Rapid performance evaluation of journal bearings

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This paper describes a rapid method for evaluating the significant design parameters such as load capacity, maximum pressure, flow, power loss, and maximum temperature in the oil film. The proposed analytical pressure expression is a modification of that given by Reason and Narang. An analytical expression for maximum pressure is presented. The accuracy of the proposed modification is validated up to an eccentricity ratio of 0.99. The effective temperature rise, which depends on the fraction of heat generation carried away by lubricant, is chosen to be a function of the eccentricity ratio. An expression for maximum temperature, based on existing experimental findings, is given. A journal bearing design table is provided to help the designer without the involvement of numerical and mathematical complexities.

Keywords: *rapid design of journal bearings, thermal effects, simplified bearing analysis*

Introduction

Journal bearings are used extensively in rotating machines because of their low wear and good damping characteristics. From a designer's perspective, a journal bearing must support the required load by occupying minimum space with minimum energy loss and slow wear out. Various techniques for analysis and checking the performance of journal bearings are available in the literature. These are either too involved because of the mathematical complexities of hydrodynamics or are based on oversimplified methods and design charts. With the continuous upgrading of machinery this latter approach is seldom adequate. An approach that predicts fairly accurate results as well as being easy in use for all practicable ranges, is the best compromise for designers[1].

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The simplest form of analytical designs are based on short and long bearing approximations which are, however, very inaccurate in most practical design ranges. To improve the solution accuracy of long bearing approximation, Warner[2] used a side flow leakage factor by arguing that the error in approximation is small compared to the errors introduced by assumptions of constant viscosity, aligned bearing, and without surface irregularities. Similarly, to improve the accuracy of short bearing approximation at high eccentricity ratios, Ritchie[3] introduced the optimized short bearing solution by using Galerkin's method. This approach is tedious and less accurate at large L/D ratios. A complex analytical model of second order approximation, which reduces to the well known short and long bearing theories for limiting values of the pair of parameters (L/D, ϵ), is given by Capone[4]. Although fairly accurate, this method can not be extended for stability analysis, without introducing further complexities. An interesting paper, presenting a simple and accurate combination of the short and the long bearing solution is presented by Reason and Narang[1]. This technique predicts good results compared with other existing analytical methods. The solution accuracy is even comparable with the finite element method (FEM) for short bearings at low eccentricity ratios. With an increase in eccentricity and slenderness ratios, however, this technique overestimates the important design variables, viz. load capacity, flow entertainment and power loss. The

Notation

C	radial clearance, m
C_O	specific heat capacity of lubricant, J/kg.°C
D	journal diameter, m
F	friction force, N
F_η	ratio of friction loss to viscosity, m^2/s
g_O, g_S	pressure correction factors for Ocvirk's and Sommerfeld bearings
H, H_{pmax}	non-dimensional film thickness, film thickness at maximum pressure location
L	bearing length, m
N	journal rotational speed, rpm
P	dimensional film pressure, Pa
P_θ, P_O, P_S	film pressure for Reason and Narang, Ocvirk and Sommerfeld bearings, Pa
P_{max}	Maximum pressure, Pa
P_{supply}	supply pressure, Pa
Q_H, Q_P	side leakage due hydrodynamic action and feed pressure, m^3/s
$Q_{leakage}$, Q_{rec}	total side flow, recirculating flow, m^3/s
R	journal radius, m
T_{in}, T_{eff}, T_{max}	inlet temperature, effective temperature, maximum temperature, °C
U	journal surface velocity, m/s
W	dimensional load capacity, N
W_ϵ, W_ϕ	dimensional load capacity along and perpendicular to line of centres, N
W_η	ratio of dimensional load capacity to viscosity, m^2/s
$W_{\epsilon\eta}$, $W_{\phi\eta}$	ratio of W_ϵ, W_ϕ to viscosity, m^2/s
z	co-ordinate in axial direction, m
Δt	temperature rise, °C
Δt_η	ratio of temperature rise to viscosity, °C/Pa.s
Λ	slenderness ratio (L/D)
β	temperature-viscosity coefficient, K^{-1}
ϵ	eccentricity ratio
ϕ	attitude angle, radian
η	viscosity coefficient of lubricant, Pa.s
η_{eff}	effective oil viscosity, Pa.s
η_{in}	inlet oil viscosity, Pa.s
θ	co-ordinate in circumfertial direction, radian
θ_{Omax}, θ_{Smax}	location of maximum pressure for Ocvirk's and Sommerfield bearings, radian
ρ_O	density of lubricant, kg/m^3
ω	angular velocity, rad/s

present trend in bearing design is to operate bearings at high eccentricity ratios (to increase load capacity without increasing bearing size), therefore a modification in the Reason and Narang[1] technique is required. Furthermore, the accurate prediction of maximum temperature and power loss is also of paramount importance for overall bearing design.

In the present study, Reason and Narang[1] pressure expression is modified by using two pressure correction factors, g_O and g_S. An analytical expression is given for the angular location of maximum pressure, using which a designer can calculate P_{max} directly. The frictional force is calculated by using various existing theories and compared. The effective temperature rise is evaluated by considering recirculating flow. To obtain maximum temperature in the oil film a model is proposed which provides results comparable with time consuming techniques such as thermohydrodynamic (THD), adiabatic, and isothermal analysis. Finally, design methodology is arranged in tabular form for accurate and easy prediction of load capacity, attitude angle, power loss, flow and maximum temperature.

Methodology

If variation in viscosity is ignored, then the bearing performance is evaluated by solving the following form of Reynolds equation:

$$\frac{\partial}{\partial\theta}\left[(1. + \epsilon\cos\theta)^3\frac{\partial P}{\partial\theta}\right] + R^2\frac{\partial}{\partial z}\left[(1. + \epsilon\cos\theta)^3\frac{\partial P}{\partial z}\right] = 12\eta\left(\frac{R}{C}\right)^2\left[\dot{\epsilon}\cos\theta + \epsilon\left(\dot{\phi} - \frac{\omega}{2.}\right)\sin\theta\right] \quad (1)$$

For steadily loaded bearings ($\dot{\epsilon} = 0, \dot{\phi} = 0$); eq. (1) will reduce to

$$\frac{\partial}{\partial\theta}\left[(1. + \epsilon\cos\theta)^3\frac{\partial P}{\partial\theta}\right] + R^2\frac{\partial}{\partial z}\left[(1. + \epsilon\cos\theta)^3\frac{\partial P}{\partial z}\right] = -6\eta\omega\left(\frac{R}{C}\right)^2\epsilon\sin\theta \quad (2)$$

The closed form solution to the Reynolds equation, for infinitely short bearing (Ocvirk's solution P_O) and infinitely long bearing (Sommerfeld solution, P_S), are readily available in the literature and are given as:

$$P_O = \frac{3\eta UL^2}{RC^2}\left[\frac{1}{4} - \left(\frac{z}{L}\right)^2\right]\frac{\epsilon\sin\theta}{(1. + \epsilon\cos\theta)^3} \quad (3)$$

$$P_S = \frac{6\eta UR}{C^2(2. + \epsilon^2)}\left[\frac{\epsilon\sin\theta(2. + \epsilon\cos\theta)}{(1. + \epsilon\cos\theta)^2}\right] \quad (4)$$

The analytical solution, P_S is applicable only when the slenderness ratio (L/D) is greater than 2.0, whereas the analytical solution, P_O is valid for a slenderness ratio less than 0.25. The ratio for most practicable design ranges is 0.25 to 1.5, where both the short bearing approximation and the long bearing approximation are inappropriate.

The heuristic closed form solution for film pressure (P_θ) given by Reason and Narang[1] is among the best compared to any other closed-form solution available in the literature. This approximation is based on a harmonic average of the short bearing (P_O) and long bearing (P_S) solution, and for π-extent film is given by the following expression:

$$\frac{1}{P_\theta} = \frac{1}{P_O} + \frac{1}{P_S} \quad (5)$$

This approximation gives a solution accuracy even comparable to FEM for short bearings at low eccentricity ratios, but with an increase in eccentricity ratios and L/D ratios, this technique overestimates the load carrying capacity. To improve the solution accuracy at high eccentricity L/D ratios, the authors have introduced two pressure correction factors g_O (= $f(\epsilon, L/D)$) and g_S (=$f(\epsilon)$) in eq. (5) and with this modification eq. (5) is rewritten as:

$$\frac{1}{P} = \frac{g_O}{P_O} + \frac{g_S}{P_S} \quad (6)$$

where

$$g_O = 1. + \epsilon\left(\frac{L}{D}\right)^{1.2}[e^{\epsilon^5} - 1.]; \; g_S = e^{(1-\epsilon)^3} \quad (7)$$

The expression for g_O and g_S were evaluated by trial and error method. The pressure P_S for infinitely long bearing grossly overestimates oil film pressure at low eccentricity ratios, but with an increase in eccentricity ratio the percentage error decreases. Therefore g_S, which modifies P_S, is chosen such that it decreases with the increase in eccentricity ratio. On other hand, P_O gives an erroneous solution at high eccentricity ratios. Ritchie[3] observed an overestimation of 2000% in load capacity predicted by short bearing approximation at an eccentricity ratio of 0.99. Therefore g_O should increase with an increase in ϵ. In addition, the pressure expression for short bearing is a function of bearing length. With an increase in L/D ratio, the error in estimating oil film pressure by short bearing approximation increases. Therefore g_O is selected in such a way that it increases with an increase in L/D ratio. Substituting eqs (3) and (4) in eq. (6), the pressure profile in terms of θ and z co-ordinates is given by:

$$P = \frac{12\eta UR}{C^2 g_O}\left(\frac{L}{D}\right)^2 \left[\frac{\left\{\frac{1}{4} - \left(\frac{z}{L}\right)^2\right\}\frac{\epsilon \sin\theta}{(1+\epsilon\cos\theta)^3}}{1 + \frac{2g_S\left(\frac{L}{D}\right)^2(2+\epsilon^2)}{g_O(1+\epsilon\cos\theta)(2+\epsilon\cos\theta)}\left\{\frac{1}{4} - \left(\frac{z}{L}\right)^2\right\}}\right]$$

The validity of pressure profile approximation, i.e., eq. (6), may be tested by calculating the load carried by lubricant film and comparing with that of rigorous numerical techniques, such as FEM. The total load supported by bearing is obtained by integrating the pressure components normal and parallel to the line of centres over the whole film, i.e.,

$$W\cos\phi = -\int_A p\cos\theta\, dA; \; W\sin\phi = \int_A p\sin\theta\, dA$$

For complete films, the integration indicated is to be carried out over the entire bearing. For ruptured films, occurring under negative pressure, it is to be carried out only over the positive portion of the pressure distribution. Equations (3) and (4) show that the positive pressure zone extends from 0 to π. Therefore load components will be calculated by the following equation:

$$W\cos\phi = -\int_0^{\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} P\cos\theta . dz . R\, d\theta;$$

$$W\sin\phi = \int_0^{\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} P\sin\theta .\, dz . R\, d\theta \quad (8)$$

Appendix A provides the details.

The comparison of the non-dimensional load capacity values obtained by the present method with those obtained by the Reason and Narang[1] technique and FEM is given in Table 1. The results of the the present method are comparable even with those obtained by using FEM for a wide range of eccentricity and L/D ratios.

Oil flow

The oil flow is an important factor to determine the operating temperature and oil viscosity. Total side flow prediction through bearing involves hydrodynamic flow Q_H caused by relative shaft rotation and the resulting film pressure, together with the feed pressure flow, Q_P, which is the direct result of oil being forced through the bearing by supply pressure. The expression for hydrodynamic flow is given by:

$$Q_H = \int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\frac{Uh}{2} - \frac{h^3}{12\eta}\frac{\partial P}{\partial x}\right]_{\theta=0} dz - \int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\frac{Uh}{2} - \frac{h^3}{12\eta}\frac{\partial P}{\partial x}\right]_{\theta=\pi} dz \quad (9)$$

The oil-feed flow Q_P depends on supply pressure, bearing geometry and oil feed configuration. Martin and Lee[5] provided a quick method for predicting feed pressure flow for various oil feed arrangements, i.e., circumferential groove, partial groove or oil hole. They derived a comprehensive curve fit equation by using geometry flux plotting techniques and accurate finite difference methods. The result shows that an algebraic summation of hydrodynamic and feed flow will slightly overestimate the total flow, but this gives designers flexibility in choosing the pressure feed device (Appendix B).

Table 1 Comparison of non-dimensional load capacity ($WC^2/6\eta ULR^2$)

L/D ratio	Various methods	Eccentricity ratio (ϵ)						
		0.1	0.5	0.8	0.9	0.95	0.98	0.99
1.0	Reason and Narang[1]	0.038	0.287	1.154	2.666	5.782	15.850	31.513
	FEM[1]	0.038	0.264	0.994	2.296	5.026	13.440	27.340
	Authors	0.038	0.270	1.034	2.294	5.056	14.392	29.419
0.25	Reason and Narang[1]	0.0032	0.0297	0.2036	0.720	2.256	9.012	21.243
	FEM[1]	0.0033	0.0290	0.1884	0.644	1.972	7.392	18.000
	Authors	0.0032	0.0295	0.1936	0.654	2.037	8.162	19.604

Friction loss

The calculation of friction loss within a bearing oil film is an integral part of the design of the bearing. The friction loss appears as heat, raises the temperature of the lubricant and lowers its viscosity, which is a key parameter of the bearing analysis. Therefore, the accurate prediction of friction loss is desired. The friction force is calculated by integrating shear stress over the journal surface, i.e.,

$$F = \int_A \tau \, dA = \int \left(\frac{\eta U}{h} + \frac{h}{2R} \frac{\partial P}{\partial \theta} \right) dA \qquad (10)$$

The frictional force contains two terms, the first is due to shear of oil film and second is due to hydrodynamic film pressure. Three different theories are available in the literature to calculate friction loss due to oil shearing. The first theory, which can be called 2π-extent, assumes that the angular extent of the oil film is 2π. According to the second theory[6], i.e., π-extent, the oil-shearing occurs only in the convergent region. The third theory[7], based on effective length (EL), treats the divergent region as an effectively convergent region by an assumption of axial contraction. In other words, the divergent region is taken to be fully filled with the lubricant in an accumulative width of the oil streamers which is less than the total bearing length. Table 2 shows the comparison of frictional force evaluated by the authors for different bearings by using these theories, with those given in brackets obtained from Khonsari *et al.*[8]. This shows that the model based on EL approximation is a better predictor of the friction force, and therefore it is used in the present design study (Appendix C).

Temperature effects

The inclusion of thermal effects basically requires a heat balance between heat produced by viscous friction (= FU) and heat carried away by the lubricant, along with heat transmitted through the lubricated surfaces. Various methods are available to take thermal effects into account. The thermohydrodynamic (THD) analysis, which considers the simultaneous solution of the Reynolds, energy and heat conduction equations, has been used to predict bearing performance accurately. However, THD theory can not be used analytically. The simplest method consists of calculating an effective temperature and the corresponding effective viscosity using isothermal theory. The effective temperature can be arrived at by balancing the power loss against the heat carried away by the oil flow. This approach can be described by the following simple relation:

Oil mass flow rate * heat capacity of oil

* effective temperature rise = power loss * σ

$$\Delta t = \frac{\sigma F U}{\rho_O C_O Q_{\text{leakage}}} \qquad (11)$$

Where σ is the fraction of the total heat generated, carried away by the oil. The value of σ taken by Reason and Narang[1] is 0.5 for lightly load bearings, by Cameron[9] 0.8 for moderately loaded bearing, and by Paranjpe[10] it is 0.9 for heavily loaded bearings. Experiments by Cole[11] show that the value of σ depends on the eccentricity ratio, which is also basically reflected in the values adopted by various references quoted above. Therefore on the basis of these observations factor σ is chosen equivalent to the eccentricity ratio. The effective viscosity corresponding to

Table 2 Comparison of frictional force calculated by using various methods

Various bearings	2π-extent (*N*)	π-extent (*N*)	EL approximation (*N*)
Dowson's bearing (61.29 *N*)	66.94	35.06	55.08
Mitsui's bearing (47.44 *N*)	52.41	26.81	44.40
Ferron's bearing 2000 rpm (45.87 *N*)	54.29	28.48	44.81
Ferron's bearing 4000 rpm (77.6 *N*)	93.24	48.55	77.58

the effective temperature can be calculated by viscosity–temperature relations. Usually, at the beginning the effective temperature is unknown, moreover, a slight change in effective temperature changes the oil viscosity and, consequently, the power loss and the oil flow change. Therefore, an iterative process is needed to establish the effective temperature.

In the journal bearing, oil recirculates across the divergent zone, i.e., π to 2π (for π-bearing), and heats up the freshly supplied oil through the feed groove. This makes the oil temperature higher (T_{mix}) compared with the supply oil temperature (T_{in}). Therefore the temperature rise Δt, is the difference in outlet temperature (T_{eff}) and T_{mix}, i.e.,

$$\Delta t = T_{eff} - T_{mix} \tag{12}$$

The mixing temperature is unknown (T_{mix}), and can be evaluated by incorporating the heat flux balance, i.e.,

$$T_{mix} = \frac{Q_{rec}T_{rec} + Q_{supply}T_{in}}{(Q_{supply} + Q_{rec})} \tag{13}$$

The supply flow is equal to the total side leakage flow ($Q_{leakage}$). To evaluate the temperature at groove location (T_{mix}), the temperature of recirculating flow is required, which can be assumed constant over the cavitated region. Many researchers, i.e., Ott and Paradissiadis[12] and Ma and Taylor[13], have concluded that there will be temperature fade in the divergent region because of (a) the "recuperator role" of the shaft in removing heat from hot film areas, and (b) the oil reverse flow into the cavitated zone from the oil feed groove. The modelling of reverse flow and heat carried by the shaft is required for physical realization of processes occurring in bearing but is not possible analytically and requires numerical methods. For simplicity we have assumed that the recirculated oil temperature is equivalent to the maximum temperature (T_{max}, occurring at $\theta = \pi$) and is constant over the cavitated region. This assumption does not much effect the accuracy of solution and makes analysis easier. Therefore, eq. (13) is modified as:

$$T_{mix} = \frac{Q_{rec}T_{max} + Q_{leakage}T_{in}}{(Q_{rec} + Q_{leakage})} \tag{14}$$

The evaluation of T_{mix} requires the value of T_{max}, which depends mainly on $Q_{leakage}$ and Q_{rec}. The experimental investigations[14] show that there is a rapid rise in the level of T_{max} at high eccentricity on one hand and on other hand at low eccentricity (small flow)[15], maximum temperature approaches T_{eff}. Therefore, it is being proposed that the maximum temperature can be modelled as:

$$T_{max} = T_{eff} + \frac{Q_{leakage}}{Q_{rec}}\Delta t \tag{15}$$

Design procedure

To facilitate the bearing design, various formulae are presented in Table 3. The supply and recirculating flow along with the location of maximum pressure can be evaluated step by step using expressions 1–7. Integration in expressions 10–11 can be determined by using Weddle's formula described in Appendix A. Usually, effective temperature is unknown at the beginning, but can be evaluated by iterating steps 16–17 along with suitable viscosity–temperature relations, such as: $\eta_{eff} = n_{in}e^{-\beta(T_{eff} - T_{in})}$, until viscosity converges. In expressions 13–15 load capacity, friction force and temperature rise are evaluated without the inclusion of viscosity, to reduce computational effort, and assigned to symbols W_{η}, F_{η}, and temperature rise Δt_{η}, respectively. Once effective temperature and corresponding effective viscosity are determined, load capacity, friction force, maximum temperature and maximum pressure can be evaluated using steps 18–21. By using the proposed design table, with the exception of supply flow, which is marginally overestimated, the prediction of other parameters have been found to be very good.

Results and discussion

To compare the results produced by the proposed design methodology with other methods, the test data given in Khonsari *et al.*[8] are used. Three different bearings are considered, namely: Mitsui's bearing and Ferron's bearings with the latter being analysed at two different speeds, viz. 209.4 rad/s and 418.9 rad/s.

Mitsui's bearing

L = 0.07 m; (L/D) = 0.7; (R/C) = 636.94; ω = 235.6 rad/s; β = 0.029 K^{-1}; $\rho_o C_o$ = 1 681 643; W = 3920; T_{in} = 40°C; η_{in} = 0.0192 Pas; P_{supply} = 98 kPa; axial groove 10° extend circumferentially and 60 mm in length.

Table 4 shows the results for Mitsui's bearing obtained from the present study along with those obtained by other methods.

Ferron's bearing

L = 0.08 m; (L/D) = 0.8; (R/C) = 344.83; ω = 209.4 rad/s and 418.9 rad/s; $\rho_o C_o$ = 1 719 576.7; W = 4000 and 6000 N; T_{in} = 40°C; η_{in} = 0·0277 Pas; β = 0.034 K^{-1}; P_{supply} = 70 kPa; axial groove 18° extends circumferentially and 65 mm in length.

The THD analysis is a simultaneous solution of Reynolds, energy, and heat conduction equations along with several boundary conditions. Although it is accurate, it is very expensive with regard to CPU time. The isothermal condition at the oil-shaft interface and adiabatic condition at the oil bush interface (ISOADI), although simpler than THD, is a numerical iterative scheme and is also time consuming. A further simplification is to use an isothermal analysis with a calculated effective oil temperature to represent the thermal effects. The principle objection to such an analysis is that it provides no information regarding the maximum temperature. Moreover, isothermal analysis results mentioned by Khonsari *et al.*[8] and in Tables 4–6, are based on Reynolds pressure boundary condition, which requires numerical iterations. Tables 4–6 show that the results obtained by the present methodology, which is simple and rapid, are close to results obtained by rigorous techniques.

Table 3 Journal bearing design table

1. $\Lambda = L/D$ $g_S = e^{(1-\epsilon)^3}$ $g_O = 1. + \epsilon\Lambda^{1.2}[e^{\epsilon^5} - 1]$	2. $B_1 = \dfrac{g_O}{2g_S\Lambda^2(2+\epsilon^2)}$ $B_{21} = B_1(1+\epsilon)(2+\epsilon)$ $B_{22} = B_1(1-\epsilon)(2-\epsilon)$
3. $B_{3j} = \sqrt{B_{2j} + 0.25}$; $B_{4j} = \dfrac{B_{2j}}{B_{3j}} \log\left(\dfrac{B_{3j}+0.5}{B_{3j}-0.5}\right)$; where $j = 1, 2$	4. $\theta_{Omax} = \cos^{-1}\left(\dfrac{1-\sqrt{1+24\epsilon^2}}{4\epsilon}\right)$ $\theta_{Smax} = \cos^{-1}\left(\dfrac{-3\epsilon}{2+\epsilon^2}\right)$
5. $\Theta = \dfrac{\Lambda^2 g_S}{3g_O} \dfrac{(2+\epsilon^2)^4(12\epsilon^2 + (2+\epsilon^2)(1-\sqrt{1+24\epsilon^2}))}{\epsilon^2(1-\epsilon^2)^2(2-\epsilon^2)\left(9-\sqrt{1+24\epsilon^2}\right)^2}$	6. $\theta_{Pmax} = \theta_{Smax} + (\theta_{Omax} - \theta_{Smax})\left(\dfrac{1}{1-\Theta}\right)$
7. $Q_{leakage} = Q_P + UCL\epsilon[1. - \dfrac{1}{g_O}\Lambda^2\{B_{21}(1-B_{41}) + B_{22}(1-B_{42})\}]$ $Q_{rec} = \dfrac{UCL(1-\epsilon)}{2} + \dfrac{UCL}{g_O}\Lambda^2 B_{22}(1-B_{42})$	
8. $H = 1 + \epsilon\cos\theta$; $B_{23} = B_1 H(1+H)$ $B_{33} = \sqrt{B_{23} + 0.25}$; $B_{43} = \dfrac{B_{23}}{B_{33}}\log\left(\dfrac{B_{33}+0.5}{B_{33}-0.5}\right)$	9. $I_{Z_\eta} = \dfrac{6ULR}{C^2 g_s}\left[\dfrac{\epsilon\sin\theta(1+H)}{(2+\epsilon^2)H^2}\right](1. - B_{43})$
10. $W_{\epsilon\eta} = -\int_0^\pi I_{Z_\eta} R\cos\theta.d\theta$	11. $W_{\phi\eta} = \int_0^\pi I_{Z_\eta} R\sin\theta.d\theta$
12. $\phi = \tan^{-1}\left(\dfrac{W_{\phi\eta}}{W_{\epsilon\eta}}\right)$	13. $W_\eta = \sqrt{W_{\epsilon\eta}^2 + W_{\phi\eta}^2}$
14. $F_\eta = \dfrac{ULR\pi}{C\sqrt{1-\epsilon^2}}\left(\dfrac{2+\epsilon}{1+\epsilon}\right) + \dfrac{C\epsilon W_{\phi\eta}}{2R}$	15. $\Delta t_\eta = \dfrac{\epsilon F_\eta U}{\rho_O C_O Q_{leakage}}$
16. $\Delta t = \eta\Delta t_\eta$	17. $T_{eff} = T_{in} + \left[2 + \dfrac{Q_{rec}}{Q_{leakage}}\right]\Delta t$
18. $W = \eta W_\eta$ $F = \eta F_\eta$	19. $T_{max} = T_{eff} + \dfrac{Q_{leakage}}{Q_{rec}}\Delta t$
20. $H_{pmax} = 1 + \epsilon\cos\theta_{pmax}$ $B_{21max} = 4.B_1 H_{pmax}(1 + H_{pmax})$	21. $P_{max} = \dfrac{3.\eta UR\Lambda^2}{C^2 g_O} \dfrac{\epsilon\sin\theta_{pmax}}{H^3_{pmax}} \dfrac{B_{21max}}{1 + B_{21max}}$

Conclusion

A simple and rapid method for evaluating load capacity, maximum pressure, flow rate, friction loss and maximum temperature is presented. The proposed analytical pressure expression agrees with the FEM solution for a wide range of design parameters. The maximum temperature calculated by the present formulation matches well with the experimental results as well as THD analysis. The values of other important parameters, i.e., friction force, recirculating flow, Sommerfield number and attitude angle, obtained by the present method lie within the scatter of the results obtained by various numerical methods. From this point of view, it can be concluded that the proposed design approach is simple and efficient.

References

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Table 4 Comparison of characteristics of Mitsui's bearing[8] obtained by the present study and established theories

Parameters (units)	Experimental data	THD solution	ISOADI solution	Isothermal solution at 51.38°C	Present study
ϵ		0.452	0.485	0.446	0.452
P_{max} (MPa)		1.29	1.27	1.28	1.27
Load (*N*)	3920	3920	3920	3920	3920
Friction force (*N*)		47.44	45.00	45.35	45.00
T_{max} (°C)	56	54.18	56.94	–	58.28
$Q_{leakage}$ (cm^3/s)		29.16	29.78	30.17	33.68
Q_{rec} (cm^3/s)		19.50	18.81	18.72	19.24
Sommerfeld number		0.392	0.379	0.375	0.380
Attitude angle ϕ (°)		61.33	61.04	60.22	60.36

Table 5 Comparison of characteristics of Ferron's bearing[8] obtained by present study and established theories (N = 2000 rpm, W = 4000 N)

Parameters (units)	Experimental data	THD solution	ISOADI solution	Isothermal solution at 45.29°C	Present study
ϵ		0.577	0.575	0.575	0.574
P_{max} (MPa)	1.3	1.25	1.25	1.27	1.25
Load (*N*)	4000	4000	4000	4000	4000
Friction force (*N*)		45.87	47.04	45.58	45.86
T_{max} (°C)	49	49.02	50.47	–	49.64
$Q_{leakage}$ (cm^3/s)		79.60	77.27	81.98	85.35
Q_{rec} (cm^3/s)		29.26	29.55	28.85	29.51
Sommerfeld number		0.191	0.189	0.183	0.189
Attitude angle ϕ (°)		53.92	54.65	52.54	52.96

Table 6 Comparison of characteristics of Ferron's bearing[8] obtained by present study and established theories (N = 4000 rpm, W = 6000 N)

Parameters (units)	Experimental data	THD solution	ISOADI solution	Isothermal solution at 48.53°C	Present study
ϵ		0.543	0.547	0.529	0.544
P_{max} (MPa)	1.9	1.78	1.78	1.80	1.80
Load (*N*)	6000	6000	6000	6000	6000
Friction force (*N*)		77.60	77.65	78.88	76.44
T_{max} (°C)	58.	58.35	60.08	–	57.15
$Q_{leakage}$ (cm^3/s)	130.6	131.1	131.2	129.9	137.04
Q_{rec} (cm^3/s)		62.94	63.43	61.56	62.68
Sommerfeld number		0.230	0.227	0.219	0.215
Attitude angle ϕ (°)		57.89	58.73	55.30	55.00

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Appendix A

Maximum pressure location

The pressure will be maximum at mid plane ($z = 0.0$), at circumferential location (θ_{Pmax}) and can be evaluated by putting pressure gradient in circumferential direction equal to zero, i.e.,

$$\left.\frac{\partial P}{\partial \theta}\right|_{\theta=\theta_{Pmax}} = 0$$

From eq. (6),

$$\frac{1}{P} = \frac{g_O}{P_O} + \frac{g_S}{P_S}$$

or

$$\frac{1}{P^2}\frac{\partial P}{\partial \theta} = \frac{g_O}{P_O^2}\frac{\partial P_O}{\partial \theta} + \frac{g_S}{P_S^2}\frac{\partial P_S}{\partial \theta} \tag{16}$$

From eq. (3),

$$\left.\frac{1}{P_O^2}\frac{\partial P_O}{\partial \theta}\right|_{z=0} = \frac{4RC^2}{3\eta UL^2\epsilon}\frac{(1+\epsilon\cos\theta)^2}{\sin^2\theta}$$

$$[(1+\epsilon\cos\theta)\cos\theta + 3\epsilon\sin^2\theta] \tag{17}$$

From eq. (4),

$$\frac{1}{P_S^2}\frac{\partial P_S}{\partial \theta} = \frac{C^2(2+\epsilon^2)}{6\eta UR\epsilon}\frac{(1+\epsilon\cos\theta)}{\sin^2\theta}$$

$$\frac{[3\epsilon + (2+\epsilon^2)\cos\theta]}{(2+\epsilon\cos\theta)^2} \tag{18}$$

Substituting eqs (17) and (18) in eq. (16) and rearranging,

$$\frac{1}{P^2}\frac{\partial P}{\partial \theta} = \frac{C^2(1+\epsilon\cos\theta)}{3\eta U\epsilon\sin^2\theta}$$

$$\left[\frac{4Rg_O}{L^2}(1+\epsilon\cos\theta)\{(1+\epsilon\cos\theta)\cos\theta + 3\epsilon\sin^2\theta\} + \frac{(2+\epsilon^2)g_S}{2R}\frac{3\epsilon+(2+\epsilon^2)\cos\theta}{(2+\epsilon\cos\theta)^2}\right]$$

Film thickness can neither be zero ($1 + \epsilon\cos\theta \neq 0$), nor negative ($1 + 1 + \epsilon\cos\theta \neq 0$).

Moreover $\sin\theta \neq 0, \Rightarrow 0 < \theta_{Pmax} < \pi$.

For maximum pressure, $\frac{\partial P}{\partial \theta} = 0$;

or

$$G|_{\theta=\theta_{Pmax}} = [a_O A_O + a_S A_S]_{\theta=\theta_{Pmax}} = 0 \tag{19}$$

where

$$a_O = \frac{4Rg_O}{L^2}(1+\epsilon\cos\theta);$$

$$a_S = \frac{(2+\epsilon^2)}{2R}\frac{g_S}{(2+\epsilon\cos\theta)^2}$$

$$A_O = (1+\epsilon\cos\theta)\cos\theta + 3\epsilon\sin^2\theta;$$

$$A_S = 3\epsilon + (2+\epsilon^2)\cos\theta$$

A_O will be zero at $\theta_{Omax} = \cos^{-1}\left(\frac{1-\sqrt{1+24\epsilon^2}}{4\epsilon}\right)$

A_S will be zero at $\theta_{Smax} = \cos^{-1}\left(\frac{-3\epsilon}{2+\epsilon^2}\right)$

The closed form solution of eq. (19) is not possible, but θ_{Pmax} can be calculated with reasonable accuracy from linear interpolation of θ_{Omax} and θ_{Smax} in the following manner,

$$\theta_{Pmax} = \theta_{Smax} + (\theta_{Omax} - \theta_{Smax})\left[\frac{G|_{\theta_{Pmax}} - G|_{\theta_{Smax}}}{G|_{\theta_{Omax}} - G|_{\theta_{Smax}}}\right]$$

$$\text{or, } \theta_{Pmax} = \theta_{Smax} + (\theta_{Omax} - \theta_{Smax})\frac{1}{1 - \dfrac{a_S A_S|_{\theta_{Omax}}}{a_O A_O|_{\theta_{Smax}}}}$$

Load carrying capacity

The load components along and perpendicular to the line of centres are given as

$$W\cos\phi = -\int_0^{\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} P\cos\theta.\mathrm{d}z.R\,\mathrm{d}\theta;$$

$$W\sin\phi = \int_0^{\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} P\sin\theta.\mathrm{d}z.R\,\mathrm{d}\theta$$

load component along the z direction can be simply carried out analytically and is given as

$$I_z = \int_{-\frac{L}{2}}^{\frac{L}{2}} P.\mathrm{d}z = \frac{6\eta ULR}{C^2 g_S}\left[\frac{\epsilon \sin\theta(2. + \epsilon\cos\theta)}{(2+\epsilon^2)(1+\epsilon\cos\theta)^2}\right]$$

$$\left[1. - \frac{B_2}{B_3}\log\left(\frac{B_3+0.5}{B_3-0.5}\right)\right]$$

where

$$B_1 = \frac{g_O(D/L)^2}{2g_S(2+\epsilon^2)};$$

$$B_2 = B_1(1+\epsilon\cos\theta)(2+\epsilon\cos\theta);$$

$$B_3 = \sqrt{B_2 + 0.25}$$

Therefore,

$$W\cos\phi = -\int_0^{\pi} I_z R\cos\theta\,\mathrm{d}\theta;\; W\sin\phi = \int_0^{\pi} I_z R\sin\theta\,\mathrm{d}\theta$$

The closed form solution of these integrals is not possible and requires a numerical method. For this either the repeated Simpson's rule or Weddle's formula is used. The Weddle's formula is most useful for journal bearings where intervals of 30° can be chosen. The Weddle's formula is:

$$\int_a^b y\,\mathrm{d}x = \frac{b-a}{20}[y_1 + 5y_2 + y_3 + 6y_4 + y_5 + 5y_6 + y_7] \tag{20}$$

where $y_1 \ldots y_7$ are seven equidistant ordinates of mutual distance $(b-a)/6$. The Simpson's rule assumes that the fourth difference is zero, and Weddle's, which is far more accurate, extended up to sixth difference. Therefore in the present study the integrals are evaluated by using Weddle's formula. The accuracy of eq. (20) depends on the eccentricity ratio. Up to an eccentricity ratio of 0.8, eq. (20) predicts accurate results. At high eccentricity ratios, due to peak of oil pressure shifting toward $\theta = \pi$, the interval of 30° gives erroneous results. Therefore to minimize the error in evaluating the integrals at high eccentricity ratios eq. (21) combined with eq. (20) should be used.

$$\int_0^{\pi} f(x)\,\mathrm{d}x = \int_0^{\frac{\pi}{2}} f(x)\,\mathrm{d}x + \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} f(x)\,\mathrm{d}x + \int_{\frac{3\pi}{4}}^{\pi} f(x)\,\mathrm{d}x \tag{21}$$

High accuracy can be obtained, if the location of maximum pressure is known previously. The Weddle's formula for intervals of $\theta_{Pmax} \pm 5°$, along with with eq. (21) gives highly accurate results, even at high eccentricity ratios.

Appendix B

Hydrodynamic oil flow at the entry is given as

$$Q_{H_{\theta=0}} = \int_{-\frac{L}{2}}^{\frac{L}{2}} \left[\frac{Uh}{2} - \frac{h^3}{12\eta}\frac{\partial P}{\partial\theta}\right]_{\theta=0} \mathrm{d}z$$

$$= \frac{UCL(1+\epsilon)}{2} - \frac{C^3(1+\epsilon)^3}{12\eta R}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\frac{\partial P}{\partial\theta}\right)_{\theta=0}\mathrm{d}z$$

$$= \frac{UCL(1+\epsilon)}{2} - \frac{UC\epsilon}{g_O}\Lambda^2$$

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{[0.25 - \bar{z}^2]}{\left(1 + \frac{2g_S(2+\epsilon^2)}{g_O(1+\epsilon)(2+\epsilon)}\Lambda^2[0.25 - \bar{z}^2]\right)}\mathrm{d}z$$

$$= \frac{UCL(1+\epsilon)}{2} - \frac{UC\epsilon B_{22}}{g_O}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{[0.25 - \bar{z}^2]}{(B_{32})^2 - \bar{z}^2}\mathrm{d}z$$

where $B_{22} = \mathrm{B}_1(1+\epsilon)(2+\epsilon)$; $B_{32} = \sqrt{\mathrm{B}_{22} + 0.25}$

$$= \frac{UCL(1+\epsilon)}{2} - \frac{UCLB_{22}\epsilon}{g_O}\Lambda^2$$

$$\left[1 - \frac{B_{22}}{B_{32}}\log\left(\frac{B_{32}+0.5}{B_{32}-0.5}\right)\right]$$

Similarly,

$$Q_{H_{\theta=\pi}} = \frac{UCL(1-\epsilon)}{2} + \frac{UCLB_{23}\epsilon}{g_O}\Lambda^2$$

$$\left[1 - \frac{B_{23}}{B_{33}}\log\left(\frac{B_{23}+0.5}{B_{23}-0.5}\right)\right]$$

where

$$B_{23} = B_1(1-\epsilon)(2-\epsilon);\; B_{33} = \sqrt{B_{23} + 0.25}$$

Therefore hydrodynamic flow of side leakage will be given as: $Q_H = Q_{H_{\theta=0}} - Q_{H_{\theta=\pi}}$

Total side leakage: $Q_{\text{leakage}} = Q_H + Q_P$

Recirculating flow $Q_{\text{rec}} = Q_{H_{\theta=\pi}}$

Appendix C

The friction force is calculated by integrating shear stress over the journal surface, i.e.,

$$F = \int_A \left(\frac{\eta U}{h} + \frac{h}{2R}\frac{\partial P}{\partial\theta}\right)\mathrm{d}A$$

$$F = \int_0^{2\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} \left(\frac{\eta U}{h} + \frac{h}{2R}\frac{\partial P}{\partial\theta}\right)\mathrm{d}z.R\,\mathrm{d}\theta$$

$$F = \int_0^{2\pi} \frac{\eta ULR}{C(1 + \epsilon \cos\theta)} \mathrm{d}\theta + \int_0^{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{h}{2} \frac{\partial P}{\partial \theta} \mathrm{d}z.\mathrm{d}\theta$$

The pressure gradient will be zero in divergent region. Usually the bearing will not run full between $\theta = \pi$ to $\theta = 2\pi$, so that this integration over the first term will give an overestimation of friction. To make a realistic prediction of friction force EL approximation is used. Assume that L' is the sum of the breadths of the lubricant streamers at any angular position in the range of π to 2π. Then by flow continuity

$$Q_{\mathrm{H}_\theta} = \frac{L'hU}{2} - \frac{LC(1 - \epsilon)U}{2}\ ; \Rightarrow L' = \frac{L(1 - \epsilon)}{(1 + \epsilon \cos\theta)}$$

Applying EL approximation to first frictional term and putting pressure gradient equal to zero in divergent portion of the bearing,

$$F = \frac{\eta ULR}{C}\left[\int_0^{\pi} \frac{\mathrm{d}\theta}{(1 + \epsilon \cos\theta)}\right.$$

$$\left. - (1 - \epsilon)\int_{\pi}^{2\pi} \frac{\mathrm{d}\theta}{(1 + \epsilon \cos\theta)^2}\right] + \int_0^{\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{h}{2} \frac{\partial P}{\partial \theta} \mathrm{d}z.\mathrm{d}\theta$$

$$= \frac{\eta ULR}{C}\left[\frac{\pi}{(1 - \epsilon^2)^{1/2}} + \frac{(1 - \epsilon)\pi}{(1 - \epsilon^2)^{3/2}}\right]$$

$$+ \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{2} Ph|_0^{\pi}.\mathrm{d}z - \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{2} \int_0^{\pi} P \frac{\partial h}{\partial \theta} \mathrm{d}\theta.\mathrm{d}z$$

$$= \frac{\eta ULR\pi}{C(1 - \epsilon^2)^{1/2}}\left(\frac{2 + \epsilon}{1 + \epsilon}\right) + 0 + \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{C\epsilon}{2R} \int_0^{\pi} P \sin\theta.R\ \mathrm{d}\theta.\mathrm{d}z$$

$$= \frac{\eta ULR\pi}{C(1 - \epsilon^2)^{1/2}}\left(\frac{2 + \epsilon}{1 + \epsilon}\right) + \frac{C\epsilon}{2R} W \sin\phi$$