A MATHEMATICAL MODEL FOR THE DISPERSION OF AIR POLLUTANTS IN LOW WIND CONDITIONS

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Abstract—For the atmospheric dispersion of pollutants in low wind conditions, a steady-state mathematical model has been formulated which takes into account the diffusion in all the coordinate directions and the advection due to mean wind. The analytical solution has been obtained by assuming constant eddy diffusivity coefficients in the advection–diffusion equation. The solution reduces to a familiar form which yields the standard Gaussian plume solution when slender or thin plume approximation is invoked. The analytical solution obtained matches with the Gaussian plume solution for ground-level concentration at the plume centreline.

The model has been evaluated with data collected from the experiments conducted at the sports ground of the Indian Institute of Technology, Delhi (India). The results are reasonably good keeping in view the limitations of the data and the model. The model performs well as indicated by the agreement between the observed concentrations and those calculated using 30 min and 3 min averaging.

Key word index: Mathematical model, low wind dispersion, tracer experiment.

INTRODUCTION

Air pollutants released from various sources affect directly or indirectly man and his environment. The resulting ground level concentration patterns have to be estimated for a wide variety of air quality analyses for social planning and industrial growth. Air pollutants emitted from different sources are transported, dispersed or deposited by meteorological and topographical conditions. Dispersion of pollutants in the atmosphere is governed by the following dominant mechanisms (Wark and Warner, 1981):

1. mean air flow that transports the pollutants downwind, and
2. turbulent velocity fluctuations that disperse the pollutants in all directions.

Wind speeds less than 2 m s\(^{-1}\) are generally considered to be low, as most of the conventional models for dispersion are to some extent suspect because of their assumptions when the wind speed falls below about 2 m s\(^{-1}\) (Smith, 1983). These are of frequent occurrence at many sites, particularly in tropical regions and have a great potential for pollution episodes. Pasquill (1961) asserted that diffusion under low wind conditions is very irregular and indefinite. Later, several diffusion experiments under light-wind, stable conditions supported Pasquill's assertion and indicated that effective estimates of dispersion coefficients (\(\sigma\)'s) can correspond to any atmospheric stability. Under low wind conditions during daytime, instantaneous plume usually does not spread much horizontally whereas during night it can meander over a wide angle (Etling, 1990). The resulting observed concentration distribution is non-Gaussian and multi-peaked. As is evident from field experiments and theoretical considerations (Kristensen et al., 1981; Hanna, 1983), averaged concentration of pollutants under meander conditions can be a factor 2–6 lower than for the situation with straight plumes downwind of the point sources. In most of the Gaussian plume models, when mean wind speed becomes very low (\(< 2\) m s\(^{-1}\)) the pollutant concentration tends to go exceptionally high because \(U\) appears in the denominator. Thus, it has become necessary to study dispersion patterns during low wind conditions.

Generally, under moderate to strong winds, diffusion in the direction of wind is neglected in comparison to advection. But this may not be the case in low winds and, consequently, diffusion could be comparable with advection in the direction of wind. Arya (1995) has shown that the conditions under which downwind diffusion might be important are essentially strong convection and weak winds (\(U < 1.5\) m s\(^{-1}\)). In the past, different modelling approaches have been adopted to deal with low winds. Various investigators have attempted modifications at the application level (i.e. in terms of the dispersion parameters).
Sagendorf and Dickson (1976) used schemes like split-sigma and segmented plume to improve upon the standard method to deal with nighttime low wind cases. For the estimation of $\sigma$, the split-sigma scheme involves the splitting of stability into horizontal and vertical whereas the segmented plume method consists in splitting the test period into sub-intervals. Zannetti (1981) has suggested a modification for computation of dispersion parameters ($\sigma$) in low winds. He introduced the concept of $U_{\text{min}}$ in order to artificially dilute the plume. Recently, Cirillo and Poli (1992) have intercompared four semi-empirical schemes with the diffusion data in low wind speed, inversion conditions. In a more recent study, Sharan et al. (1995a) have suggested "short-term averaging" approach which is quite useful in the absence of detailed wind data.

Sirkov and Djolov (1979) made an attempt to model the dispersion of pollutants from a continuous source in the absence of the wind. However, the parameterization of turbulent exchange coefficients to compute concentrations for practical purposes is left undone. Demuth et al. (1978) have proposed an analytical model for calm wind situations when there is a finite mixing height. For unstable cases, they have taken the model by Berlyand and Kurenbin (1970) defined by diffusivity coefficients varying in space, and introduced a reflection at the top of the mixing layer. For the stable cases, they have taken a model with constant diffusivity coefficients. Yokoyama et al. (1979) have derived a puff formula for computing the concentration of smoke emitted from a point source in calm wind conditions by expressing the dispersion parameters as linear functions of time. However, it requires a prior knowledge of the time for which calm conditions are persistent. In some cases, an arbitrary value of the persistent time was assumed. Yamamoto et al. (1986) have simulated the lateral profiles of concentration in weak and variable wind conditions by calculating the trajectories of the virtual particles emitted from a ground level source. These trajectories are computed using wind speed and direction obtained from tower observations. However, this method provides the proportional (relative) concentration at a point rather than the actual concentration. They also developed a non-Gaussian model for low and variable wind conditions by assuming that the lateral concentration profiles are proportional to the angular distribution of the wind direction and inversely proportional to the product of wind speed and distance from the source. Okamoto and Shiozawa (1987) have proposed a trajectory plume model to handle calm and weak wind conditions. They have integrated three-dimensional puff model with respect to time by assuming the growth rate of spreading plume to be linear with diffusion time. Various aspects of atmospheric diffusion in low winds have recently been reviewed by Yadav et al. (1995).

The currently available dispersion parameters are based on field experiments conducted mostly in mid-latitudes. These may not be applicable to tropical conditions. Therefore, a series of SF$_6$ (tracer) field experiments were conducted at IIT, Delhi, in order to formulate the horizontal dispersion parameter for different stabilities in tropical conditions. At this location, low winds occur frequently during day as well as nighttime. Another objective is to develop a suitable mathematical model to study dispersion patterns in low wind conditions. In the present study, an attempt has been made in this direction. A mathematical model for low wind dispersion is formulated and is evaluated with the field data.

**MATHEMATICAL FORMULATION**

Dispersion of a pollutant in the atmosphere is governed by the basic atmospheric diffusion equation. Under non-isotropic conditions, the atmospheric diffusion equation satisfying the equation of continuity can be written as

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = \frac{\partial}{\partial x} \left( K_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial C}{\partial z} \right) + S + R \quad (1)$$

where $C$ is the pollutant concentration, $S$ represents the source term, $R$ is the removal term, and $u$, $v$, and $w$ are the wind components and $K_x$, $K_y$, and $K_z$ are the eddy diffusivity coefficients along the $x$, $y$ and $z$ directions, respectively.

Equation (1) forms the basis of most air pollution models. On the left-hand side of equation (1), the first term is time-dependent which accounts for non-stationary conditions whereas the remaining terms represent the transport due to wind. The first three terms on the right-hand side of equation (1) represent eddy diffusion in the $x$, $y$ and $z$ directions, respectively.

The following assumptions are made:

1. Steady-state conditions are considered.
2. There is no removal or washout of the pollutants, i.e. $R = 0$.
3. As the magnitude of the vertical component of velocity is smaller than that of the horizontal, advection in the vertical direction is neglected in comparison to that in the horizontal direction. This assumption is valid except for particulate pollutants (with appreciable settling velocities) under low wind speed conditions.
4. The emission source is assumed to be located at the origin and accordingly

$$S = q \delta(x) \delta(y) \delta(z) \quad (2)$$

where $q$ is the emission rate, and $\delta$ is the Dirac's delta function.
5. Eddy diffusivities are considered to be constant.
Based on the above assumptions, equation (1) becomes

\[ u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = K_x \frac{\partial^2 C}{\partial x^2} + K_y \frac{\partial^2 C}{\partial y^2} + K_z \frac{\partial^2 C}{\partial z^2} + q \delta(x) \delta(y) \delta(z). \]  

(3)

**Boundary conditions**

Equation (3) is subject to the following boundary conditions:

1. Pollutant concentration tends to zero at large distances from the source

\[ C \to 0 \quad \text{as} \ |x|, \ |y|, \ z \to \infty. \]  

(4)

2. The pollutant is not absorbed at the ground and therefore there is no diffusion flux at the surface, i.e.

\[ -K_z \left( \frac{\partial C}{\partial z} \right) = 0 \quad \text{at} \ z = 0. \]  

(5)

Equation (3) is a three-dimensional linear elliptic partial differential equation. We solve it analytically, using the boundary conditions (4) and (5), in the next section.

**SOLUTION**

Using the transformation:

\[ X = xK_x^{-1/2}, \quad Y = yK_y^{-1/2}, \quad Z = zK_z^{-1/2} \]

equation (3) becomes

\[ U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} + \frac{\partial^2 C}{\partial Z^2} + q \delta(X) \delta(Y) \delta(Z) \]  

(6)

where \( U = uK_x^{-1/2}, \ V = vK_y^{-1/2}, \ q' = q(K_xK_yK_z)^{-1/2}. \)

Using the cosine transform

\[ C(X, Y, \lambda_3) = \frac{2}{\sqrt{\pi}} \int_0^\infty C(X, Y, Z) \cos(\lambda_3 Z) dZ \]

equation (6) reduces to

\[ U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} - \lambda_3^2 C + \frac{q'}{2\pi} \delta(X) \delta(Y) \]  

(7)

Now, using double Fourier transform

\[ C(\lambda_1, \lambda_2, \lambda_3) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(X, Y, \lambda_3) e^{-2\pi i (\lambda_1 X + \lambda_2 Y)} dX dY \]

equation (7) becomes

\[ C = \frac{q}{\sqrt{\pi}} \frac{1}{4\pi^2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)} \frac{1}{2\pi i (\lambda_1 U + \lambda_2 V)}. \]  

(8)

On inverting (8) with respect to \( \lambda_1 \) and \( \lambda_2 \), using \( \xi = 632, 207 \) and \( 917 \) (Campbell and Foster, 1957, pp. 74, 39, 125) we obtain

\[ C(X, Y, \lambda_3) = \frac{q}{2\pi} \frac{1}{\sqrt{\pi}} \exp \left[ \frac{UX + VY}{2} \right] \]

\[ \times K_0 \left[ (X^2 + Y^2)^{1/2} \left( \lambda_3^2 + \frac{1}{4}(U^2 + V^2)^{1/2} \right) \right]^2 \]  

(9)

where \( K_0 \) is the modified Bessel function of the second kind of zero order.

Inverting equation (9) with respect to \( \lambda_3 \)

\[ C(X, Y, Z) = \frac{q}{2\pi} \frac{1}{\sqrt{\pi}} \exp \left[ \frac{UX + VY}{2} \right] \]

\[ \times \int_0^\infty K_0 \left[ (X^2 + Y^2)^{1/2} \left( \lambda_3^2 + \frac{1}{4}(U^2 + V^2)^{1/2} \right) \right]^2 \cos(\lambda_3 Z) d\lambda_3. \]  

Using \( \xi = 6677.5 \) (Gradshetein and Ryzhik, 1981, p. 736), we get

\[ C(X, Y, Z) = \frac{q}{2\pi (X^2 + Y^2 + Z^2)} \exp \left[ \frac{UX + VY}{2} \right] \]

\[ \times \exp \left[ -\frac{1}{2}(U^2 + V^2)^{1/2} \right] \times \left( X^2 + Y^2 + Z^2 \right)^{1/2}. \]  

(10)

In original coordinates \((x, y, z)\)

\[ C(x, y, z) = \frac{q}{2\pi (K_xK_yK_z)^{1/3}} \exp \left[ \frac{1}{2} \left( \frac{ux}{K_x} + \frac{vy}{K_y} \right) \right] \]

\[ - \frac{1}{2} \left( \frac{u^2}{K_x} + \frac{v^2}{K_y} \right)^{1/2} \]  

(11)

where

\[ r = \left[ \frac{x^2 + y^2 + z^2}{K_x + K_y + K_z} \right]^{1/2}. \]

Equation (11) allows us to compute the concentration at any point \((x, y, z)\) with reference to fixed coordinate axes. Assuming \(x\)-axis to be in the direction of wind, i.e. \(v = 0\), and taking, \(u = U\), we have

\[ C(x, y, z) = \frac{q}{2\pi (K_xK_yK_z)^{1/3}} \]

\[ \times \exp \left[ \frac{UX}{2K_x} - \frac{U}{2(K_x)^{1/2} r} \right] \]  

(12)

where \( r \) is same as in equation (11).
The solution (12) can also be obtained by integrating the solution (i.e., puff solution) of the corresponding time-dependent equation with respect to time from 0 to $\infty$. For a similar problem, Roberts (1923) and Seinfeld (1986) have used the boundary condition for $z$ from $-\infty$ to $\infty$, probably in analogy to puff approach, which does not seem to be realistic physically in the case of a plume.

Slender plume approximation (a particular case)

For the case of emission from a point source, we can use the slender or thin plume approximation when the crosswind spread of the plume is small compared to the downwind distance it has travelled. In other words, slender plume approximation indicates that only the concentrations close to the plume centerline are of importance. Equation (12) can be rewritten as

$$C(x, y, z) = \frac{q}{2\pi(K_yK_z)^{1/2}} \times \exp \left[ \frac{U_x(1 + p)^{1/2}}{2K_x} \right]$$

(13)

where

$$p = \frac{K_x}{x^2} \left( \frac{y^2}{K_y} + \frac{z^2}{K_z} \right).$$

For thin plume approximation, we should have

$$p \ll 1$$

(14)

which is equivalent to having

(1) \( \frac{y^2 + z^2}{x^2} \ll 1 \)

(2) \( \frac{K_y}{K_x} \leq O(1), \quad \frac{K_z}{K_x} \leq O(1) \)

(15)

where “O” represents the order.

The condition (15) seems plausible under moderate to strong winds. Equation (13) can be simplified using binomial expansion and condition (15) to give

$$C = \frac{q}{2\pi(K_yK_z)^{1/2}} \times \exp \left[ -\frac{U}{4x} \left( \frac{y^2}{K_y} + \frac{z^2}{K_z} \right) \right]$$

(16)

which is the classical solution in the Gaussian form. The same result can also be obtained as a limiting case by taking $K_x \rightarrow 0$ which is the same as neglecting downwind diffusion.

Thus, it is worthwhile to examine theoretically the situations of overprediction/underprediction by formula (16) in treating weak wind cases. This becomes important as formula (16) is simple, conceptually appealing, and computationally efficient. Referring to equation (13) which is just a rearrangement of equation (12), we can write the ratio, $R$, of equations (16) and (12) as

$$R = \sqrt{(1 + p) \exp \left[ \frac{\beta}{2} \left( \frac{(1 + p)^{1/2} - (1 + p/2)}{2K_x} \right) \right]}$$

(17)

where $p$ and $\beta$ are dimensionless parameters defined by

$$p = \frac{K_x}{x^2} \left( \frac{y^2}{K_y} + \frac{z^2}{K_z} \right) = \frac{y^2 + z^2}{X^2}, \quad \beta = \frac{U_x}{K_x}.$$  

(18)

Notice that $\beta$ resembles the well known Peclet number, $Pe$, and it essentially represents the ratio of advective (convective) transport to diffusive transport. Here, both $p$ and $\beta$ can take only positive values. Regarding the physical interpretation of $p$, one can think of the magnitude of $p$ as giving the region of interest relative to the plume axis. The values of $p$ close to zero represent the region in the proximity of the plume centreline. For application purposes, one can safely restrict oneself to the range (0,1) for $p$. On the other hand, $\beta$ can be physically interpreted as the parameter whose magnitude indicates the atmospheric conditions in terms of the strength of winds. Small values of $\beta$ can be related to the weak winds when the downwind diffusion becomes important and the region of interest remains close to the source, whereas large values of $\beta$ imply moderate to strong winds when the downwind diffusion is neglected in comparison to the advection and the region of interest extends to a larger distance from the source. For $p = 0$, we have $R = 1$ for all finite values of $\beta$. However, $p$ equals zero in two cases: (1) $y = 0$ and $z = 0$, which implies that the ground-level concentration along the plume centreline is not affected by the inclusion of the downwind diffusion term in the formulation; (2) $K_x \rightarrow 0$ (neglecting diffusion along the axis of the wind).

For other cases, we have studied the sensitivity of $R$ with respect to the parameters $p$ and $\beta$. Figure 1 shows the variations of $R$ with the parameter $\beta$ for various values of $p$. The figure reveals that for values of $\beta$ less than two, the ratio $R$ increases with the increase in $p$. That is, for the condition of weak winds (when the downwind diffusion is expected to contribute) the application of formula (16) leads to overestimation, for example, to the extent of about 25% when $p = 0.6$. Even for the higher values of $\beta$, there is an overpredicting trend for a small range of $p$ (upto 0.4 in this figure). Finally, the situation with higher values of $\beta$ and $p$ (close to 1) are physically uncommon and the variation of $R$ is found to be in reverse order.
PARAMETERIZATION OF EDDY DIFFUSIVITIES

For computing the concentration using formula (12), we require suitable parameterization of the eddy diffusivity coefficients $K_x$, $K_y$, and $K_z$. We express $K_y$ and $K_z$ in terms of $\sigma_x$ and $\sigma_z$, the standard deviations of the crosswind and vertical Gaussian concentration distribution respectively, as follows (Rao, 1983):

$$K_y = \frac{U}{2x} \sigma_x^2, \quad K_z = \frac{U}{2x} \sigma_z^2. \quad (19)$$

In analogy to $K_y$ and $K_z$, we express $K_x$ in terms of $\sigma_y$ (Llewelyn, 1983).

In order to make use of the large amount of empirical data on the dispersion parameters, $\sigma_x$, $\sigma_y$, and $\sigma_z$, available in the literature for various meteorological and terrain conditions, $K_y$ and $K_z$ can be expressed as (Rao, 1983)

$$K_y = \frac{U}{2x} \sigma_y^2, \quad K_z = \frac{U}{2x} \sigma_z^2. \quad (20)$$

The studies related to dispersion parameters have been reviewed extensively by Gifford (1976), Hanna et al. (1977), Irwin (1983) and Yadav et al. (1995). Thus, in terms of $\sigma_x$, $\sigma_y$, and $\sigma_z$, solution (12) can be written as:

$$C(x, y, z) = \frac{q}{\pi U \sigma_x \sigma_y \sigma_z} \exp \left[ \frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2} - \frac{z^2}{\sigma_z^2} \right] \quad (21)$$

where

$$s = \left[ \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2} \right]^{1/2}.$$

It may be noted that solution (12) does not resemble the normal distribution. The parameterization of eddy diffusivity in terms of dispersion parameters through equation (20) is usually invoked, in a mathematical sense, when the solution is of Gaussian type. However, the Fickian diffusion approximation (i.e. equation (20)) which is also the solution to Taylor's statistical diffusion equation at large $t$ is used in this study.

EXPERIMENTAL SETUP FOR VALIDATION

As mentioned in the Introduction, the field experiment was conducted at the sports ground, IIT Delhi to formulate the horizontal dispersion parameter in the tropics (Singh et al., 1991; Agarwal et al., 1995). We have utilized this diffusion data set to validate our model. For the sake of completeness, we describe here briefly some of the relevant experimental details.

The experimental site lies in the city, has flat terrain but surrounded by buildings and other structures on almost all sides, and hence has been chosen to study dispersion in urban terrain. The sampling grid involved receptors on 50, 100 and 150 m and in some cases on 200 m circular arcs with 45° angular spacing between them. The layout plan is shown in Fig. 2. Samples were collected at various times of the day to cover almost all atmospheric stabilities. SF$_6$ tracer was released at a height of 1 m above the ground and the samplers were also placed approximately at the same height. Thus, for computational purposes the release can be considered as a ground source. On the first day, tracer was released continuously at the rate of 50 ml rain$^{-1}$. On subsequent days, release periods were of 60 min duration with sampling during the later half, and the release rate varied between 30 to 50 ml min$^{-1}$. The relevant details regarding the experiment are given in Table 1. Meteorological data (wind speed, wind direction, etc.) have been obtained from a 30 m multi-level micrometeorological tower installed close to the experimental site. It was instrumented mainly with cup anemometers, wind vanes and temperature sensors at 1, 2, 4, 8, 15 and 30 m levels. A sonic anemometer was mounted at 4 m but unfortunately it had some technical problems regarding its operations during the tracer experiments. Wind instruments supplied mean variables every 3 min.
Fig. 2. Layout of the site for tracer diffusion experiments.

Table 1. Details of the tracer experiments conducted at the sports ground, IIT Delhi in February 1991

<table>
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<tr>
<th>Run no.</th>
<th>Sampling time (h)</th>
<th>Date (m-d-y)</th>
<th>Release period</th>
<th>Release rate (ml min⁻¹)</th>
<th>Layout</th>
<th>W/S (m s⁻¹)</th>
<th>W/D (deg)</th>
<th>P-G stab. class</th>
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<tr>
<td>1</td>
<td>1200-1230</td>
<td>02-13-91</td>
<td>continuous</td>
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<td>W/S</td>
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<td>1530-1600</td>
<td>02-13-91</td>
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<td>02-13-91</td>
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*a 15 m level meteorological data are used in the computations.

*bRelease point has been shifted 100 m towards northwest.

The existing formulations for the dispersion parameters can be broadly classified into the following three groups:

(a) methods based on power law functions (Briggs, 1973);
A mathematical model for the dispersion of air pollutants

(b) methods based on statistical parameters such as horizontal and vertical wind direction variances ($\sigma_{u}, \sigma_{v}$) (Draxler, 1976);
(c) methods based on similarity theory (Hanna et al., 1982).

Methods based on similarity theory are difficult to use as they require the knowledge of parameters such as friction velocity ($u_{*}$), convective scaling velocity ($w_{*}$), mixed layer height ($Z_{i}$). The formulations based on statistical methods involve $\sigma_{u}$ and $\sigma_{v}$ which need large amount of meteorological data for their computations. Thus, in the present study, we have adopted dispersion parameters for urban terrain by Briggs which are based on power law functions. These are analytical expressions depending upon downwind distance and atmospheric stability. The atmospheric stability has been calculated from Pasquill’s turbulence typing scheme (Gifford, 1976) based on wind speed, solar insolation, and cloud cover.

RESULTS AND DISCUSSION

The concentration is computed using data collected at 4 m height (wherever 4 m data are not available, 15 m data are used) of a 30 m multi-level micrometeorological tower. In all, 16 test runs were conducted for the purpose of computation, out of which 2 have been dropped due to loss of data. Some of the receptors falling close to the well, stadium and the wall (Fig. 2) in the field have been discarded because of the possibility of unusual accumulation, stagnation or re-circulation, etc. Minimum threshold value of the instrument used for wind speed is 0.3 m s$^{-1}$ and the observations below this value have been replaced by 0.3 m s$^{-1}$.
Table 2. Observed and predicted concentrations for Run 7 obtained with 3 and 30 min averaging

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>Observed conc. (ppt)</th>
<th>Predicted conc. (ppt)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>30 min</td>
</tr>
<tr>
<td>1</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>248</td>
<td>411</td>
</tr>
<tr>
<td>3</td>
<td>190</td>
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<td>18</td>
<td>143</td>
<td>46</td>
</tr>
<tr>
<td>19</td>
<td>72</td>
<td>13</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

ppt—parts per trillion.

The micrometeorological tower provided the average wind data for every 3 min period. However, the sampling period for each test run was 30 min. Thus, the concentration at a receptor can be computed in the following two ways:

(i) applying formula (21) with $z = 0$ for every 3 min and then averaging out for the test period. This is referred to as 3 min averaging.

(ii) using formula (21) with $z = 0$ for half hourly average of the wind data. This is 30 min averaging.

As an illustration, results computed from these approaches are shown in Table 2, for a typical test Run 7. This shows that short-term averaging (i.e. 3 min) covers a larger number of receptors in comparison to 30 min averaging. This is due to the fact that wind direction variation is taken into account by dividing the sampling period into smaller intervals. We point out that there is no sanctity about the 3 min averaging period. Depending upon the nature of the available wind data, the averaging period can be so chosen as to satisfy the condition $t_a > x_{max}/U$, where $t_a$ is the averaging time, $U$ is the wind speed averaged over the period $t_a$, and $x_{max}$ is the distance of the receptor farthest from the release point in the direction of the wind.

The maximum concentrations computed using 3 and 30 min averaging are shown in Fig. 4 along with the corresponding observed maximum values. The comparison shows that in most of the cases predictions are reasonably close to the observations for both the approaches. Also, there is no significant difference between the predictions by the two methods (except in one or two cases). Because of the wide scatter in the field data, we could not capture properly the role of short-term averaging. However, it has been observed that this approach is quite often helpful in dealing with weak and variable wind conditions (Sagendorf and Dickson, 1976; Sharan et al., 1995a). Therefore, in the rest of the analysis, we restrict ourselves to 30 min averaging. Here, we would like to mention that in one or two cases, the predicted maximum does not coincide with the observed one. This may be due to the fact that wind measuring instruments such as wind
vane and cup anemometer do not respond reliably for very low wind speeds. This can be improved upon by making correction in the wind direction. Furthermore, in Runs 2, 7 and 14, the maximum concentration is observed at the outer arc (100 m). Stagnation, re-circulation, puddling and meandering are significant processes observed in low wind conditions which might have been responsible for concentration at the outer arcs being greater than at the inner arcs.

Not only the maximum concentration, but also the overall situation is reasonably good as evident from the scatter diagram of predicted vs observed concentration (Fig. 5(a)). Majority of the points lie around the line of perfect correlation within a factor of 2. This is also evident from the plot of the ratio (predicted/observed) against the observed values (Fig. 5(b)). Figure 6 shows that more than 65% of the predicted cases are within a factor of 2.

As pointed out earlier, the concentration distribution given by equation (12) is non-Gaussian. After comparing the concentrations computed from our model with those obtained from the Gaussian model, we found that the difference between the two is not
Factor > 6

spheric conditions. Thus, the highest concentrations are expected near the ground for a ground-level non-buoyant source. The concentration trend is not uniform with the atmospheric stabilities for higher values of $z$. This is justified since the area under the concentration distribution curve is constant due to mass conservation for all stabilities.

Similarly, the crosswind and the downwind behaviour of the concentration distribution can be plotted and examined for the general characteristics.

LIMITATION AND IMPROVEMENTS

There are some limitations of this study from experimental and modelling points of view. These are:

(a) less number of samplers,
(b) angular spacing between the samplers being too large,
(c) 30 m tower not on-site,
(d) poor response of the wind measuring instruments during light winds, and
(e) using the classical approach of converting eddy diffusivities ($K$'s) to Gaussian dispersion parameters, although the concentration distribution is non-Gaussian.

Smaller number of samplers and large angular spacing between the adjacent samplers would be less useful in situations when there is almost a fixed plume direction with little lateral spread. So, to capture the concentration pattern better, we should have more samplers with lesser spacing between the consecutive ones. Although not on-site, 30 m micrometeorological
A mathematical model for the dispersion of air pollutants

tower could be taken as representative since it is located only 300 m (approx.) away from the release point. Data from fast responding and more sensitive instruments such as sonic anemometer could have improved the reliability of the meteorological information in low wind conditions. With the availability of faster data, we could have explained phenomena like meandering, puddling and re-circulation, etc., which are generally responsible for observation of higher concentration at the outer arcs than at the inner arcs and non-zero concentration in the upwind side. Moreover, with the calculation of $\sigma_\theta$ and $\sigma_u$ we could have used more realistic estimation of dispersion parameters based on direct observations.

The approach of taking $K$'s to be constant initially in the formulation and relaxing it later, although mathematically inconsistent, it has been widely accepted from the application point of view. As a simple case, Sharan et al. (1995b) have solved the problem with eddy diffusivities as linear functions of the downwind distance.

CONCLUSIONS

A mathematical model has been developed for low wind conditions by taking into account the diffusion in the downwind direction. The analytical solution so obtained turns out to be non-Gaussian. However, for practical application of the model we have used the conventional conversion from eddy diffusivities to Gaussian dispersion parameters. The standard Gaussian plume solution has been shown as a limiting case of the model when the concentration close to the plume centreline is desired. Equivalent is the situation when the downwind diffusion is neglected ($K_z \to 0$). The ratio $R$ of the two solutions has been expressed as a function of the dimensionless parameters $p$ and $\beta$ and it is suggested that theoretically the Gaussian plume equation leads to considerable overestimation during low wind conditions. The model has been evaluated with tracer data from IIT-SF experiment. Although the data set collected from this experiment has some limitations, we could get reasonable agreement between the observed concentrations and those calculated from the model.

Short-term averaging (3 min) was not very useful in the present study; however, it provides good explanation for the situations where the observed concentration pattern is wide and has multiple peaks (Sharan et al., 1995a) due to fluctuations in the wind. In the existing framework, the model performed well with the given data set. However, this model needs testing with more data sets under different conditions for better understanding of its performance and domain of applicability.

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REFERENCES


