

Parametric study of an irreversible regenerative Brayton cycle with isothermal heat addition

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Abstract

A parametric study of an irreversible regenerative Brayton heat engine with isothermal heat addition has been performed with external as well as internal irreversibilities. The external irreversibility is due to the finite temperature differences between the heat engine and the external reservoirs, while the internal irreversibilities are due to other processes, viz. nonisentropic compression and expansion processes in the compressor and turbine, respectively, and the regenerative heat loss. The power output is maximized with respect to the working fluid temperatures, and the effects of different parameters on the maximum power output and the corresponding thermal efficiency have been studied. There is a significant improvement in the thermal efficiency (above 15%) of a Brayton cycle with isothermal heat addition over the conventional one. It is seen that the effect of the isobaric side effectiveness is rather pronounced for the power output and the corresponding thermal efficiency. The effect of the turbine efficiency is found to be more than that of the compressor on both power output and thermal efficiency. Also, it is seen that there are optimal values of the various heat capacitance rates between the different reservoirs and the heat engine.

Keywords: Brayton cycle; Isothermal heat addition; Regular combustion chamber; Converging combustion chamber

1. Introduction

Thermal efficiency is an important performance characteristic parameter of Brayton cycles and, in practice, has a major impact on operating cost. It is well known that reheating in gas turbine engines limits the extent to which an isothermal heat addition is approached. With respect to

Nomenclature

A	area (m ²)
M	Mach number
P	power output (kW)
T	temperature (K)
U	overall heat transfer coefficient
c	heat capacitance rates (kW/K)
N	number of heat transfer units
Q	heat transfer rates (kW)
V	velocity (m/s)
η	thermal efficiency
ϵ	effectiveness

Subscripts

c	compressor
H, H1	hot side/heat source
L	sink/cold side
R	regenerator
S	isentropic
t	turbine
W	working fluid
1, 2S, 2, 2R, 3, 4, 5, 5S, 5R	state points

The different parameters

x	$\dot{C}_{HEH} = \dot{C}_W$
y	$C_{LeL} = C_W$
$x1$	$C_{H1EH1} = C_W$
a_1	$\dot{C} + \dot{O} - \dot{M}i - f/c$
b_1	$(1 - y)(1 - \epsilon_R)(1 - \eta_c)$
$c1$	$yT_{L1}(1 - \eta_c)$
a_2	$(1 - \epsilon_R)(1 - x)(1 - \eta_t^{-1})$
b_2	$\eta_t^{-1} + \epsilon_R(1 - x)(1 - \eta_t^{-1})$
c_2	$xT_{H1}(1 - \eta_t^{-1})$
a_3	$\epsilon_R(1 - y)$
h	$(1 - \epsilon_R)(1 - y)$
c_3	yT_{L1}
a_4	$(1 - x)(1 - \epsilon_R)$
b_4	$\delta(1 - X) \delta_R$
c_4	xT_{H1}
a_5	$a_1b_2 + a_2b_1 - \alpha^{-1}(a_3b_4 + a_4b_3)$
b_5	$b_1b_2 - \alpha^{-1}b_3b_4$

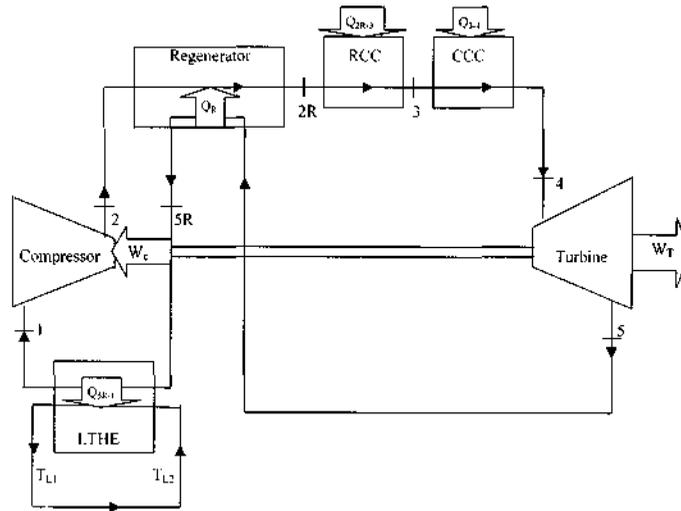
c_5	$c_1 c_2 - 0 c^{-1} c_3 c_4$
a_6	$a_1 c_2 \text{ p } a_2 c_1 - c C^x \delta a_3 c_4 \text{ p } a_4 c_3 \text{ p}$
b_6	$b_1 c_2 \text{ p } b_2 c_1 - a^{x1} \delta b_3 c_4 \text{ p } b_4 c_3 \text{ p}$
A	$A a_1 a_2 - a r^x a T, a_4$
B	$a \$ T \$ - \setminus - a (\,$
C	$b_5 T^{\wedge} + b_6 T_5 \text{ p } c_5$
a_j	$C_w \{ x (\setminus - e_R) + J \text{ £ } R \setminus - ; c_i (1 - x) (1 - \text{ £ }_R) \}$
h	$C_w \{ j (1 - \text{ £ }_R) + x \text{ £ } R \cdot \text{ p } e_R \delta 1 - x) x_x \}$
a_8	$\delta a_5 a_7 - 2 A b_7 \text{ p } = a_7$
a_9	$a_5^2 - 4 A b_5$
aw	$a_5 a_6 - 2 A b_6$
A_1	$a_9 - (a_9 / a_5 f$
B_1	$a_7 0 - a_9 a_1 0 = a_2$
C_1	$a_1 - 4 A c_5 - 8 a_1 0 = a_8 \text{ p }^2$

simple heat addition, when a compressible gas with subsonic velocity flows through a frictionless constant area duct with heat addition, the temperature of the gas increases along the duct. Also, with respect to a simple area change, when a compressible fluid/gas with subsonic velocity flows through a frictionless adiabatic duct with decreasing area, the temperature of the gas decreases along the duct. Based on the nature of these two flows, simple heat addition (under Rayleigh flow) and simple area change (under isentropic flow) may be combined in such a way as to yield an isothermal heat addition process [1]. The idealized isothermal process consists of a compressible gas with subsonic velocity flowing through a frictionless converging duct such that while being heated all along the duct, any infinitesimal decrease in temperature due to the simple area change is exactly compensated by the simple heat addition. It is noted that since the temperature of the gas is constant during the isothermal heat addition, the kinetic energy of the gas (and hence, the Mach number) must increase in order to satisfy conservation of energy. The appropriate application of the idealized isothermal process is to gas turbine engines operating with air. It is equally desirable that the Brayton cycle be modified by the isothermal heat addition.

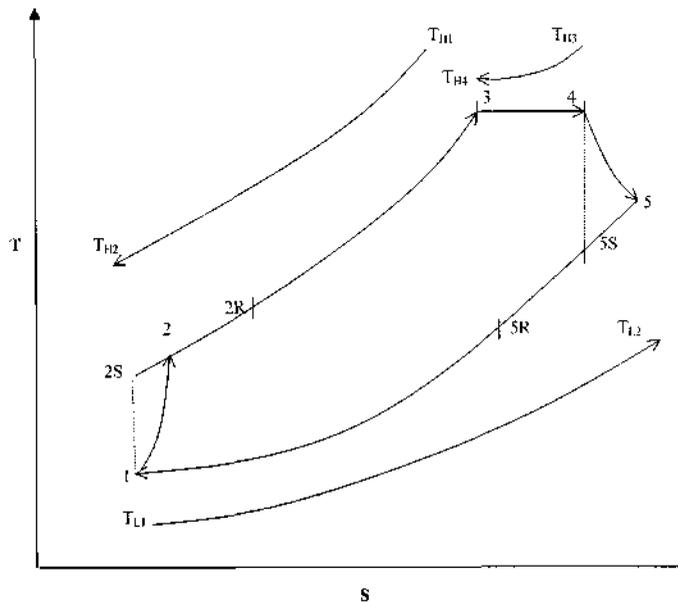
Vecchiarelli et al. [1] indicated that the hypothetical modification of gas turbine engines to include two heat additions (rather than one) may result in some efficiency improvement as compared with conventional engines. In recent years, many workers [2-4] studied the effect of isothermal heat addition using open/closed cycle regenerative Brayton heat engines and showed that there is a significant improvement in the thermal efficiency (over 10%) compared with conventional engines. In this paper, we have presented a more general and detailed analysis of a regenerative closed cycle Brayton heat engine with isothermal heat addition and nonisentropic compression and expansion processes for finite heat capacity of the external reservoirs.

2. System description

The schematic and T - s diagrams of the closed cycle regenerative Brayton heat engine with isothermal heat addition are shown in Fig. 1(a) and (b), respectively. The basic components of this



(a)



(b)

Fig. 1. (a) Schematic of regenerative Brayton cycle with isothermal heat addition. (b) T - s diagram of regenerative Brayton cycle with isothermal heat addition. Horizontal axis: Specific entropy in kJ/K; vertical axis: Temperature in K.

cycle are the compressor, regenerator, regular combustion chamber (RCC), converging combustion chamber (CCC), turbine and low temperature heat exchanger (LTHE). The gas enters the compressor at state 1 and is compressed to state 2 (nonisentropically) or to state 2S (isentropically). At state 2 or 2S, the cold gas leaving the compressor enters the regenerator, where it is heated to state 2R. In an ideal regenerator, the gas will leave the regenerator at the temperature of the turbine exhaust (state 5), i.e. $T_{2R} = T_5$. The isobaric heat addition process takes place between

states 2R and 3 in the RCC from a finite heat capacity source whose temperature varies from T_m to T_m . Further heat addition is accomplished in the CCC isothermally between states 3 and 4 from a finite heat capacity source whose temperature varies from T_{H3} to T_{H4} . When the gas leaves the CCC at state 4, it has a lower pressure than at state 3, but the velocity and, hence, the kinetic energy of the gas has increased enormously due to the nature of the CCC. The gas enters the turbine at state 4 and expands nonisentropically to state 5 (ideally to state 5S). The hot gas leaves the turbine at state 5 or 5S and enters the regenerator where it is cooled to state 5R isobarically by supplying heat to the compressor outlet gas and finally enters the LTRE at state 5R and is cooled to state 1 at constant pressure, rejecting the heat to a heat sink of finite heat capacity whose temperature varies from $TL1$ to $TL2$, completing the cycle. Thus, we have considered here the theoretical model of a modified regenerative Brayton cycle 1-2-2R-3-4-5-5R-1 with real processes.

3. Thermodynamic analysis

Let QH , QL and QR be the heat transfer rates to and from the heat engine and the regenerative heat transfer rate, respectively, then

$$QH = Q_{2-3} + Q_{3-4} \tag{1}$$

$$QL = Q_{5R-1} \tag{2}$$

$$QR = Q_{2-2R} = Q_{5-5R} \tag{3}$$

where

$$Q_{IR-3} = C_w(r_3 - T_{2R}) = C_H(T_m - T_m) = [U A_H (LMTD)_H] \tag{4}$$

$$Q_{%A} = m(V_4 - V_3)/2 = CH(T_m - T_m) = U_m A_m (LMTD)_{H1} \tag{5}$$

$$Q_{R-i} = C_w(r_{5R} - T_i) = C_L(T_{L2} - T_u) = [U A_L (LMTD)_L] \tag{6}$$

$$QR = C_w(T_5 - T_{5R}) = C_w(T_{2R} - T_2) = U A_R (LMTD)_R \tag{7}$$

where CH , $CH1$, CL and CW are the heat capacitance rates in the external fluids on the RCC, the CCC and the cold side reservoirs and the working fluid, respectively. $U A_H$, $U A_{H1}$, $U A_L$ and $U A_R$ are the overall heat transfer rates and areas products between the external reservoirs and the RCC, the CCC and the heat sink and in the regenerator, respectively. V_3 and V_4 are the velocities of the working fluid at state points 3 and 4, respectively, and m is the mass flow rate of the working fluid.

The isothermal heat addition between states 3 and 4 can also be defined in terms of Mach number (M) as:

$$M = V/V_s \tag{8}$$

where V_s is the speed of sound and for a perfect gas:

$$V_s = \delta k R T P^{1=2} \tag{9}$$

where k is the specific heat ratio and R is the universal gas constant of the working fluid. From Eqs. (9) and (5), we have:

$$Q^{\wedge} = C_w(k - 1)P T_3 \delta M_4^2 - M I / 2 \tag{10}$$

From Eqs. (4)-(7), we have:

$$Q_{2R-3} = C_w \delta T_3 - T_{2R} P = C_H e_H \delta T_{H1} - T_{2R} P \tag{11}$$

$$Q_{3-4} = m(V_4^2 - V_3^2) / 2 = C_{H1} \varepsilon_{H1} (T_{H3} - T_3) \tag{12}$$

$$f_{iR} = C_w (r_{5R} - T_x) = C_{LSL} (T_{5K} - T_w) \tag{13}$$

$$Q_R = C_w \delta T_5 - T_{5R} P = C_w \delta T_{2R} - T_2 P = C_w e_R \delta T_5 - T_2 \tag{14}$$

where the e 's are the effectivenesses of the various heat exchangers and are defined as:

$$\varepsilon_H = \frac{1 - e^{-N_H \left(1 - \frac{C_{H,min}}{C_{H,max}}\right)}}{1 - \frac{C_{H,min}}{C_{H,max}} e^{-N_H \left(1 - \frac{C_{H,min}}{C_{H,max}}\right)}} \tag{15}$$

$$\varepsilon_L = \frac{1 - e^{-N_L \left(1 - \frac{C_{L,min}}{C_{L,max}}\right)}}{1 - \frac{C_{L,min}}{C_{L,max}} e^{-N_L \left(1 - \frac{C_{L,min}}{C_{L,max}}\right)}} \tag{16}$$

$$e_m = \frac{m}{1 + m} \tag{17}$$

$$\varepsilon_R = A W (1 + m) \tag{18}$$

where N_H , N_{H1} , N_L and N_R are the number of heat transfer units based on the minimum thermal capacitance rates $N_H = U_H A_H / C_H$, $N_{H1} = U_{H1} A_{H1} / C_{H1}$, $N_L = U_L A_L / C_L$ and $N_R = U_R A_R / C_w$.

The compressor and turbine efficiencies are defined as below:

$$\eta_c = (T_{2S} - T_1) / (T_2 - T_1) \tag{19}$$

$$\eta_t = \delta T_4 - T_5 P = \delta T_4 - T_{5S} P \tag{20}$$

Now, from Eqs. (11)-(20), we get:

$$T_{5R} = (1 - \varepsilon_R) T_5 + \varepsilon_R T_2 \tag{21}$$

$$T_{2R} = (1 - \varepsilon_R) T_2 + \varepsilon_R T_5 \tag{22}$$

$$T_x = (1-y)T_{5K} + yT_u \tag{23}$$

$$T_3 = \delta_1 - x\beta T_{2R} + xTH_1 \tag{24}$$

$$r_{2S} = (1 - \beta/c) T_1 + \beta/c T_2 \tag{25}$$

$$T_{5S} = (1 - n^{-x}) T_4 + J^{-1} T_5 \Rightarrow r_{5S} = (1 - J^{-1}) r_3 + J^{-1} r_5 \tag{26}$$

where $x = C_{HeH} = C_W$, $x_1 = CH1EH1 = C_w$ and $y = C_{LeL} = C_W$ and $T_3 = T_4$.
 However, in this case, for the Brayton cycle

$$T_{2S}/T_1 = R_T = (r_p f^{-1})^k \quad \text{and} \quad T_3/T_{5S} = T_4/T_{5S} = (r_t r_p f^{-1})^k \tag{27}$$

where R_T is the cycle temperature ratio, $r_t (= p_4 = p_3 < 1P)$ is the isothermal pressure drop ratio and $r_p (= p_2 = p_1 > 1P)$ is the cycle pressure ratio and p denotes the pressure. From Eq. (27), we have:

$$a^{1/S} / r_0 = T_4/T_{5S} \Rightarrow T_{2S} T_{5S} = a T^x T_A T_x = a T^x T_3 T_1 \tag{28}$$

where $a = (r_p f^{-1})^k$.

Substituting the values of T_1 , T_{2S} , T_3 and T_{5S} from Eqs. (21)-(26) in Eq. (28), we get a quadratic equation in T_2 as:

$$AT_2^2 + BT_2 + C = 0 \tag{29}$$

Solving for T_2 [treating T_5 as a constant] yields:

$$T_2 = \{-B \pm \sqrt{B^2 - 4AC}\} / 2A \tag{30}$$

where A , B and C depend on T_5 and are fixed for a set of operating conditions. Now, by the first law of thermodynamics:

$$P = Q_H - Q_L = Q_{HR} - Q_{LR} = c_{HeH}(r_{H1} - r_{2R}) + c_{HeH}(r_{H3} - r_3) - c_{LeL}(r_{5R} - r_{L1}) \tag{31}$$

Substituting Eqs. (21)-(24) and Eq. (30) into Eq. (31), we get:

$$P = C_w \{x(1 - x_1)T_m + x_1 T_m + y T_{L1}\} - C_w \{x(1 - e_R) + j e_R + x_1(l - x)(l - e_R)\} \\ \times \{[-B \pm \sqrt{B^2 - 4AC}] / 2A\} - C_w \{j(1 - \xi_R) + \xi_{RX} + \xi_R(1 - x)x_x\} T_5 \tag{32}$$

We see from Eq. (32) that P is a function of the single variable T_5 (as T_2 is also a function of T_5). Thus, maximizing "P" w.r.t. T_5 , viz. $\frac{dP}{dT_5} = 0$, yields:

$$A_1 T_5^2 + B_1 T_5 + C_1 = 0 \tag{33}$$

Solving Eq. (33) for T_5 , we get:

$$T_{5,opt} = \{-B_1 \pm \sqrt{B_1^2 - 4A_1 C_1}\} / A_1 \tag{34}$$

where the various parameters are given in the nomenclature.

Substituting the value of $T_{5;opt}$ into Eq. (30), we find $T_{2;opt}$, and from Eqs. (21)-(26), we can obtain the optimum temperatures. The maximum power output (P_{max}) and the corresponding thermal efficiency (g_m) can be calculated for a typical set of operating conditions.

4. Discussion of results

In order to have a numerical appreciation of the results, we consider the heat sources/sink inlet temperatures as $TH_1 = 900-1350$ K, $TH_3 = 1100-1500$ K and $T_{L1} = 290-325$ K, the effectivenesses of the various heat exchangers (e_H , e_m , ER and e_L) in the range 0.40-1.00, the turbine and compressor efficiencies (g_t and g_c) in the range 0.60-1.00 and the heat capacitance rates (C_H , CH_1 , C_L and C_W) of the various fluids in the range 0.60-1.30 kW/K. We have studied the effect of each of

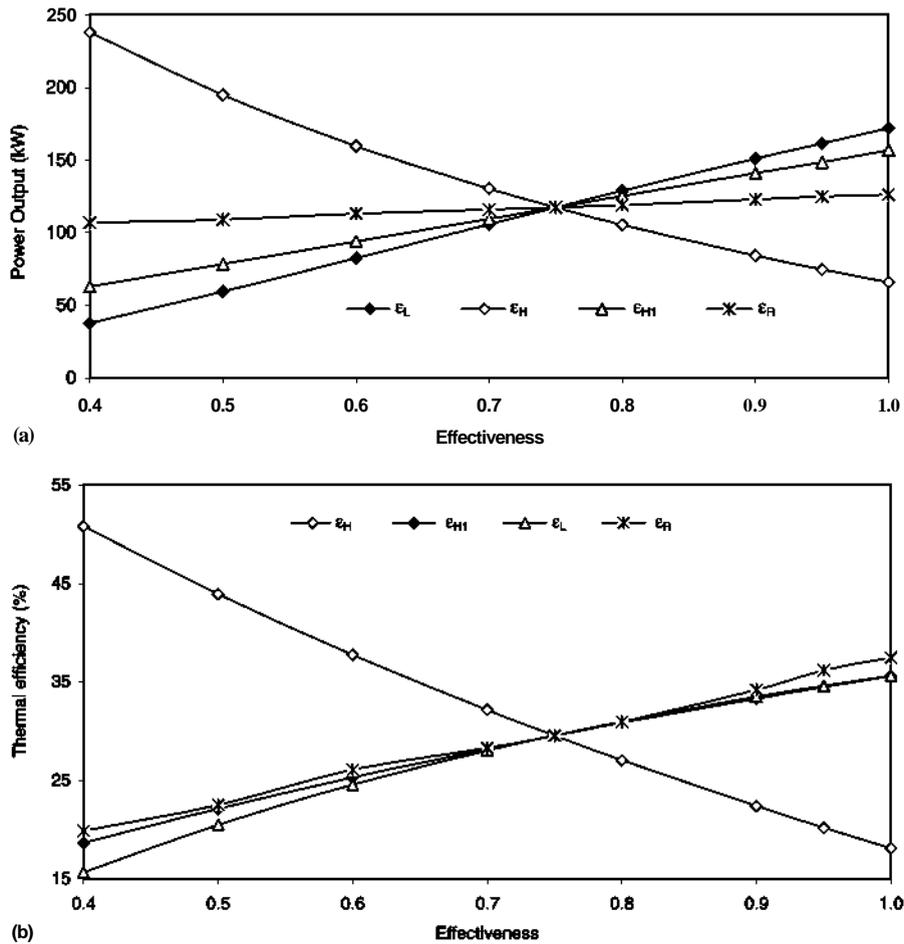


Fig. 2. (a) Power output vs effectiveness. (b) Thermal efficiency vs effectiveness.

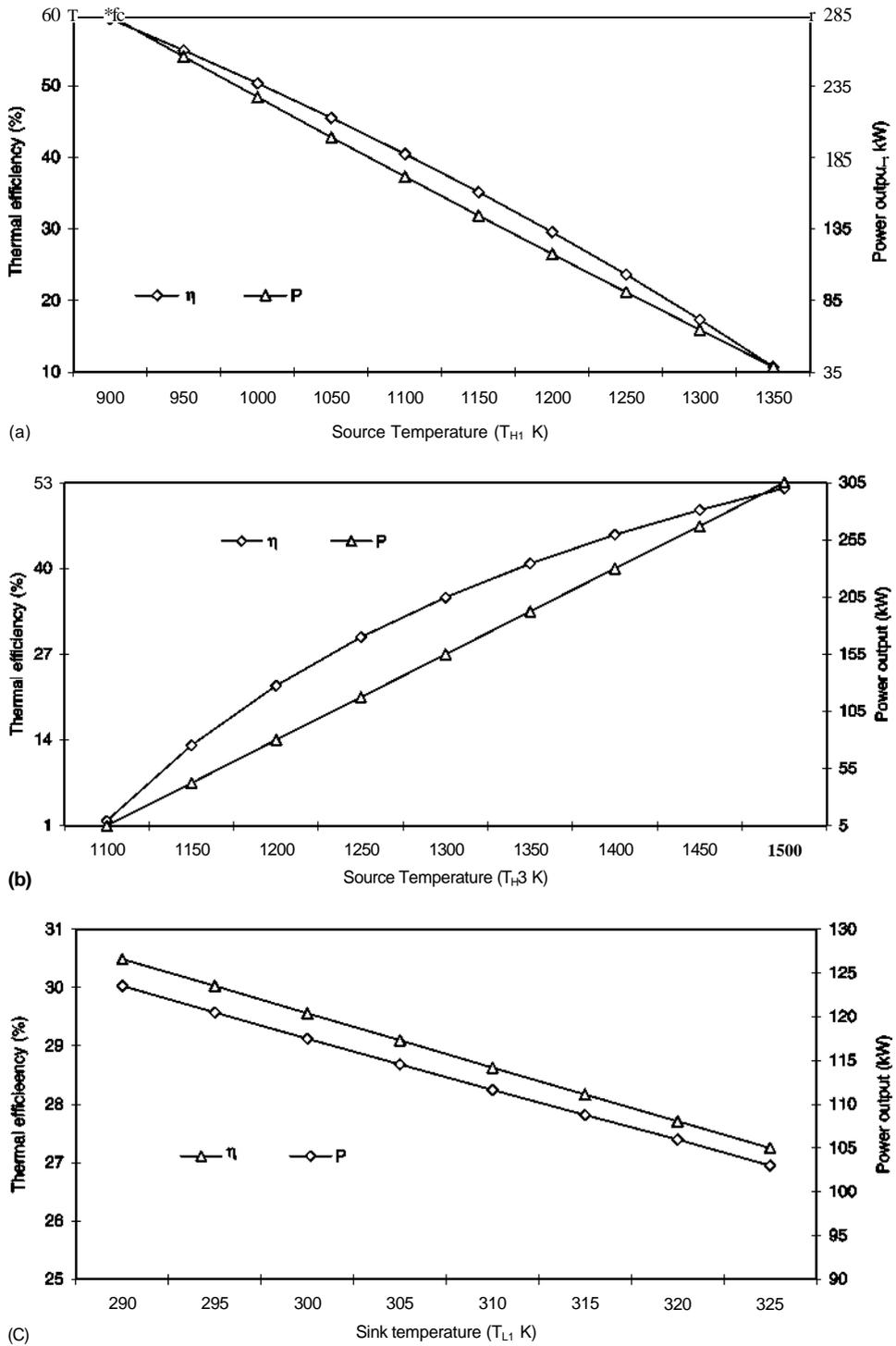


Fig. 3. (a) Power output and the thermal efficiency vs T_{H1} . (b) Power output and thermal efficiency vs T_{H3} . (c) Effect of sink temperature on the power output and thermal efficiency.

these parameters (while keeping the others constant) on the various state points temperatures, heat transfer rates, maximum power output, thermal efficiency and the cycle temperature ratio, and the discussion of the results is given below:

4.1. Effect of various effectivenesses

The effects of the effectivenesses of the various heat exchangers on the power output and the corresponding thermal efficiency of an irreversible regenerative Brayton cycle heat engine are shown in Fig. 2(a and b). It is seen from these figures that the power output and the corresponding thermal efficiency increase as the effectivenesses on the sink side (e_L), the hot side (isothermal) (e_{H1}) and the regenerative side (e_R) heat exchangers increase, while both parameters decrease as the

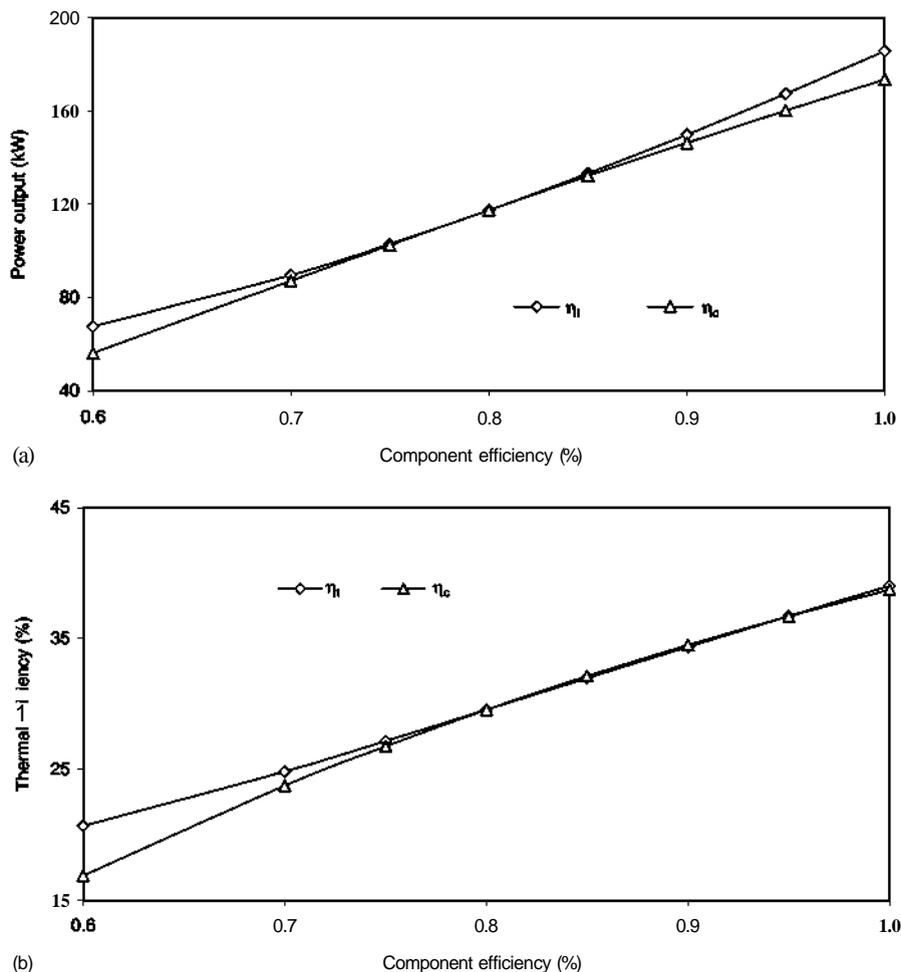


Fig. 4. (a) Power output vs component efficiency. (b) Thermal efficiency vs component efficiency.

effectiveness of the isobaric side (e_H) heat exchanger increases. The effect of the isobaric side effectiveness is rather pronounced for both parameters.

Since, the higher values of ϵ_H , ϵ_m and ϵ_L decrease the external irreversibilities on their respective sides heat exchangers by forcing the working fluid to transfer heat to and from the sink/sources reservoirs by increasing the heat transfer rates and decreasing the temperature differences. However, a higher value of ϵ_L increases power more than a higher value of ϵ_H , while a higher value of ϵ_H decreases not only the power output but also the corresponding thermal efficiency. So, it is desirable to have the relation $\epsilon_L > \epsilon_m > \epsilon_H$ for better performance of the cycle.

Although the Brayton cycle with ideal regenerator is more efficient than the Brayton cycle with real regenerator, no ideal regenerator is available in practice. Hence, it would be impossible to conclude any new results in the analysis of the Brayton cycle if the regenerative losses were not considered.

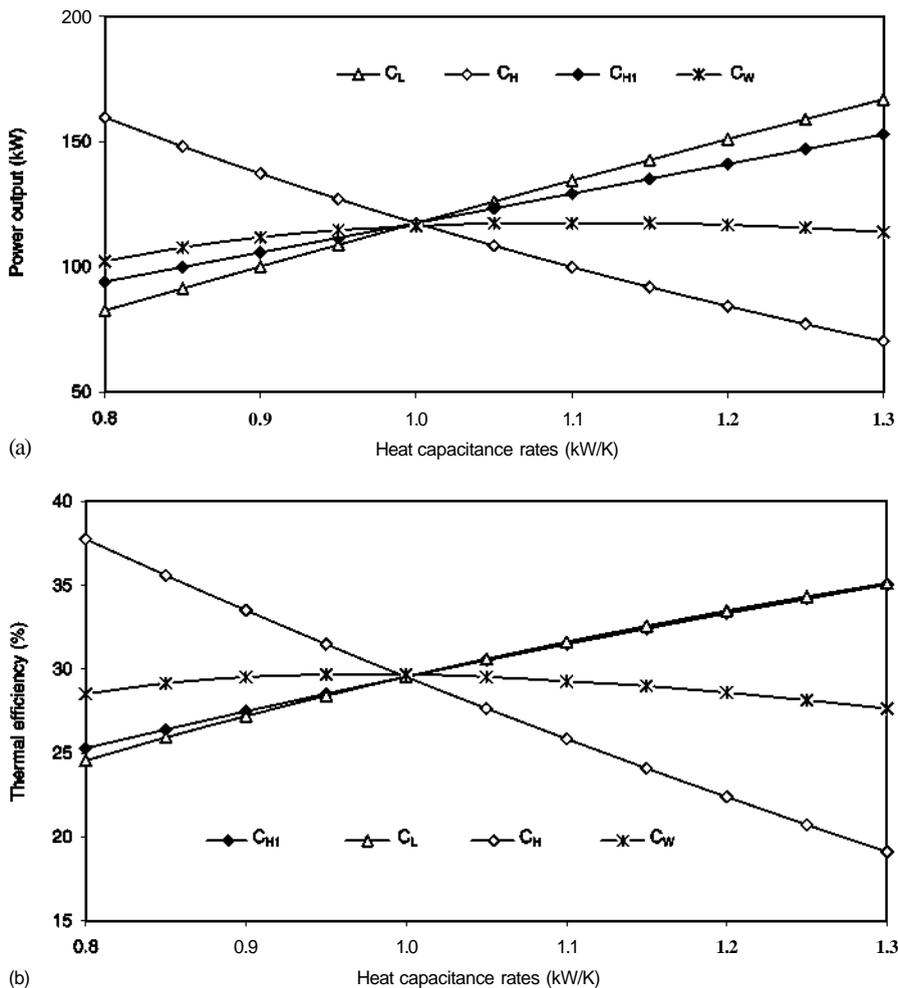


Fig. 5. (a) Power output vs heat capacitance rates. (b) Thermal efficiency vs heat capacitance rates.

4.2. Effect of reservoir temperatures

The effects of the reservoir temperatures on the power output and the corresponding thermal efficiency of an irreversible regenerative Brayton cycle are shown in Fig. 3(a)-(c). It is seen from these figures that as the temperatures of the isobaric side (TH1) and the sink side (T_L1) heat exchangers increase the power output, as well as the corresponding thermal efficiency, decreases, while both parameters increase as the temperature of the isothermal side (T_H3) heat exchanger increases.

4.3. Effect of turbine and compressor efficiencies

Fig. 4(a) and (b) show the effect of the turbine and compressor efficiencies on the power output and the corresponding thermal efficiency of an irreversible regenerative Brayton cycle. It is seen from these figures that the power output and the corresponding thermal efficiency increase as the efficiency of either component increases but the effect of turbine efficiency is slightly more pronounced than the compressor efficiency on both parameters.

4.4. Effect of heat capacitance rates

The effects of the various heat capacitance rates on the power output and the corresponding thermal efficiency are shown in Fig. 5(a) and (b). It is seen that the power output and the corresponding thermal efficiency increase as the heat capacitance rates on the isothermal side (CH1), the sink side (C_L) and, to a lesser degree, the cycle fluid (C_W) increase, while both the parameters decrease as the heat capacitance rate on the isobaric side (C_H) heat exchanger increases. The effect of the isobaric side heat capacitance rate (C_H) is more pronounced than the other heat capacitance rates not only on the power output but also on the corresponding thermal efficiency.

The higher values of heat capacitance rates decrease the external irreversibilities on their respective side heat exchangers by forcing the working fluid to transfer heat to and from the reservoirs by increasing the heat transfer rates and/or decreasing the temperature differences. However, a higher value of C_L increases the power more than higher values of the other heat capacitance rates (viz. CH1, CH, or CW), while a lower value of CH increases not only the power output but also the corresponding thermal efficiency. It is found that these heat capacitance rates should be in the order $C_L > CH_1 > CW > CH$ for better performance of these cycles.

5. Conclusions

An irreversible regenerative Brayton cycle with isothermal heat addition for finite heat capacities of the external fluids with real processes (in the compressor, turbine and regenerator) has been studied. The power output is maximized with respect to cycle temperatures, and the maximum power output and corresponding thermal efficiency are calculated for the various operating conditions. The effects of various parameters on the performance of the Brayton heat engine have been studied. There is a significant improvement in the performance of the Brayton cycle (above 15%) with isothermal heat addition over the conventional Brayton cycle. It is seen that the ef-

fectivenesses of the various heat exchangers should be high except that in the isobaric heat addition. The inlet temperature for the isobaric heat addition should be less than the inlet temperature of the isothermal heat addition for better performance because the heat addition in the former increases the quantity of energy, while in the latter, it increases the quality of energy (as the isothermal heat addition increases the kinetic energy rather than the thermal energy unlike the former source). The effect of turbine efficiency is more than that of the compressor on the performance of the cycle. The heat capacitance rates also play an important role in the performance of Brayton cycles, and it is found that there is a required order in choosing the heat capacitance rates of the various reservoirs and the working fluid. For this case, the order is found to be $CL > CH1 > CW > CH$. Thus, the present parametric study is useful to understand the deviation of the performance of a real cycle from an ideal one and could be useful to design the theoretical Brayton cycles model with one or two additional heat sources.

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