New Fourier methods for generation system production costing

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Abstract: Two new, robust, and computationally efficient Fourier methods for solving the probabilistic production costing problem are presented. The method first proposed yields closely matching results with those of the standard recursive method, which is an exact method. The second method that is proposed is less accurate, but about three times as fast as the first method. Both methods offer substantial computational savings over the standard recursive method. A model test system is used to illustrate the performance of the proposed methods vis-a-vis that of the standard recursive method.

1 Introduction

The major step in the production costing procedure is the estimation of the quantum of energy generated by each unit in a generation system as the units are committed sequentially, according to their merit order. In this analysis both the availabilities of the generating units and the system load demands are to be modelled in the probabilistic sense. An individual unit may be committed in whole or in several blocks, depending on the relative marginal cost of the different units. The quantum of energy to be supplied by each of these blocks is determined. Thereafter, these quanta of energy are multiplied by the respective heat rates, each of which is constant over a loading block. Further multiplication by the fuel costs for each unit finally yields the cost of energy produced by that unit. The cost of energy produced by the different units can be summed up and the total production cost of a particular generation system plan can be evaluated.

The production costing results form the basis of choosing the most economical plan from among a number of feasible plans.

The objective of this paper is to present new, robust, accurate, and computationally efficient methods for computing the expected values of energy generated by different generating units in a system.

A method for probabilistic production costing was developed in a paper by Booth [1] in which the equivalent load duration curve (ELDC) was constructed by a recursive algorithm using the system data for the load duration curve (LDC) and generating unit availabilities. The computational requirement of this method increases enormously if the power system consists of a large number of dissimilar units.

Rau et al. [2] and Stremel et al. [3] concurrently proposed new probabilistic methods that utilised statistical moments to speed up the computation. These methods are quite fast, but introduce approximations into the calculation on two counts. First, the Fourier transforms of the unit outage density functions and of the LDC are represented by their first few moments only. Secondly, the Gram-Charlier expansion is used to fit a finite set of normal distributions to represent the various non-normal distributions encountered in the problem. These approximations have been found to introduce inaccuracies, and the results from these methods have been shown to be in considerable error for small generation systems with unit ratings varying over a wide range, and also for systems having highly reliable units [3 (Closure), 4, 51.

Billinton et al. [6, 7] have solved the problem by building, through a recursive algorithm, the capacity outage probability table, that contains the probability of occurrence of various magnitudes of capacity outage of the generation system. For each state of the capacity model, the energy not supplied is calculated by the procedure described in Reference 8. The expected value of energy not served is then calculated by the sum of the energy not supplied for each state of the capacity model, weighted by the probability of occurrence of that state, over all the capacity states. The expected value of energy produced by each generating unit is given by the difference between the expected values of energy not served and after committing the unit. The method is exact because it can consider all possible exact capacity outage states of the generation system, and also the actual hourly or daily peak loads arising in the period of study. This method is, therefore, referred to in this paper as the standard recursive method. The drawback of this method is that if the generation system has a number of dissimilar units with capacity sizes that are not integral multiples of a fairly large common factor (which is normally the case in most practical systems), the number of possible capacity outage states becomes enormous and the formulation of the capacity outage probability table requires considerable computer time.

Two new Fourier methods are proposed for production costing in this paper. The first method incorporates a direct extension of the techniques used in Reference 9 and yields accurate results. The second method uses some of the techniques of Reference 9, but avoids, in a novel way, the necessity of taking an inverse Fourier transform. This method provides fairly accurate results and a three-to-one speed advantage over the first method. The superior accuracy of the techniques
described in Reference 9 over the method of statistical moments has been brought out [10] for systems with unit ratings varying over a wide range and containing some highly reliable units. Both of the proposed methods are benchmarked against the standard recursive method, and the IEEE reliability test system is used to compare the three methods. The computation time taken by the proposed methods is much less than that taken by the standard recursive method for the reliability test system and for practical systems. The proposed methods are not limited in application to generation systems of any particular type.

2 Description of new methods

The load duration curve of a power system contains statistical information about the load on the system. In both proposed methods the load duration curve of the system is transformed by using the fast Fourier transform (FFT) algorithm [11].

The probability density function of outage, or availability, of a generating unit contains statistical information about the forced outage rates, complete or partial, of the unit. In the first method the outage probability density functions of the generating units are transformed into the Fourier domain by using the algorithm [9] which fully exploits the fact that these functions are impulse trains that have just a couple of impulses located at the possible unit capacity outage states. The ELDC in the capacity domain is computed after convolution of the outage density function of each unit with the LDC [Z]. This convolution is done by multiplying the Fourier transforms of the outage density functions and the LDC, and taking the inverse transform using the inverse FFT algorithm.

In the second method the availability density functions of the generating units are transformed into the Fourier domain using the algorithm described in Reference 9. An equivalent load duration curve (ELDC) is constructed in this method. However, unlike the first method, the ELDC represents the load seen by the generating units that have not yet been committed [11]. In other words, the ELDC, after committing each unit, is obtained by deconvolving that unit’s availability density function from the ELDC that exists before committing that unit. The transformed ELDC is computed by multiplication of the conjugate of the transforms of each of the unit availability density functions with the transform of the LDC. This is the convolution and correlation procedure that corresponds to deconvolution in the capacity domain. The equivalence between deconvolution and correlation is shown formally in Appendix 8.1. Subsequently, the expected value of energy generated by each unit is computed by finding the area under an appropriate curve in the Fourier domain itself. The method thus operates entirely in the Fourier domain, and is therefore substantially faster than the first method. However, certain approximations are introduced into the calculations.

In both the methods proposed, multiple units with the same outage or availability behaviour are handled efficiently, and the transformed probability density function for these units is calculated once only. Multiple state outage/availability models are handled in a similar way as the two state models.

The input data for the production costing problem consists essentially of the LDC of the power system, the probability density function of outage/availability of each generating unit of the system, together with the merit order of loading of the units. The merit order is required to be determined before the production costing problem is calculated. The merit order of loading is determined by the relative incremental costs of energy generation by the different units or by multiple blocks of the units, wherever applicable.

2.1 Method 1: Using inverse fast Fourier transform algorithm

The transform of the LDC is multiplied by the transform of the outage density function of the first generating unit in the merit order. The function thus obtained is inversely transformed using the inverse FFT algorithm back to the capacity domain to obtain the ELDC.

The expected value of energy not served after committing the first unit is obtained from the area under the ELDC [Z]. The value of the abscissa of the curve from where the area computation is commenced is the capacity of the generating unit that is committed. The expected value of energy not served is the area under the ELDC to the right of this abscissa, and is evaluated by the trapezoidal rule.

The amount of energy that is expected to be generated by the first committed unit is the difference between the expected value of energy not served before this unit was committed, i.e. the total area under the LDC, and the current expected value of energy not served after this unit has been committed.

This procedure is continued for the generating units subsequently appearing in the merit order. Fig. 1 shows how the expected value of energy not served is calculated after a certain number of generating units, whose capacity aggregates to TC, have been committed.

Multiblock representation of any unit may be handled by first convolving the partial outage density function of that unit, including the first loading block, and calculating the amount of energy that is expected to be generated by that first block [Z]. When the second block of that unit is to be committed, the partial outage density function including the first loading block is deconvolved from the ELDC, and the partial outage density function including the first two loading blocks is then convolved. This procedure may be performed again when later blocks of the unit are to be committed. A brief description of the algorithm is as follows:

Step 1: Read the LDC values and assign them successively to the real and imaginary parts of a function. Calculate the area under the LDC; this is the expected value of energy not served before any unit is committed.

Step 2: Transform the complex function formed in Step 1 using the FFT algorithm.

Step 3: Read the generating unit data one by one and calculate the transformed outage probability density

Fig. 1 Calculation of expected value of energy not served

TC = total capacity of generating units convolved
EVENS = expected value of energy not served
ELDC = equivalent load duration curve
function of each unit. For similar units this calculation is done once only.

Step 4: Multiply the transformed LDC values in Step 2 with the transformed outage probability density function of the first generating unit in the merit order. The product function is the transformed ELDC after committing the first unit.

Step 5: Inverse transform the latest transformed ELDC by using the FFT algorithm.

Step 6: Recover the real values of the ELDC from the complex inverse transformed function.

Step 7: Evaluate the area under the ELDC to the right of the point corresponding to the total capacity of the generating units so far convolved. This area corresponds to the expected value of energy not served.

Step 8: Calculate the difference in the expected values of energy not served before and after a generating unit is committed. This is the expected value of energy generation of that unit.

Step 9: Multiply the transformed ELDC with the transformed outage probability density function of the next generating unit in the merit order. This product function becomes the latest transformed ELDC.

Step 10: Repeat steps 5-9 until all the generating units have been committed in the merit order.

22 Method 2: Avoiding inverse Fourier transformation

In Method 1 the transformed ELDC has to be inverse transformed after committing each generating unit, and the area under a part of it has to be calculated after each inverse transformation. The motivation for devising the second method was that the inverse transformation would be unnecessary, if the area under the ELDC in the capacity domain could be obtained by calculating the area under an appropriate curve in the transform domain itself. The technique for achieving this is described in Appendix 8.2.

Deconvolution of the availability density function of the generating unit, that is first in the merit order, from the LDC is done by the procedure indicated at the beginning of this Section. After the deconvolution operation, the area under the ELDC is calculated by the procedure described in Appendix 8.2. This gives the expected value of energy not served after committing the first unit.

In this method deconvolution is necessary because the technique for calculating the area directly in the Fourier transform domain computes the area under a curve in the capacity domain over the entire width specified by T, and not the area that is over just a part of this width.

Multiblock representation of a generating unit may be handled by first deconvolving the partial availability density function of that unit, including the first loading block, and calculating the amount of energy that is expected to be generated by that block. When the second block of that unit is to be committed, the partial availability density function, including the first loading block, is first convolved with the ELDC, and the partial availability density function, including the first two loading blocks, is then deconvolved. This procedure may be performed again when more blocks of the unit, if any, are to be committed. A description of the algorithm is as follows:

Step 1: Read the LDC values and transform the LDC using the FFT algorithm.

Step 2: Calculate the area under the load duration curve. This area corresponds to the expected value of energy not served before committing any unit.

Step 3: Calculate the SINC R and SINC I functions.

Step 4: Read the generating units data one by one and transform the availability probability density function of each unit. For similar units the calculation is done once only.

Step 5: Multiply the transformed LDC values by the complex conjugate of the transformed availability probability density function of the first generating unit in the merit order. This is the transformed ELDC after deconvolving the first unit.

Step 6: Interpolate the latest transformed ELDC, then multiply it by the complex sum of the SINC R and SINC I functions.

Step 7: Evaluate the area under the product function computed in step 6. This area corresponds to the expected value of energy not served.

Step 8: Calculate the difference in the expected values of energy not served before and after a unit is committed. This is the expected energy generation of that unit.

Step 9: Multiply the latest transformed ELDC function by the complex conjugate of the transform of the availability probability density function of the generating unit that is next in the merit order. This becomes the latest transformed ELDC.

Step 10: Repeat steps 6-9 till all the generating units have been committed in the merit order.

3 Guidelines for choice of parameters for the two methods

The sampling interval in the capacity domain is required to be kept small enough so that aliasing, or overlap in the Fourier transform domain, is negligible [12]. Some numerical experimentation is necessary for arriving at the right capacity step size for a particular system. Too large a step size will result in accuracy due to aliasing, and too small a step size will increase the computer time. The widely used technique in any numerical method, namely, the successive halving or doubling of the step size and analysing the effect on the results, will be useful for arriving at the right value.

The sampling interval in the transform domain is fixed once the sampling interval and the number of sampling points in the capacity domain are fixed [12]. The sampling interval in the transform domain must be small in method 2 to obtain a high degree of accuracy. The number of sample points in both the domains is the same.

The capacity range considered, this is the product of the capacity domain, must equal or exceed the sum of the capacity of the system installed and the peak load in method 1 to meet the requirements of discrete Fourier transform convolution [9,123.

4 Example

The two proposed methods are tested on the IEEE reliability test system [13]. This is a 32-machine system with 3405 MW installed capacity. The details of the machines are shown in Table 1.

Table 1 shows that for this system all the unit ratings are not multiples of any capacity step other than 1 MW. Some of the units also have small forced outage rates.

The transforms of the unit outage/availability density functions are obtained as described in Reference 9. The cosine term that constitutes the real part of the transform
Table 1: IEEE RTS unit data

<table>
<thead>
<tr>
<th>Step</th>
<th>Unit rating</th>
<th>Number of identical units</th>
<th>Forced outage rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>2</td>
<td>0.12</td>
</tr>
<tr>
<td>2</td>
<td>350</td>
<td>1</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>197</td>
<td>3</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>155</td>
<td>4</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>3</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>76</td>
<td>4</td>
<td>0.02</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>6</td>
<td>0.01</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>4</td>
<td>0.10</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>5</td>
<td>0.02</td>
</tr>
</tbody>
</table>

is sampled with a step size determined by the 5 MW sampling interval in the capacity domain and the number of sample points.

The annual peak load for the system is 2850 MW. The individual hourly peak loads are arranged in descending order and a cumulative load model, the load duration curve, is developed using these data.

The load duration curve is sampled with 5 MW step size to avoid overlap in the transform domain. Sample points considered for method 1 are 2048, and 4096 points for method 2. In method 2, integration in the transform domain is performed over 660 interpoint intervals.

The order of loading the units is taken to be the same as in Reference 8.

5 Results and discussion

The unitwise energy generation obtained from the two proposed methods is presented in Table 2. The results obtained from the standard recursive method [8] are also shown in the Table.

The results obtained from method 1 are seen to be very nearly the same as the results from the recursive method. The results from method 2 agree with the results from the recursive method (to a good approximation). The computation time taken by method 1 is 198 s, by method 2 it is 72.8 s, and by the recursive method the time is 1723.8 s on an ICL 2960 computer system.

6 Conclusion

In this paper two new Fourier domain methods have been described for solution of the production costing problem. The methods yield the expected value of unitwise energy generation in a power generation system, and are robust and efficient. They yield results that match closely with those from the standard recursive method (that is an exact method) and take much less computer time.

The new methods offer advantages for handling small sized systems with unit ratings varying over a wide range, and for systems containing some highly reliable units.

In method 1 an extension is made of techniques that have been developed earlier. The method is accurate.

In method 2 a novel technique is proposed for avoiding the inverse transformation from the Fourier domain to the capacity domain in solving the production costing problem. This method yields acceptable estimates of energy produced by each unit. Computationally it is very fast, and takes only about one-third the computation time taken by method 1.

The monetary cost of energy production on each generating unit can be obtained by multiplying the quantum of energy generated by each unit with the appropriate average cost of such generation.

7 References

8 Appendixes

8.1 Equivalence between deconvolution and correlation

Deconvolution is performed when the difference of two independent random variables is the random variable whose density is to be obtained.

Consider independent random variables \(A\) and \(B\), and the density to be obtained for random variable \(C = A - B = A + (-B)\).

From the fundamental theorem of probability theory [14]:

\[
M_C(x) = \int_{-\infty}^{\infty} f_A(a) f_C(x-a) \, da
\]

where \(f_C(x)\) is the probability density function of random variable \(C\).

Now the cumulative probability function

\[
F_{-A}(x) = \text{prob} (-B < C < x) = \text{prob} (B > -x) = 1 - \text{prob} (B < x) = 1 - F_{-A}(x)
\]

Therefore

\[
M_x(x) = \int_{-\infty}^{x} f_A(a) f_C(x-a) \, da
\]

Substituting \(c' = -c\) and \(f(-c) = f_c(c)\). Therefore

\[
M_x(x) = \int_{-\infty}^{x} f_A(a) f_C(-x+a) \, da
\]

this is correlation.

8.2 Area under load duration curve

Let \(H(f)\) and \(X(f)\) be the respective Fourier transforms of the functions \(h(t)\) and \(x(t)\) in the capacity domain. At this point, emphasis can be laid on the fact that the variable \(t\) does not denote time.

From the frequency convolution theorem [12] it is known that the convolution integral of \(H(f)\) and \(X(f)\) is equal to the Fourier transform of the product \(h(t)x(t)\), i.e.

\[
\int_{-\infty}^{\infty} h(t)x(t)e^{-j2\pi ft} \, dt = \int_{-\infty}^{\infty} H(f)X(f) \, df
\]

where \(U\) is a dummy variable.

Putting \(U = 0\) in eqn. 1

\[
\int_{-\infty}^{\infty} h(t)x(t)dt = \int_{-\infty}^{\infty} H(f)X(-f)df
\]

Eqn. 2 suggests that if \(x(t)\) is chosen to be a unit pulse of width \(T\), occupying the interval 0 to \(T\), the infinite integral on the left side of eqn. 2 will become just the integral of the function \(h(t)\) over the interval 0 to \(T\). Evaluation of this integral in the capacity domain is what is desired.

Eqn. 2 shows that this may be done by performing the integration on the right-hand side of eqn. 2, which is entirely in the Fourier transform domain.

Now, if \(x(t)\) is a unit pulse of width \(T\),

\[
x(t) = u(T) - u(t-T)
\]

The Fourier transform of \(x(t)\) is

\[
x(f) = \int_{-\infty}^{\infty} e^{-j2\pi ft} \, df
\]

Substituting eqn. 3 into eqn. 2

\[
\int_{-\infty}^{\infty} x(f)H(f-M)df
\]

Simplifying and substituting eqn. 4 into eqn. 5

\[
\int_{-\infty}^{0} H(f)df = \int_{-\infty}^{0} H(f) \left[ \sin \frac{2\pi ft}{2} + j \frac{\sin \frac{\pi ft}{2}}{\frac{\pi ft}{2}} \right] df
\]

Define

\[
\text{SINCR} = \frac{j\pi}{\pi f} \quad \quad \text{SINCI} = \frac{1}{\pi f}
\]

In eqn. 6 the integral in the Fourier transform domain is an infinite one. However, the absolute values of the SINCR and SINCI functions, that are signon functions, diminish very rapidly as the transform domain variable increases, and these functions have been found to be negligibly small for large values of the transform domain variable. Therefore this integral need be evaluated only in a small range near the origin.

Thus the area under the curve of the function \(h(t)\) in the capacity domain may be obtained by integrating an appropriate function directly in the transform domain.

The \(h(t)\) function corresponds to the LDC to start with, and subsequently corresponds to the ELDC in the problem under study. The pulse width \(T\) is chosen to be the capacity value to which the LDC extends. The integral in the Fourier transform domain is evaluated numerically. Acceptable accuracy is obtained by limiting the numerical integration to about just 10% of the range considered for the discrete Fourier transform.

It is necessary that the \(H(f)\) function is interpolated, and the interpolated values are used along with the corresponding values of the SINCR and SINCI functions while evaluating the integration of eqn. 6. Since the SINCR and SINCI functions assume large values near the origin, and the values diminish away from the origin, the interpolation is done intensively near the origin and progressively less intensive away from the origin. To obtain a good degree of accuracy, the first interval between the sample points in the Fourier transform domain may be divided into 100 subintervals, the next 5 intervals into 20 subintervals each, and the subsequent 1% of the total number of intervals into 5 subintervals each. For the remaining part of the spectrum over which integration is done, no interpolation may be required. Integration is performed using the trapezoidal rule.