Dynamics of Rotor Shaft System on Flexible Supports with Gyroscopic Effects

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1 Introduction

An analysis of gyroscopic effect due to asymmetrically placed rotor and overhung rotors is given in [1-2]. Dynamic analysis of an elastically mounted single rotor shaft system with an asymmetrically placed rotor has been reported in [3]. The effect of support flexibility and viscous damping on the synchronous response of a single mass flexible rotor has been analyzed in [4]. Viscoelastic damping at the supports is considered to be useful in controlling the unbalance response and stability of a rotor shaft system [5] and this has been confirmed in some recent studies [6-7]. The analysis [6] is applicable to symmetrically placed rotors and gyroscopic effect of the rotor has been ignored. The present work deals with the determination of the eigenvalues of the rotor shaft system with asymmetrically placed rotor on elastic supports and unbalance response for the system with viscoelastic supports. Gyroscopic effect has been incorporated in the analysis and its effect on the eigenvalues and the response has been studied.

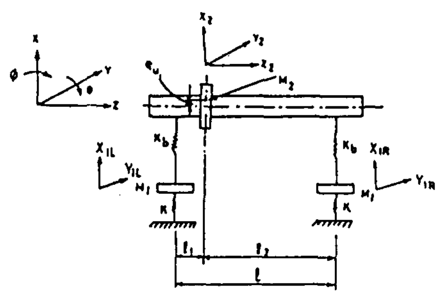

FIG. 1 THE SYSTEM CONFIGURATION

2 System with Elastic Supports

Figure 1 shows the schematic diagram of the configuration of the system analyzed. The rotor disc mass M_2 is placed asymmetrically along the length of the shaft. The bearings are modelled as elastic elements of stiffness K_b each. M_1 is the mass and K is the stiffness of each support assumed elastic. The expression for kinetic energy T, strain energy V and the dissipation energy D can be obtained from [3] after adding the terms due to each support mass, flexibility and internal damping of the shaft in terms of displacements x_2, y_2, of the rotor disc, x_{1L}, y_{1L}, x_{1R}, y_{1R} of the support masses as in Figure 1, deformations x_{j1}, y_{j1} of the left journal x_{j2}, y_{j2} of the right journal, angular deflections ϕ and θ of the rotor axis about x and y axes respectively and the angular orientations α_o and β_o of the rotor axis about x and y axes with the shaft taken as rigid body. Other parameters are x_s and y_s which are the deflections of the shaft at the rotor disc location relative to the ends of the shaft in the x and y directions respectively. I_p and I_t are the polar and transverse moments of inertia of the disc respectively and are taken into account to incorporate the gyroscopic effect of the rotor. K_s is the stiffness of the shaft at the rotor disc location in the transverse direction and ω is the angular velocity of the rotor. The expressions for T, V and D are as below.

$$T = 1/2 \; [\; M_2\dot{x}_2^2 + M_2\dot{y}_2^2 + M_1\dot{x}_{1L}^2 + M_1\dot{y}_{1L}^2 + M_1\dot{x}_{1R}^2 + M_1\dot{y}_{1R}^2 + I_p\omega^2 + I_t\dot{\theta}^2 + I_t\dot{\phi}^2 + 2I_p\omega\theta\dot{\phi} \;] \quad (1)$$

$$V = 1/2 \; [K_s(x_s^2+y_s^2) + C_{22}(\theta-\alpha_o)^2 + C_{22}(\phi-\beta_o)^2 + K_b(x_{j1}^2 + y_{j1}^2) + K_b(x_{j2}^2 + y_{j2}^2) + K(x_{1L}^2 + y_{1L}^2 + x_{1R}^2 + y_{1R}^2) - 2C_{12}(\theta-\alpha_o)x_s - 2C_{12}(\phi-\beta_o)y_s \;] \quad (2)$$

where C_{22} is the angular stiffness of the shaft at the rotor disc location (moment/angular deflection) and C_{12} is force/angular deflection or moment/deflection of the shaft at the rotor disc location.

$$D = C_i \; [\; (\dot{x}_s^2 + \dot{y}_s^2)/2 + \omega(y_s\dot{x}_s - x_s\dot{y}_s) \;] \quad (3)$$

where C_i is the viscous internal damping coefficient and the terms α_o, β_o, x_s and y_s are not independent variables but can be written as

$$\begin{aligned} \alpha_o &= (x_{j_2} + x_{1R} - x_{j_1} - x_{1L})/\ell \\ \beta_o &= (y_{j_2} + y_{1R} - y_{j_1} - y_{1L})/\ell \\ x_s &= x_2 - (x_{j_1} + x_{1L})e_2 - (x_{j_2} + x_{1R})e_1 \\ y_s &= y_2 - (y_{j_1} + y_{1L})e_2 - (y_{j_2} + y_{1R})e_1 \end{aligned} \quad (4)$$

where $e_1 = \ell_1/\ell$ and $e_2 = \ell_2/\ell$

Substituting equations (4) in the equations 1,2 & 3 and using Lagrange's equations expressed in terms of generalized co-ordinates 'q' we get 12 equations of motion in co-ordinates x_2, y_2, x_{1L}, y_{1L}, x_{1R}, y_{1R}, x_{j_1}, y_{j_1}, x_{j_2}, y_{j_2}, θ and ϕ. These can be reduced to 6 equations in terms of $z_n = y_n + ix_n$ where n in general denotes the subscripts 2, j_1, j_2, 1L, 1R and in terms of $\psi = \theta + i\phi$. Putting the value of $C_i = 0$ and ignoring the support mass, the equations are seen to be identical to those in [3]. Again, the equations can be reduced to those in [4] if the gyroscopic effect is ignored and the rotor is assumed to be in the middle of the shaft.

To find the undamped eigen values of the system, C_1 is taken to be zero. Assuming $z_n = Z_n e^{i\omega_{n1}t}$ and $\psi = \Psi e^{i\omega_{n1}t}$, we get the characteristic equation the roots of which give the eigenvalues. The eigenvalues are plotted in Figures 2 and 3 for the following parameters: $M_1/M_2 = 0.4$, $K_b/K_s = 5.0$, $\ell_1/\ell = 0.1$, $K/K_s = 0.1$, $I_p/I_t = 1.8$ and $C_{22}/I_t = 0.1$. It is seen that four eigenvalues are positive corresponding to the forward whirl and the other four are negative which signify the backward whirl of the rotor.

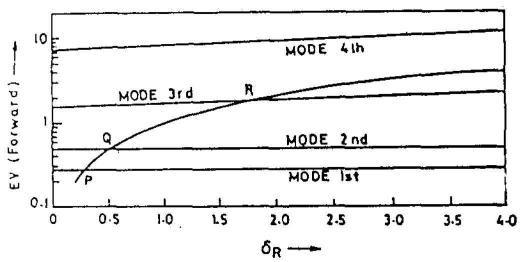

FIG. 2 VARIATION OF FORWARD MODE EIGEN VALUES WITH ROTOR SPEED PARAMETER

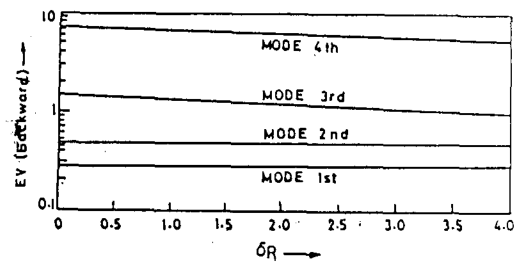

FIG. 3 VARIATION OF BACKWARD MODE EIGEN VALUES WITH ROTOR SPEED PARAMETER

These are shown in Figures 2 and 3 respectively where $E_v = \omega_{n1} / \omega_n$ denotes the nondimensional eigenvalue , the nondimensional rotor speed $\delta_R = \omega/\omega_n$, where ω_n, the transverse natural frequency of the system is expressed as $(K_s/M_2)^{1/2}$. As the rotor speed parameter δ_R increases, the stiffening due to gyroscopic effect increases and the eigenvalues E_v for the forward whirl increases and for the backward whirl, decreases, Whenever the rotor speed coincides with any of the eigenvalues, resonance occurs and thus we can confirm the critical speeds. Thus in Figure 2, where the whirling is in the same direction as the direction of rotation of the rotor, a line drawn at an angle of 45^0 to the δ_R axis gives the values of the critical speeds at the points of intersection with the plots of eigenvalues for various modes. The line is curved since the logarithmic scale is used for the vertical axis. The points P , Q and R give the three critical speeds and the fourth one is not obtainable while the rotor rotates.

3 System with Viscoelastic Supports

For finding the unbalance response of a system with viscoelastic supports due to unbalance e_u, the parameter K is replaced by $K(1+i\eta)$ where η is the loss factor of the viscoelastic material and K is the in-phase stiffness. The amplitude of the force due to unbalance is $M_2 e_u \omega^2$ and is easily incorporated in the equations derived. The frequency of excitation is ω and the unbalance response will have the same frequency.

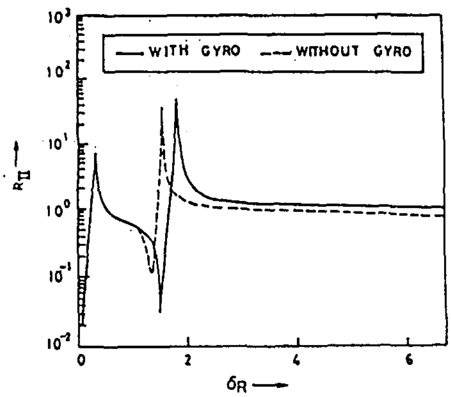

FIG. 4 VARIATION OF UNBALANCE RESPONSE AMPLITUDE OF ROTOR WITH ROTOR SPEED PARAMETER

The equations can be solved to obtain nondimensional unbalance response amplitude of the disc $R_D = |Z_2|/e_u$, which is plotted in Figure 4 for the value of $\eta = 0.1$, the other parameters being the same as for the Figures 2 and 3. The unbalance response has been plotted for two cases viz. with and without consideration of the gyroscopic effect of the rotor. The peaks occur at values of δ_R corresponding to the points P and R in Figure 2. The second mode corresponding to the point Q , which is predominantly due to angular rotation effect is not excited due to the unbalance excitation. The gyroscopic effect is seen to affect the value of δ_R corresponding to the second peak and the corresponding peak amplitude. A higher value of η can reduce the peak amplitude considerably.

Thus in the system with an undamped support, four forward and four backward whirling speeds are obtained. The gyroscopic effect affects the values of the critical speeds and unbalance response at high speeds in the case of unsymmetrically located disc. The unbalance response of the disc can be minimized by choosing a damped viscoelastic support.

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