A DECENTRALISED VARIABLE STRUCTURE MODEL FOLLOWING CONTROLLER FOR ROBOT MANIPULATORS

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ABSTRACT
A new algorithm for the trajectory control of robot manipulators by decentralised feedback is proposed, by dealing with the manipulator and joint actuator dynamic models in a model following framework. A variable structure control law is used with or without force measurement. The controller is simple, computationally easy, and robust to parametric uncertainties and payload variations.

INTRODUCTION
A well-known approach for the control of interconnected systems is to consider the interactions as disturbances, and to reject them by disturbance rejection methods. In this paper we develop a new control algorithm for stabilisation of interconnected systems by decentralised feedback, based on the above approach, and apply it to the trajectory control of robot manipulators.

We treat the stable (or locally stabilised) isolated subsystem as a 'model' for the interconnected subsystem, so that the stabilisation of the latter becomes a problem of model following. Incorporation of the model following behaviour requires an adaptive control mechanism, and we make use of the variable structure control (VSC) approach for this purpose. Controller design is proposed for cases with and without force/torque measurement.

The proposed controller is simple, computationally easy, and robust to parametric changes and external disturbances. It compares favourably with some of the recently proposed control methods based on model following and/or variable structure control approaches.

CONTROL OF ROBOT MANIPULATORS
The complexity of the robot control problem arises due to the nonlinear dependence of system parameters on variables such as displacement and velocity, on the geometry and inertia of the links, uncertainties associated with gravity, Coriolis and centrifugal forces, variations in payload handled by the manipulator, and environmental influences.

Non-adaptive control techniques such as optimal control and computed torque methods have serious drawbacks, namely complex controller structure, excessive on-line computation, and sensitivity to non-linearities and uncertainties in the system. These difficulties are compounded further by their being centralised approaches. Adaptive control methods tackle the robot control problem quite effectively, but this comes at the cost of complexity of the controller. In much of the works on adaptive control of manipulators, the justification for this added complexity is not addressed.

Formulating the variable structure control problem in a decentralised model following framework will help overcome many of the difficulties encountered in controlling manipulators by the above methods. The VSC approach, non-adaptive, but exhibits the model following capabilities of model reference adaptive techniques, with its gains being discontinuous across a specified switching plane. Once the system is in the sliding mode, it is insensitive to parametric variations and disturbances.

THE PROPOSED CONTROLLER
The above decentralised control problem is simplified, if we deal with the dynamic models of the joint actuators along with that of the manipulator.

Let the dynamics of the actuator associated with i-th link in an N-link manipulator be given by the linear, time-invariant system:

\[ \dot{x}_i = A_{ii} x_i + b_{ii} u_i + d_i f_i \]

where \( x_i \) is an \( n_i \)-vector, and \( u_i \) is a scalar control. Neglecting the actuator friction torque, \( f_i \) is the driving torque acting on the actuator and is given by the nonlinear, uncertain dynamics of the manipulator:

\[ f_i = H_i(q,d) \dot{q}_i + h_i(q, \dot{q}_i, d) \]

where \( q = (q_1, q_2, ..., q_N) \) is the N-vector of joint angles or displacements and \( d \) is an \( J \)-vector of parameters such as link masses and inertias. \( H_i \) is the N-vector of inertia and \( h_i \) represents Coriolis, centrifugal and gravity forces. In each \( x_i \), two coordinates coincide with \( q_i \) and \( \dot{q}_i \).

In trajectory tracking applications, the control task is to make the system track the desired or nominal trajectories \( (q_1(t), q_2(t), ..., q_N(t)) \) over \( t \in [0,T] \). The nominal
velocity trajectories \( \dot{\mathbf{q}}_i(t), \dot{\mathbf{q}}_2(t), \ldots, \dot{\mathbf{q}}_N(t) \) readily obtained. From (3) below, we also specify nominal trajectories for the remaining variables in \( x_i \), \( i=1,2,\ldots, N \).

For example, for a dc motor actuator, as used in many industrial robots, \( n = 3 \) and the third coordinate is the armature current. Let \( x_{n1}(t) \) be the corresponding nominal state trajectory.

We assume that there exists a nominal control \( u_i(t), t \in [0,T] \), which forces the isolated subsystem in (f) to track the nominal state trajectory, i.e.,

\[
\dot{x}_i(t) = A_i x_i(t) + b_i u_i(t) + \mathbf{f}_i(t)
\]

(3)

The error in tracking is \( e_i(t) = x_i(t) - x_{n1}(t) \).

From (1) and (3), the dynamics of the tracking error is given by

\[
\dot{e}_i = A_i e_i + b_i u_i + \mathbf{f}_i
\]

(4)

where \( \mathbf{f}_i = u_i - u^\text{nom} \).

The control \( u_i = u_i^\text{nom} + u_i^\text{corr} \) thus has two components: \( u_i^\text{nom} \) for tracking the nominal trajectories in the absence of uncertain dynamics, and \( u_i^\text{corr} \) for compensating the effects of nonzero initial error \( e_i(t=0) \) and the effects of interlink coupling torques, Coriolis, centrifugal and gravity forces and of payload variations.

The trajectory control problem is solved by choosing a control law for \( u_i \) in (4), such that \( e_i \) tends to zero asymptotically. Two solutions to this problem are proposed below using the VSC approach.

a) Control with Force Feedback

By incorporating force sensors to feedback joint forces, \( \mathbf{f}(x) \) in (4) can be treated as a measurable disturbance. Accordingly the variable structure controller takes the form

\[
u_i = u_i^\text{nom} + u_i^\text{corr}
\]

(5)

where the discontinuous error feedback is

\[
& e_i = -S_j^i e_j, \quad j \neq i
\]

(6)

and the discontinuous force feedback is

\[
\mathbf{f}_i^\text{corr} = -\mathbf{f}_i
\]

(7)

and the gains are switched across the sliding plane defined by

\[
s_i = c_i^\top e_i = 0
\]

(8)

according to

\[
\epsilon_i = \begin{cases} 
\ldots, e_i, s_i, \ast = 0 \\
\ldots, e_i, s_i, \ast \neq 0
\end{cases}
\]

and

\[
\ast = c_i^\top e_i
\]

(9)

The condition for existence of a sliding mode on (11) is given by

\[
\dot{s}_i = c_i^\top e_i > 0, \quad s_i = 0
\]

(10)

Starting with \( s_i(t=0) = 0 \), the error trajectory will reach the sliding plane, if the condition (10) is satisfied in the entire error space. Then, from (4) and (11) we have,

\[
\dot{s}_i = c_i^\top e_i = \lambda_i^\text{nom} (A_i e_i + b_i u_i)
\]

Substituting for \( u_i \) from (9), and rearranging,

\[
\dot{s}_i = c_i^\top \left( \lambda_i^\text{nom} A_i e_i + b_i u_i^\text{nom} + c_i d_i \mathbf{f}_i + c_i^2 b_j y_j \text{sgn} (s_j) \right)
\]

(11)

The constant \( n_i \)-vector \( c_i \) is chosen to specify the properties of sliding mode and the gains \( \lambda_i^\text{nom}, \lambda_i^\text{nom} \), and \( \lambda_i^\text{nom} \) are chosen to ensure the existence and reachability of the sliding mode ^\text{\textsuperscript{c}}. The force feedback scheme suffers from the drawbacks of high cost of precision sensors and introduction of noise in the transducers. Where these considerations are overriding, the following controller configuration may be adopted.

b) Control with Relay Gain

In (4), we note that the 'disturbance' \( \mathbf{f}_i \) is a global state- and parameter-dependent nonlinearity, and so is difficult to compensate by simple linear continuous or discontinuous state feedback. Hence, for simplicity, we treat \( \mathbf{f}(x,d) \) as a piecewise-continuous, time-variant, bounded disturbance \( \mathbf{f}(t) \) bounded by, say, \( | \mathbf{f}(t) | \leq F \), where \( F \) is the (absolute) maximum torque level specified for the joint actuator.

The assumption of bounded disturbance would be justified if the state \( x_i \), hence the error \( e_i \), is shown to be bounded. For the VSC methodology, if the conditions for existence and stability of sliding mode are satisfied, error boundedness follows.

To compensate for the disturbance \( \mathbf{f}_i \), the controller should incorporate a relay component:

\[
u_i = -V_i e_i - \mathbf{f}_i^\text{corr}
\]

(9)

where the error feedback gains are switched according to

\[
\phi_i = \begin{cases} 
\mathbf{1}_i^\top e_i > 0 \\
\mathbf{1}_i^\top e_i < 0
\end{cases}
\]

and \( \mathbf{1}_i \) is the relay gain.

The switching hyperplane in the state space is specified as

\[
s_i = c_i^\top e_i = 0
\]

(11)

The condition for existence of a sliding mode on (11) is given by

\[
\dot{s}_i = c_i^\top e_i > 0
\]

(12)

Substituting for \( s_i(t=0) = 0 \), the error trajectory will reach the sliding plane, if the condition (12) is satisfied in the entire error space. Then, from (4) and (11) we have,

\[
\dot{s}_i = c_i^\top \left( \lambda_i^\text{nom} A_i e_i + b_i u_i^\text{nom} \right)
\]

Substituting for \( u_i \) from (9), and rearranging,

\[
\dot{s}_i = c_i^\top \left( \lambda_i^\text{nom} A_i e_i + b_i u_i^\text{nom} + c_i d_i \mathbf{f}_i + c_i^2 b_j y_j \text{sgn} (s_j) \right)
\]

(13)
where $A^j_1$ is the jth column of matrix $A_1$. From (10), (12) and (13), it can be seen that if the conditions

$$
(\text{sgn}(c_1 b_1))_{11} > |c_1 b_1|^T |c_1 A_1^j|
$$

$$
(\text{sgn}(c_1 b_1))_{12} > |c_1 b_1|^T |c_1 A_2^j|
$$

$$
(\text{sgn}(c_1 b_1))_{11} v_1 > |c_1 b_1|^T H c_1 d_1
$$

are satisfied by appropriate choice of gains $c_1$, $b_1$ and $v_1$, then the sliding mode exists on the hyperplane $s_i = 0$, and is reachable.

\[\text{Chatter Reduction by High Gain}\]

Once system (4) is in the sliding mode, it will be invariant to the disturbance $f_i$, provided the invariance condition

$$\text{rank}(b_i) = \text{rank}(b_i d_i) \quad (15)$$

is satisfied. For the actuator in (1), this condition is, in general, not valid.

Because of the discontinuous nature of control in (9), the error trajectory 'chatters' about the sliding plane. Chattering involves high power dissipation in the actuator and may excite the unmodelled high frequency dynamics. So to eliminate chattering, the VSC is replaced by a high gain PI controller in the vicinity of the sliding plane.

For high gain feedback, the conditions for decoupling the state vector from the disturbance are the same as (15), and are satisfied in our case. Hence we effect a coordinate conversion of error dynamics (4), such that the invariance conditions hold good. Now, in the vicinity of sliding plane, $|s_i| < e_i$, where $e_i > 0$ is small, the VSC is replaced by a high gain PI controller of the form

$$\dot{\hat{s}}_i = c_i \hat{s}_i + \hat{s}_i \sum_{n} (q_i - q_{i,n}) \delta \tau_i \quad (16)$$

Here $c_i > 0$ is a large scalar gain factor, $\hat{s}_i$ is the transformed state, and $c_i$ and $k_i$ are chosen to ensure stable motion of the slow and fast components of the augmented high gain system. This choice of $c_i$ can be used in (11), to form the stable switching surface. The integral feedback term is added to eliminate the steadystate tracking error.

\[\text{AN APPLICATION TO TRAJECTORY CONTROL}\]

We now illustrate the application of the above control algorithm to the UM5-2 cylindrical manipulator with minimal configuration ($N=3$), considered by Stokic and Vukobratovic. Here, dc motor actuators are used, with $n=3$, $i$. A's are constant 3x3 matrices, $b_i=0 \ 0 \ I_{3 \times i}$ and $d_i=0 \ -1/j_i \ 0$. The invariance conditions do not hold good in this case.

The manipulator dynamics are given by

$$I_i = \frac{e_i}{1} (b_i + h_i)$$

where, $^v A = J_1^{1/2} J_1^{1/2} \ W V \ W^T$.

The nominal displacement trajectories are specified as shown in Fig.1. The $c_{1i}$ are chosen as in Table 1, and the corresponding gains $(c_{1i} = \text{sgn}(b_i))$ are chosen to satisfy (14).

\[\text{Table 1}\]

<table>
<thead>
<tr>
<th>Link</th>
<th>$c_{1i}$</th>
<th>$a_{1j}$</th>
<th>$v_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(100, 20, 1)</td>
<td>(1, 4.7, 0.1)</td>
<td>2.8</td>
</tr>
<tr>
<td>2</td>
<td>(10000, 200, 1)</td>
<td>(5, 100, 1.0)</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>(10000, 141, 1)</td>
<td>(10,500,2.23)</td>
<td>0.1</td>
</tr>
</tbody>
</table>

To examine the robustness of the proposed controller to initial errors and payload uncertainties, we have assumed $e_i = (0.1, 0.01, 0.01)$, $c_{1i} = (0) = e_i$ at $t = 0$, and a change from the nominal payload ($M_0 = 5.6$ kg, $J = 0.32$ kg m$^2$) to an off-nominal payload ($M_0 = 9.6$ kg, $J_0 = 1.0$ kg m$^2$).

The simulation results are shown in Figs.2 to 4. The chattering errors of links 2 and 3 have been shown in Figs. 5 and 6 on an expanded scale for the off-nominal case with zero initial error. As is obvious from these figures, the tracking performance is quite satisfactory and is robust to payload variations.

\[\text{DISCUSSIONS}\]

For implementation of the proposed controller, the nominal state and control trajectories are to be stored in memory, as is the case with other explicit or implicit model following designs. However, our VSC design is decentralised, with simple off-line design computation.

In contrast, the results ' ' ' give a centralised controller; in 13 control components for nonlinearity and uncertainty compensation are additionally required. The decentralised controller design using combined actuator and manipulator models, introduced in ' ' ' suffers from the requirements of additional global force feedback or adaptive feedback, if the local feedback fails to stabilise the system against expected internal and payload uncertainties.

The use of operating range bounds $F_i$ assumed on torques $f_i$ in (14), which greatly simplifies the control law, is a fairly realistic assumption. The control gains (14) ensure, for any bounded nonzero initial error (which can be minimised suitably), reachability of the sliding mode; if we wish to eliminate chattering motion a high gain control law takes over, so the error asymptotically goes to zero. As $n$ rpsnT, $fi$ and $i$ may be expected to be bounded within $F_i$, and so the reachability condition is always satisfied by (10).

A major limitation of the proposed method is that the number of joint actuator model variables required for
feedback is generally larger than the number of link variables (namely, joint position and velocity) which are the state variables in a majority of the existing robot control schemes. For example, electrical (dc motor) and hydraulic actuators are modelled with good accuracy by third and fifth order models.

For the third order model of the electrical actuator, if coordinate conversion of error dynamics is employed to satisfy the conditions(15), joint acceleration error substitutes for armature current error as the third state variable. So, accelerometers may have to be used on the joints, or atleast numerical differentiation of position/velocity measurements will be needed.

Reasonable accuracy of modelling can, however, be maintained by dealing with reduced order actuator models and this also simplifies the controller design. For example, if a second order dc motor model is used, joint position and velocity are the only variables to be measured, and the invariant conditions hold good directly.

**CONCLUSION**

We have proposed a decentralised model following controller with variable structure control law, which bypasses the complexities posed by the nonlinear, uncertain dynamics of the manipulator, by dealing instead with the simpler actuator dynamics, and by treating the joint torques as disturbances.

In practice, the actuator dynamics also involve uncertainty, nonlinearity and disturbances (due to motor parameter drift, torque saturation, dry friction etc.),and may have to be modelled by not only the nominal dynamics of (3) and torque f, but also additional uncertain and nonlinear dynamics.

This complicates the controller design to a certain extent. We are presently working on a method for estimating this uncertain dynamics and the torque, by utilising the sliding mode concept.

**REFERENCES**

Fig. 1 NOMINAL POSITION TRAJECTORIES

Fig. 2 POSITION TRACKING : LINK 1

Fig. 3 POSITION ERROR TRAJECTORY : LINK 2

Fig. 4 POSITION ERROR TRAJECTORY: LINK 3

Fig. 5 POSITION ERROR TRAJECTORY: LINK 2

Fig. 6 POSITION ERROR TRAJECTORY : LINK 3