the interaction region helps in the generation of higher powers in several ways. First, it allows beam currents in excess of the vacuum current limit ($\simeq$ 17 KA) via charge and current neutralization [5]. Second, the electron bunching process can be enhanced by tuning the wave frequency close to the plasma frequency. Third, one could employ an externally launched slow electromagnetic wave as a wiggler to operate the device in an explosive mode [6], [7]. Fourth, a density depleted plasma channel provides strong radiation guiding.

Benford *et al.* [8] have carried out extensive experiments on the propagation of high current (/;, $\geq$ 40 KA) relativistic electron beams through dense gases. The beam ionizes the gas forming a plasma channel for it's propagation. It also drives a space charge wave via two stream instability. Under appropriate conditions, the space charge wave may get upconverted to high frequency radiation observed in many experiments. Lalita and Tripathi [9] have explained some of these results invoking the Raman regime FEL mechanism. The role of a plasma in the efficiency enhancement of microwave generation is most spectacularly seen in two experiments. Kuzelev *etal.* [10] have operated a device, viz., a cylindrical waveguide loaded with a thin coaxial plasma shell using a REB to produce 10 G.Hz. radiation with 35% efficiency. Carmel *et al.* [11] have operated a backward wave oscillator at 8.6 G.Hz. using a REB. The efficiency of the device in the absence of a plasma is $\simeq$ 5%. When a plasma is introduced, the efficiency increases with the plasma density attaining a maximum of 40% at $UJ_P \simeq u>_I/2$. At higher densities, the efficiency falls rather sharply.

In this paper, we examine the possibility of operating a FEL in the whistler mode using a static magnetic wiggler and a strong axial guide field. In recent years, a lot of effort has been devoted to the study of electron orbits, radiation spectra and growth rates of FEL's with axial guide fields **[12]—[16].** These studies have shown that operation of FEL's near cyclotron resonance $uj_cho \sim k_o v_b$, where k_o is the wiggler wave number and v_b is the beam velocity, results in an enhancement in the electrons' transverse velocity giving larger growth rate, bandwidth, linear gain and nonlinear saturation efficiency for the FEL instability. The presence of stable and unstable electron orbits has been shown by Freund [13] in

Manuscript received April 6, 1992; revised May 10, 1993. The authors acknowledge the financial support of the C.S.I.R. and the D.S.T., India.

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IEEE Log Number 9400860.

his numerical study of the adiabatic injection of a relativistic electron beam into a combined axial guide field and helical wiggler. He has also observed that in a certain parameter regime, i.e., for certain values of wiggler and axial guide fields and the beam energy, the wave-particle resonance condition in FEL's can be destroyed due to the fluctuations in the axial velocity of the electrons. In other regimes, however, the study shows an exponential growth of the radiation field with substantial enhancements in the efficiency of interaction over the zero-guide-field limits. In their computer simulation of a FEL with a guide field, Kho and Lin [17] have observed that the electron-wave interactions support a distinct hybrid FEL-cyclotron instability having a higher growth rate and efficiency than conventional FEL's. In all these earlier studies, however, the frequency of radiation is much larger than the cyclotron frequency. Hence, the radiation frequency is not significantly affected by the guide field.

Operation of the FEL in the whistler regime restricts it's frequency to below the cyclotron frequency *(u>i < OJ_C)*. However, with existing technology, it is possible to achieve magnetic fields of the order of 10-15 Tesla. The operation frequency of the device can, therefore, be scaled upto 200-250 G.Hz.. The plasma can significantly slow down the radiation mode thereby relaxing the beam energy requirement considerably. Recent studies on gas loaded and plasma loaded FEL's **[18]—[19]** have demonstrated the operation of FEL's at shorter wavelengths for a particular wiggler and beam energy. In their computer simulation of a plasma filled FEL, Wen-Bing *et al.* [19] have observed an enhancement in the gain and frequency of the device in the presence of a high density plasma. They have, however, not considered a guide magnetic field as a result of which the phase velocity of the FEL wave is rather large. Hence, a shorter wiggler period or a higher beam energy is required to generate radiation at a particular frequency.

In Section II of this paper, we study the Compton regime operation of the FEL neglecting kinetic and boundary effects. In Section III, the treatment is generalized to include beam space charge effects. A discussion of the results is given in Section IV.

II. COMPTON REGIME

Consider the interaction region of a FEL with a static magnetic wiggler

$$\vec{B}_w = B_w(\hat{x} + i\hat{y})e^{ik_oz}, \quad (3)$$

a guide magnetic field $B_s\hat{z}$ and a plasma of density n_{op}. An electron beam of density n_ob and velocity $v_b\hat{z}$ propagates through the interaction region. To a relativistic electron, the wiggler field appears as an electromagnetic wave with frequency i_ok_ovb- The guide magnetic field imparts to the electrons, cyclotron motion at a frequency u_c. On solving the equation of motion for the electrons, the equation being the same as that of a resonant oscillator at frequency w_c driven at the frequency "$/_ok_ovb$, under the assumption that there is no radiation by the electrons under the effect of the magnetostatic fields, the transverse wiggle velocity is obtained as

$$\vec{v}_{o\pm} = \frac{e\vec{B}_{w..} \quad Vb}{\cdot y_o mc(k_ov_b + Ucho)} \quad (4)$$

where m is the electron rest mass, 7_O is the relativistic gamma factor and $LO_C/\wedge_0$ is the electron cyclotron frequency where $oj_c = eB_o/mc$.

Under the influence of a perturbing electromagnetic whistler mode (u>i,fci);

$$\vec{E}_1 = A_\pm(\hat{x} + i\hat{y})e^{-i(\omega_1 t - k_1 z)}$$
$$\vec{B}_1 = c\frac{\vec{k}_1 \times \vec{E}_1}{\omega_1}, k_1 = \frac{\omega_1}{c}\mu_1 \quad (5)$$

where $u\backslash = \left[1 + \frac{\omega_p^2}{\omega_1(\omega_c - \omega_1)}\right]^{1/2}$ is the refractive index of the whistler mode, the electrons acquire a transverse velocity

$$v_{1\perp} - \frac{k_1 v_b/\omega_1) \quad e\vec{E}_1}{(\omega_1 - k_1 v_b - \omega_c/\gamma_o)\text{im}\gamma_o}. \quad (6)$$

The beating of the whistler and wiggler fields exerts a ponderomotive force on the electrons

$$\vec{f}_1 = "\frac{e}{S}<\vec{v}_o^* \times \vec{B}_1 + \vec{v}_1 \times \vec{B}^*_w) = iek<f>_p\hat{z} \quad (7)$$

where

$$\phi_p = \frac{eB_wAiA}{imck\gamma_o\omega_1}e^{-i(\omega_1 t - kz)}, \quad (8)$$

$k = ki - k_0$ is the wave number of the ponderomotive wave, $k_o = -|k_o|$ because to an electron moving with a velocity $v\backslash\hat{z}$, the static magnetic wiggler appears as a backward propagating electromagnetic wave having wave vector $- \mid k_o \mid \hat{z}$, and

$$A = \left[\frac{\omega_1 - k_1 v_b}{(\omega_1 - k_1 v_b - LJ_c/"/_o)} - \frac{k\backslash Vb}{(k_0V_b + Uc/l_o)}\right]. \quad (9)$$

Resonant interaction of the beam with the ponderomotive wave requires $uJi/(ki + |k_o|) = v_b$ which can be satisfied here permitting FEL interaction to take place. Under the influence of the ponderomotive force, the electrons acquire an axial velocity

$$V2z = \sim\frac{ek(f>_p}{m\gamma_o^3(\omega_1 - kv_b)}. \quad (10)$$

The resulting density perturbation, obtained by solving the equation of continuity

$$\vec{\nabla} \cdot \vec{J} + dp/dt = 0 \quad (11)$$

where $\vec{J}$ is the electron current density and p is the charge density, is given as

$$n_{2b} = -\frac{n_{ob}ek^2\phi_p}{m\gamma_o^3(\omega_1 - kv_b)^2}. \quad (12)$$

The oscillatory axial velocity $V2_Z$ interacts with the wiggler field $\vec{B}_w$ generating a nonlinear force $-(e/2c)v_{2z}\hat{z} \times \vec{B}_w$. The electrons' response to this force is given as

$$\vec{v}_{2\perp} = -\frac{e}{27_o mc}\frac{v_{bz}\vec{B}_u}{(\omega_1 - \omega_c/\gamma_o)}. \quad (13)$$

The total transverse beam current density driving the whistler radiation at (ω_1, k_1) is given by

$$\vec{J}_{1\perp} = -n_{ob} e \vec{v}_{1\perp} - \frac{1}{2} n_{2b} e \vec{v}_{o\perp} - n_{ob} e \vec{v}_{2\perp}$$
$$= \frac{i n_{ob} e^2(\omega_1 - k_1 v_b)}{\gamma_o m \omega_1(\omega_1 - k_1 v_b - \omega_c/\gamma_o)} \vec{E}_{1\perp} - \frac{iA \quad n_{ob} e^2 k v_b \omega_{c}^2 A_1(\hat{x} + i\hat{y})}{2\gamma_0^5 m \omega_1(\omega_1 - kv_b)^2 (k_o v_b + \omega_c/\gamma_o)} + \frac{e^2 \omega_1}{2c\ k} \frac{n_{2b} \vec{B}_w}{\gamma_o m(\omega_1 - \omega_c/\gamma_o)} \tag{14}$$

where $\omega_{co} = e B_w/mc$.

The contribution of the background plasma to the current density is obtained from $\vec{J}_{1\perp}$ by considering $\gamma_o = 1, v_b = 0$ and by replacing n_{ob} and n_{2b} by n_{op} and n_{2p} respectively.

$$\vec{J}_{1p\perp} = \frac{i n_{op} e^2}{m(\omega_1 - \omega_c)} \vec{E}_{1\perp} + \frac{e^2}{2c} \frac{\omega_1 \vec{B}_w n_{2p}}{km(\omega_1 - \omega_c)} \tag{15}$$

where

$$n_{2p} = -\frac{n_{op} e k^2}{m\omega_p^2} \phi_{pp} \quad \text{and} \quad \phi_{pp} = \frac{eB_w}{(\ldots \omega_c)} E_{1\perp}. \tag{16}$$

Using Eqns. (14) and (15) in the wave equation:

$$(d^2/dz^2 + \omega_1^2/c^2)\vec{E}_{1\perp} = -(4\pi i \omega_1/c^2)\vec{J}_{1T}, \tag{17}$$

where $\vec{J}_{1T} = \vec{J}_{1\perp} + \vec{J}_{1p\perp}$, we obtain the nonlinear (NL) dispersion relation:

$$D \cdot (\omega_1 - kv_b)^2 = R \tag{18}$$

where

$$D = \omega_1^2 - c^2 k_1^2 - \frac{\omega_{pb}^2(\omega_1 - k_1 v_b)}{\gamma_o(\omega_1 - k_1 v_b - \omega_c/\gamma_o)} + \frac{\omega_p^2 \omega_1}{(\omega_1 - \omega_c)} \tag{19}$$

and

$$R = \frac{\omega_{co}^2}{2} \left[\frac{\omega_p^2(\omega_1 - kv_b)^2}{(\omega_1 - \omega_c)^2} + \frac{\omega_{pb}^2 A}{\gamma_o^5} \left[\frac{\omega_1}{(\omega_1 - \omega_c/\gamma_o)} - \frac{kv_b}{(k_o v_b + \omega_c/\gamma_o)} \right] \right] \tag{20}$$

The simultaneous zeros $\omega_1 = \omega_{1r}$ of the left hand side of (18) give the frequency of operation of the FEL:

$$D = 0.$$
$$\omega_{1r} = k \cdot v_b. \tag{21}$$

As shown in Fig. 1, one can plot ω_1/ω_p versus kv_b/ω_p for the beam mode and ω_1/ω_p versus $k_1 v_b/\omega_p$ for the radiation mode. A horizontal line with a length intercept $v_b|k_o|/\omega_p$ between the two dispersion curves and intersecting the ω_1/ω_p axis gives the frequency of operation of the device.

To determine the growth rate of the FEL instability, we use the first order perturbation technique. In the absence of the right hand side terms, (18) reduces to

$$D \cdot (\omega_1 - kv_b) = 0. \tag{22}$$

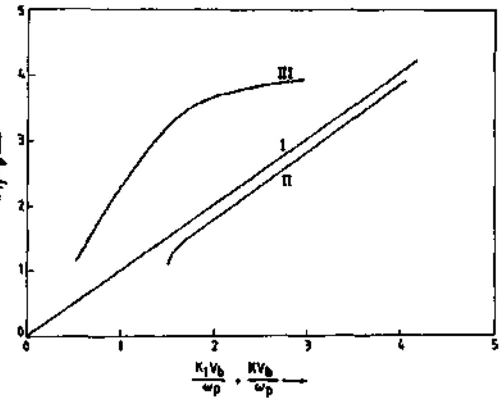

Fig. 1. Normalized dispersion curves for the radiation and beam modes for the parameters: $\omega_{pb}/\omega_p = 0.2, \omega_c/\omega_p = 4$ and $v_b/c = 0.4$. Curves I and II represent the beam mode $(\omega_1/\omega_p \text{ vs } kv_b/\omega_p)$ in the Compton regime and Raman regime respectively and curve III represents the Radiation mode $(\omega_1/\omega_p \text{ vs } k_1 v_b/\omega_p)$.

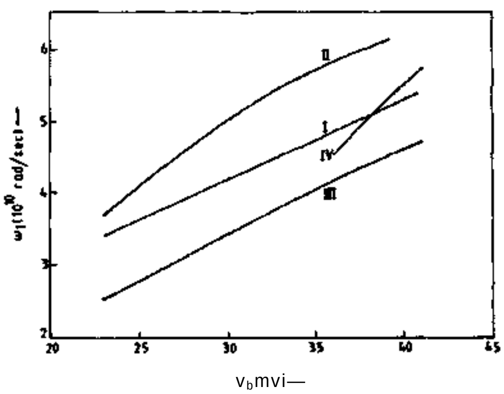

Fig. 2. Frequency ω_1 vs beam voltage V_b for the parameters $\omega_{pb}/\omega_p = 0.2, \omega_c/\omega_p = 4$ and $k_o = 2.5\,\text{cm}^{-1}$. Curves I and II represent Compton regime operation with $\omega_p = 2 \times 10^{10}$ rad/sec and $\omega_p = 3.5 \times 10^{10}$ rad/sec respectively. Curves III and IV represent Raman regime operation for the same plasma frequencies.

Let this represent the unperturbed state with eigen function E_n and eigen frequency ω_{1r}. In the presence of the right hand side terms, we assume that the eigen functions are not modified but their eigen values are. On substituting $\omega_1 = \omega_{1r} + i\Gamma$ into (18), Γ being the growth rate of the instability, and solving for Γ we obtain:

$$\Gamma = R^{1/3} \times \left[2\omega_1 + \frac{\omega_{pb}^2 \omega_c}{\gamma_o^2(\omega_1 - k_1 v_b - \omega_c/\gamma_o)^2} - \frac{\omega_p^2 \omega_c}{(\omega_1 - \omega_c)^2} \bigg|_{\omega_1 = \omega_{1r}} \right]^{-1/3} \times \sin\frac{2\pi}{3}. \tag{23}$$

A. *Gain Estimate*

The FEL instability may saturate via the trapping of the electrons in the ponderomotive wave. Under the influence of

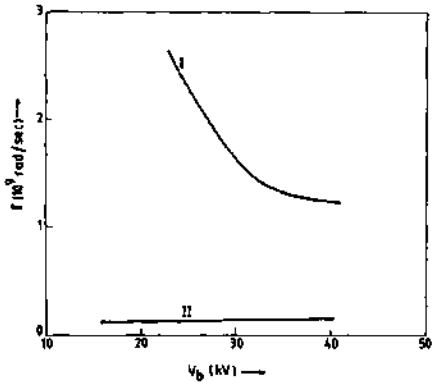

Fig. 3. Growth rate *Tvs* beam voltage Vj, for the parameters *Upb/u$_p$ = 0.2,UJ$_C$/U$_P$ = 4,LJ$_P$* = 2 x 10^{10}rad/sec and k_o = 2.5 cm"[1]. Curves I and II represent Raman regime and Compton regime operations respectively.

the electric field E_p of the ponderomotive wave:

$$E_p = -F_p/e = \frac{eB_w A_1 A}{\gamma_o mc\omega_1} e^{-i(\omega_1 t - kz)}, \quad (24)$$

the single particle equation of motion for the electron is

$$\frac{d}{dt}(\gamma_e) = -\frac{eE_p}{m}\cos(\omega_1 t - kz). \quad (25)$$

where γ_e is the gamma factor of an electron. For $v \simeq$ c. (25) reduces to

$$\frac{d}{dz}(\gamma_e) = -\frac{eE_p}{mc^2}\cos(\omega_1 t - kz). \quad (26)$$

Defining variables $\Delta\gamma_e = \gamma_e - \gamma_r$ and $\psi = kz - \omega_1 t$, where $\gamma_r = (1 - v^2_{ph}/c^2 - \omega^2/\omega^2)^{-1/2}$ is the gamma factor of an electron moving with the phase velocity of the ponderomotive wave and ψ is the phase of the wave as seen by the electron, (26) modifies to

$$\frac{d\Delta\gamma_e}{dz} = -\frac{eE_p}{mc^2}\cos\psi. \quad (27)$$

Also,

$$\frac{d\psi}{dz} = k - \omega_1/v_z \simeq \frac{\omega_1 \Delta\gamma_e}{2c(\gamma_r^2 - 1)^{3/2}}. \quad (28)$$

Equations (27) and (28) combine to give the well known pendulum equation for the electrons moving under the influence of a spatially periodic ponderomotive potential [1,2]:

$$\frac{d^2\psi}{dz^2} = -\Omega_L^2 \cos\psi \quad (29)$$

where $\Omega_L^2 = \frac{e E_p \omega_1}{2mc^3(\gamma_r^2 - 1)^{3/2}}$. An electron trapped in the potential well executes nearly harmonic motion at a frequency Ω_L. Dimensionalizing z by the length of the interaction region L, Eqs. (27) and (28) give

$$\frac{d\psi}{d\xi} = P \quad (30)$$

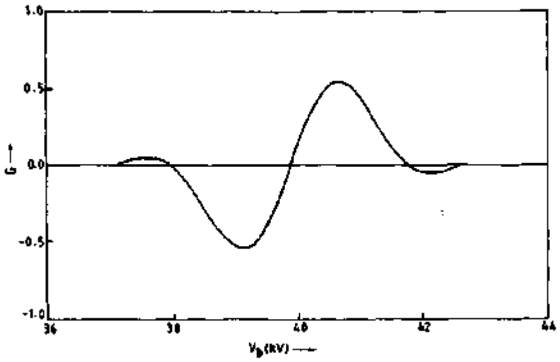

Fig. 4. Gain *Gvs* beam voltageVj, for the parameters ω_1 = 6.2 x 10^{10}rad/sec and L = 40 cms.

and

$$\frac{dP}{d\xi} = -A\cos\psi \quad (31)$$

where $P = \frac{L\omega_1 \Delta\gamma_e}{2c(\gamma_r^2 - 1)^{3/2}}$ and $A = \frac{L^2\omega_1 eE_p}{2mc^3(\gamma_r^2 - 1)^{3/2}}$.

In the small signal approximation ($A \simeq$ const), the above equations can be solved to obtain the phase space trajectories of the electrons.

$$P^2 = P_{in}^2 - 2A(\sin\psi - \sin\psi_{in}) \quad (32)$$

where $P_{in} = P_{\xi=0}$ and $\psi_{in} = \psi_{\xi=0}$. The gain 'G' can be determined by solving Eqs. (30) and (31) by expanding P and ψ to different orders in A:

$$\begin{aligned}\langle \Delta P\rangle &= \langle -(P_1 + P_2)\rangle_{\xi=1} \\ &= \frac{A^2 G}{8} \\ &= \frac{A^2}{P_{in}^3}\left[1 - \cos P_{in} - \frac{P_{in}}{2}\sin P_{in}\right] \quad (33)\end{aligned}$$

Fig. 4 shows the variation of the gain G with the beam voltage for the parameters ω_1 = 6.2 x 10^{10} rad/sec, k_1 = 2.375 cm"[1] and ω_p = 1.9 x 10^{10} rad/sec. For these parameters and for beam voltage $V_b \geq$ 40 KV, there is a net transfer of energy from the beam electrons to the ponderomotive wave.

The electrons can transfer energy to the wave as long as $\gamma_e > \gamma_r$ and the efficiency of energy exchange is given by $\eta_o = \frac{\Delta\gamma_e}{\gamma_e - 1}$. The energy exchange efficiency can be significantly enhanced by slowing down the ponderomotive wave adiabatically along the length of the interaction region. The enhancement in the exchange efficiency is given by

$$\eta_1 = \frac{\gamma_{ro} - \gamma_{r1}}{\gamma_{ro} - 1} \quad (34)$$

where γ_{ro} and γ_{r1} are the resonant energies of the electrons at $\xi = 0$ and $\xi = 1$ respectively. The total efficiency $\eta = \eta_o + \eta_1$. For operation at v = 10 G.Hz., $v_b/c \ll 1$ and (34) reduces to

$$\eta_1 = \frac{v_{ro}^2 - v_{r1}^2}{v_{ro}^2} = 1 - \frac{\mu_o^2}{\mu_1^2} \quad (35)$$

where v_{ro} and v_{r1} are the resonant velocities at $\xi = 0$ and $\xi = 1$ respectively and μ_0 and μ_1 are the refractive indices at

$\ell = 0$ and $\ell = 1$ respectively. Apart from tapering the wiggler field, the slowing down of the ponderomotive wave can also be brought about by tapering the guide magnetic field (i.e. changing ω_c) or by changing the plasma frequency ω_p along the interaction region.

If ω_c^o and ω_c^1 are the cyclotron frequencies at $\ell = 0$ and $\ell = 1$ respectively, then for a fixed plasma frequency and frequency of operation, (35) reduces to:

$$\eta_1 = \left(1 - \frac{\omega_c^1}{\omega_c^o}\right) \Big/ \frac{\omega_1}{\omega_c^o}\left(\frac{(\omega_c^1 - \omega_1)(\omega_c^o - \omega_1)}{\omega_p^2} + \frac{(\omega_c^o - \omega_1)}{\omega_1}\right) \tag{36}$$

If the ponderomotive wave is slowed down by varying the plasma frequency along the interaction region, then, (36) reduces to:

$$\eta_1 = \frac{\omega_{p0}^2 - \omega_{p1}^2}{\omega_1(\omega_c - \omega_1) + \omega_{p1}^2} \tag{37}$$

where ω_{po} and ω_{p1} are the plasma frequencies at $\ell = 0$ and $\ell = 1$ respectively.

III. A RAMAN REGIME OPERATION

At high beam currents, the electrons experience a self-consistent space charge potential $\phi e^{-i(\omega_1 t - kz)}$ apart from the ponderomotive potential. The electrons' response to these potentials can be obtained from Eqns. (10) and (12) on replacing ϕ_p by $\phi + \phi_p$ as:

$$v_{2z} = \frac{-ek}{m\gamma_o^3(\omega_1 - kv_b)}(\phi + \phi_p) \tag{38}$$

$$n_{2b} = \frac{-n_{ob}ek^2}{m\gamma_o^3(\omega_1 - kv_b)^2}(\phi + \phi_p) \tag{39}$$

The nonlinear current density at ω_1, $\vec{k}_1$ can be written as

$$\vec{J}_{1\perp} = \frac{in_{ob}e^2(\omega_1 - k_1v_b)}{\omega_1 m\gamma_o(\omega_1 - k_1v_b - \omega_c/\gamma_o)}\vec{E}_{1\perp} + \frac{k^2\phi\omega_{pb}^2\vec{v}_{o\perp}}{8\pi\gamma_o^3(\omega_1 - kv_b)^2} + \frac{e^2\omega_1 n_{2b}\vec{B}_w}{2ck\gamma_o m(\omega_1 - \omega_c/\gamma_o)} \tag{40}$$

where $\vec{v}_{o\perp}$ is the same as in (4).

The nonlinear current of the background plasma can be obtained from (40) by taking $\gamma_o = 1$, $v_b = 0$ and on replacing n_{ob} and n_{2b} by n_{op} and n_{2p} respectively.

$$\vec{J}_{1p\perp} = \frac{in_{op}e^2}{m(\omega_1 - \omega_c)}\vec{E}_{1\perp} + \frac{e^2\omega_1 n_{2p}}{2ckm(\omega_1 - \omega_c)}\vec{B}_w \tag{41}$$

where

$$n_{2p} = \frac{-n_{op}ek^2}{m\omega_1^2}(\phi + \phi_{PP}). \tag{42}$$

Using the total current density $\vec{J}_{1\perp} = \vec{J}_{1\perp} + \vec{J}_{1p\perp}$ in the wave equation (i.e. in (17)), we obtain:

$$D_1 \cdot E_{1\perp} = R_1 \tag{43}$$

where $D_1 = \omega_1^2 - c^2k_1^2 - \frac{\omega_{pb}^2(\omega_1 - k_1v_b)}{\gamma_o(\omega_1 - k_1v_b - \omega_c/\gamma_o)} - \frac{\omega_p^2\omega_1}{(\omega_1 - \omega_c)}$, $R_1 = \frac{-i\omega_{co}\omega_1 k^2\phi}{2}\left[\frac{-\kappa_b v_b}{\gamma_o(k_o v_b + \omega_c/\gamma_o)} + \frac{\omega_1\kappa_p}{k(\omega_1 - \omega_c)} + \frac{\omega_1\kappa_b}{k\gamma_o(\omega_1 - \omega_c/\gamma_o)}\right]$

$\kappa_b = -\omega^2 \ldots$ and $\kappa_p = \ldots$; the values of n_{2b} and n_{2p} from (39) and (42) in the Poisson's equation, we obtain:

$$\varepsilon_R\phi = -\frac{\omega_{co}}{ik}\left[\frac{\kappa_b}{\gamma_o\omega_1}\left(\frac{\omega_1 - k_1v_b}{(\omega_1 - k_1v_b - \omega_c/\gamma_o)} - \frac{k_1v_b}{(k_ov_b + \omega_c/\gamma_o)}\right) + \frac{\kappa_p}{(\omega_1 - \omega_c)}\right]E_{1\perp} \tag{44}$$

where $\varepsilon_R = 1 - \frac{\omega_p^2}{\omega_1^2} - \frac{\omega_{pb}^2}{\gamma_o^3(\omega_1 - kv_b)^2}$. From (43) and (44), we obtain:

$$D_1\varepsilon_R = Q \tag{45}$$

where $Q = \frac{\omega_{co}^2\omega_1 k}{2}\left[\frac{\kappa_b}{\gamma_o\omega_1}\left(\frac{\omega_1 - k_1v_b}{(\omega_1 - k_1v_b - \omega_c/\gamma_o)} - \frac{k_1v_b}{(k_ov_b + \omega_c/\gamma_o)}\right) + \frac{\kappa_p}{(\omega_1 - \omega_c)}\right]\left[\frac{\omega_1\kappa_p}{k(\omega_1 - \omega_c)} - \frac{\kappa_b v_b}{\gamma_o(k_ov_b + \omega_c/\gamma_o)} + \frac{\omega_1\kappa_b}{\gamma_o k(\omega_1 - \omega_c/\gamma_o)}\right]$

The simultaneous zeros of the left hand side of (45) give the frequency of operation of the device.

$$\varepsilon_R = 0, D_1 = 0. \tag{46}$$

The frequency of operation can be determined following the same procedure as in the case of Compton regime operation. Considering a wiggler of wave number $k_0 \simeq 3\ \mathrm{cm}^{-1}$ and a plasma frequency $\omega_p \sim 1.9 \times 10^{10}$ rad/sec, the frequency of operation as designated by the dispersion curves in Fig. 1 for a beam of velocity $v_b = 0.4c$ is $\omega_{1r} = 6.2 \times 10^{10}$ rad/sec.

The growth rate of the FEL in the Raman regime is given by:

$$\Gamma = \left[-Q\left(\frac{\partial\varepsilon_R}{\partial\omega_1}\cdot\frac{\partial D_1}{\partial\omega_1}\right)^{-1}_{\omega_1 = \omega_{1r}}\right]^{1/2} \tag{47}$$

where ω_{1r} is the solution of (45).

A. *Plasma Resonance*

There exists an exciting possibility of operating the device close to plasma resonance:

$$\omega_1 \simeq \omega_p = kv_b = \omega_a \tag{48}$$

Where ω_a is the root of (46). Around $\omega_1 = \omega_p$, the plasma nonlinearity exceeds the beam nonlinearity. Following the treatment of Tripathi and Liu [21], on substituting $\omega_1 = \omega + \delta = \omega_a + \delta = kv_b + \delta$ in (45), we obtain:

$$\delta^3 - \frac{Z\omega_p^2}{2}\delta - \frac{\omega_{pb}^2\omega_p}{2\gamma_0^3} = 0 \tag{49}$$

Where $Z = \frac{\omega_{co}^2}{4(\omega_c - \omega_p)^2}$. This equation can be solved analytically for δ and the growth rate $\Gamma = Im(\delta)$ can be obtained for operation near plasma resonance.

IV. RESULTS AND DISCUSSION

In Fig. 2, we have plotted the frequency of FEL emission as a function of the beam voltage for the following parameters: $\omega_{pb}/\omega_p = 0.2$, $\omega_c/\omega_p = 4$, $v_b/c = 0.4$ and $k_o = 3\ \mathrm{cm}^{-1}$. Here, a word about the choice of the ratio ω_{pb}/ω_p. When the beam density is greater than or comparable to the density of the background plasma, the phenomena of charge neutralization and the formation of an optical waveguide takes place which can guide the radiation signal. At very high beam densities, however, the device is almost similar to a vacuum FEL. When the density of the background plasma is much higher than the beam density in the presence of a strong guide field, the plasma effects are more pronounced and the radiation signal (FEL wave) is slowed down quite considerably. The growth rate of the instability is also enhanced. The emphasis in this study is on operation of the device in this regime to generate radiation at a frequency $u>\backslash \simeq$ 10 G.Hz. using mildly relativistic electron beams. The frequency of radiation is higher in the Compton regime than in the collective Raman regime (c.f. Fig. 2). The growth rate, however, is larger in the Raman regime than in the Compton regime (c.f. Fig. 3). As the beam voltage is increased, radiation occurs at a higher frequency. In the Raman regime, the growth rate decreases as the operation frequency increases, while it remains virtually unaffected in the Compton regime. It can be noted from Fig. 2 that the wave frequency is enhanced by increasing the plasma density. Although the frequency can also be enhanced by using more energetic electron beams or by shortening the wiggler period, very high beam energies are not easily accessible and very small wiggler periods are not very practicable using magnetic wigglers. Thus, the plasma density can help in tuning the FEL to higher frequencies.

Fig. 4 shows the variation of the gain function G with the beam voltage in the Compton regime. For a beam voltage $V_b \geq$ 40 KV, there is a net transfer of energy from the beam electrons to the ponderomotive wave. Slowing down the ponderomotive wave adiabatically along the interaction region enhances the efficiency of energy transfer quite significantly. When the tapering is brought about by varying the guide magnetic field, it is observed that a reduction of about 10% along the interaction region enhances the energy transfer efficiency by about 17%. If the tapering is brought about by varying the plasma density, then, a change of 20% in the plasma density results in an efficiency enhancement by about 5%. However, the axial inhomogeneity in the guide magnetic field in the interaction region,introduced by the tapering, can influence beam dynamics a little but may not be detrimental to the FEL instability.

The operation of the device at plasma resonance can greatly enhance the electron bunching process resulting in a larger growth rate for the instability. For $Vf,$ = 0.4c, wi $\simeq u_p$ = 6 x 10^{10}rad/sec the B_s required to maintain triple resonance (c.f. (48)) is B_s 3*: 9.5 KG and the growth rate T of the instability is 7.8 x 10^{10}rad/sec which is about 4—5 times the growth rate away from plasma resonance. Fig. 5 shows the variation of the frequency of operation and the growth rate of the instability, at plasma resonance, with the cyclotron frequency. As the cyclotron frequency is enhanced, the growth rate of the instability increases very slowly but the frequency satisfying the triple resonance condition (48) falls quite significantly. Hence, variation in the guide magnetic field at plasma resonance has only a marginal effect on the growth rate of the instability.

The instabilities inherently supported by a plasma generate a lot of undesired noise in the output of the FEL. Although the

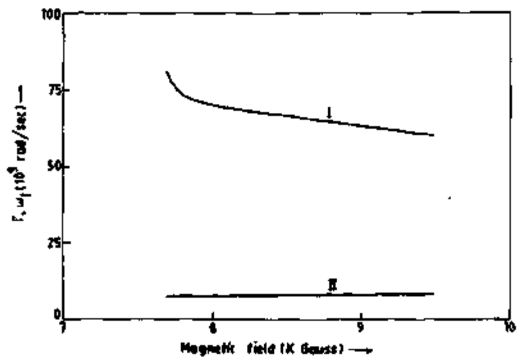

Fig. 5. Growth rate T and frequency uii vs guide magnetic field$B_s$at plasma resonance for the parameters u;, = 0.4c, to_pb = 6x 10^9rad/sec and k_o = 2.5 cm"^{1}. Curve I represents $w\backslash vsB_s$ and curve II represents $TvsB_o$.

two stream instability can be helpful in enhancing the growth rate of the FEL instability at plasma resonance, the plasma return current can drive ion acoustic waves unstable over a long time scale and this may limit the operation of the device to a pulsed mode. When the oscillatory velocity of the electrons is of the order of the thermal velocity, parametric processes become important and the device parameters need to be tuned such that the growth rate of these processes is very small.

In conclusion, a plasma loaded FEL operating in the slow whistler mode can be an efficient device for generating radiation in the 10 G.Hz. band using mildly relativistic electron beams. The FEL instability does not require an axial component of the electric field as is necessary in the conventional Cerenkov process. The advantage of a plasma filled FEL with a guide field over a conventional FEL is most pronounced in the high power outputs because of the higher beam currents that can be propagated. Further, as in FEL's using a guide field **[16]—[18],** recent results on plasma filled FEL's have also shown an enhancement in the growth rate and efficiency of the device over conventional FEL's [19]-[20]. Apart from this, the presence of a plasma also helps in generating radiation at shorter wavelengths for a particular beam energy and wiggler and in lowering the beam energy requirement for radiation at a particular frequency. The plasma density can serve as a mechanism for tuning the frequency of the device.

A plasma filled FEL may not compete with a gyrotron which is shown to possess a high efficiency **[21]—[23].** However, in the FEL configuration, one does not require transverse beam energy which is a significant advantage over a gyrotron. Further, factors like frequency tuning, beam energy relaxation and higher beam currents contribute in favor of a plasma filled FEL.

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