Measurement of vibration with Young's fringe modulated speckle patterns in a photorefractive correlator

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Abstract

A technique has been proposed to analyze small in-plane component of vibrations by measuring the fringe shift, and analysing the correlation output of captured vibrating Young's fringes with a photorefractive correlator. Fringes are formed due to interference between the reconstructed speckled beam recorded for stationary object beam, and vibrated speckled beam transmitting through a $BaTiO_3$ crystal after introducing a small shift in the diffuser position. The correlation peak intensity is affected due to mismatch of vibrations between two piezo-mirrors in the object beam. High correlation output gives an indication of the frequency matching of known reference and unknown vibrations, thus giving information on the frequency of vibration. Vibration amplitudes of the order of a wavelength of light have been measured by counting the number of fringes passing past a line in the observation plane.

Keywords: Speckle metrology; Young's fringes; Photorefractive correlator

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1. Introduction

A rough surface illuminated with coherent light leads to the formation of speckles in free space as well as at the image plane. Speckle techniques find applications in engineering metrology, viz., measurement of surface roughness, small displacement, tilt, strain and vibration, etc. [1–3]. Speckle photography, being a non-contact method of assessment, provides an engineering tool for testing and measurement of deformations in materials, components and devices. Many investigations have been reported on displacement and vibration measurement [4–12].

Silver halides have been widely used as recording materials. However, these need processing and thus are non-real-time. Quasi-real-time assessment can be done using dynamic optical processing techniques. Development of spatial light modulators (SLMs) and photorefractive (PR) non-linear materials has recently become of special interest for recording of speckle patterns in quasi-real-time [11–21]. Consequently, for displacement and tilt measurement, speckle photography using correlation techniques has recently been investigated [18–21] in a two-wave mixing configuration with PR $BaTiO_3$ crystal.

Although, methods are available to measure large amplitudes of in-plane vibration using time-average speckle photography, measurement of small amplitude of vibrations poses difficulties due to diffraction halo. Measurement of small vibrations has been attempted by using classical interferometric techniques [22,23] which, however, necessitate the use of polished surfaces. Therefore, it is desirable to develop speckle photography methods for small vibration measurement.

In this paper, we report a simple approach for quasi-real-time measurement of vibrations of a rough surface with small amplitude of the order of wavelength of light, and frequency using PR speckle correlator. Double-exposure method is adopted for recording the speckle patterns in a $BaTiO_3$ crystal, yielding straight parallel speckle fringes at the observation plane. Correlation peaks for fringes with poor visibility can also be obtained, thus increasing the range of measurement. Experimental results and simulation results are presented.

2. Method of operation

Fig. 1 shows basic recording geometry of speckle patterns. A rough surface (a strong diffuser) illuminated with laser light produces speckles and a $BaTiO_3$ crystal is used to record the interference pattern between the object and a strong pump beam. Two-beam coupling in PR $BaTiO_3$ leads to energy exchange between the interfering beams. As a result, speckles with sufficient intensity gain are observed in the observation plane. Stable fringes are recorded when the frequencies of both object and pump waves are equal.

Frequency of the object beam can be changed by reflecting the beam from a piezo-mirror (Fig. 2) which is vibrated by exciting with a periodic sinusoidal or saw tooth voltage from a signal generator. The difference of frequency in the object- and pump beams in a two-beam coupling process leads to running grating in the PR crystal. Consequently, the amplified speckles appear vibrating. Young's fringe-modulated speckle patterns are first generated by giving an appropriate displacement to the diffuser. The moving fringes are produced due to moving grating being recorded in the crystal. A further frequency shift in the object beam is introduced by vibrating the

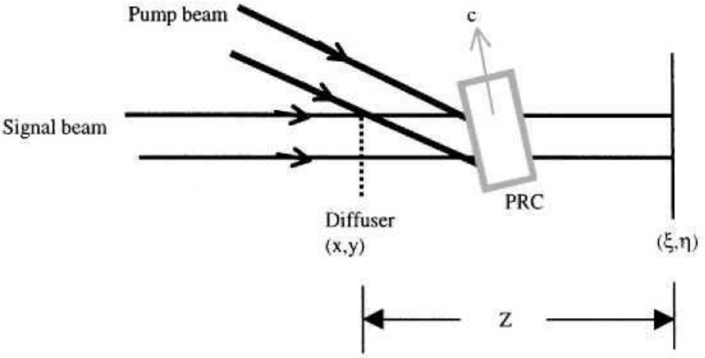

Fig. 1. Basic layout of speckle pattern recording geometry.

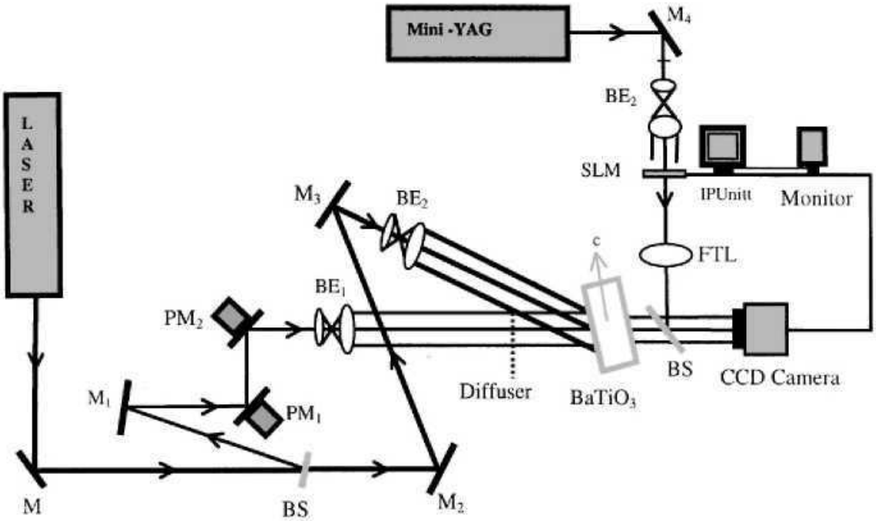

Fig. 2. Schematic layout of experimental set-up: M, M_1, M_2, M_3, M_4, mirrors; BE_1,BE_2, BE_3, beam expanders; BS, beam splitter ; PM_1, PM_2 piezo-mirrors ; FTL Fourier transforming lens and SLM, spatial light modulator.

second piezo-mirror. By canceling the frequency shift due to the vibration of mirror PM_2, the running fringes become stationary. The frequency shift and amplitude of vibration due to PM_1 in the object beam fully match with that of the piezo-mirror PM_2. This gives a direct measurement of the unknown frequency of vibration.

Stationary and vibrating interference fringes are detected by a CCD camera using a frame grabber with a computer. Fringes are then displayed onto an electrically addressed SLM (EASLM). A Fourier transform of the fringe pattern yields correlation output with its peak value varying with the value of vibration frequency mismatch.

Thus, the frequency of vibration of a surface can be measured in real-time by using correlation techniques.

Measurement of a small vibration amplitude has been carried out by counting the fringes that move past a line in the observation plane. To accomplish this task, one of the two piezo-mirrors is switched off. Interference pattern is generated in the manner mentioned above. Variation in the voltage applied to the piezo-mirror, changes the number of fringes passing past a line on the observation plane. This number multiplied by the fringe width gives a direct measure of the amplitude of vibration.

3. Theoretical background

The diffuser is illuminated (Fig. 1) with a beam of coherent light ($\lambda = 632.8\,\mathrm{nm}$), giving amplitude distribution $A_1(\xi, \eta)$ at the observation plane

$$A_1(\xi,\eta) = \frac{\exp(\mathrm{i}2\pi z/\lambda)}{\mathrm{i}\lambda z}\iint a(x,y)\exp\left[-\mathrm{i}\frac{\pi}{\lambda z}(x^2+y^2)\right]$$

$$\exp\left\{\mathrm{i}\frac{\pi}{\lambda z}[(\xi - x)^2 + (\eta - y)^2]\right\}\mathrm{d}x\,\mathrm{d}y, \tag{1}$$

where $a(x, y)$ is the diffuser function, Z is the distance between diffuser and observation plane, and λ is the wavelength of light used. It can be further written as

$$A_1(u,v) = C\iint a(x,y)\exp\{-\mathrm{i}2\pi(ux + vy)\,\mathrm{d}x\,\mathrm{d}y\} = \mathrm{CFT}[a(x,y)]_{\xi/\lambda z\ \eta/\lambda z} \tag{2}$$

with

$$C = \frac{\exp(\mathrm{i}2\pi z/\lambda)}{\mathrm{i}\lambda z}\exp\left[-\mathrm{i}\frac{\pi}{\lambda z}(\xi^2+\eta^2)\right],\quad u = \xi/\lambda z \text{ and } v = \eta/\lambda z.$$

In order to keep the analysis simple, we do not consider the PR crystal parameters such as gain of the material, etc., and this does not affect the final outcome of the above expression.

During the second exposure, amplitude distribution $A_2(u, v)$ after a displacement x_0 of the diffuser can be expressed as

$$A_2(u,v) = \mathrm{C}\iint a(x + x_0, y)\exp\{-i2\pi(ux + vx)\}\,\mathrm{d}x\,\mathrm{d}y$$

$$= \mathrm{C}\iint a(x,y)\exp\{\mathrm{i}2\pi(ux_0)\}\exp\{-\mathrm{i}2\pi(ux + vy)\}\,\mathrm{d}x\,\mathrm{d}y$$

$$= \mathrm{CFT}[a(x,y)]_{\xi/\lambda z\ \eta/\lambda z}\exp\{\mathrm{i}2\pi(ux_0)\}. \tag{3}$$

The resultant amplitude $A_\mathrm{R}(u, v)$ due to the two exposures is

$$A_\mathrm{R}(u,v) = A_1(u,v)[1 + \exp\{\mathrm{i}2\pi(ux_0)\}]. \tag{4}$$

Thus, the intensity distribution as detected by the CCD camera in the observation plane is given as

$$I_R(u, v) = \frac{2}{\lambda^2 z^2}|A_1(u, v)|^2[1 + \cos(2\pi u x_0)] = I_0[\cos^2(\pi u x_0)]. \tag{5}$$

3.1. Measurement of vibration amplitude

When x_0 is changed due to vibration of the piezo-mirror, the fringe pattern starts moving provided the amplitude of vibration is less than x_0. In this condition a small change $\Delta x_0/\cos\theta$ in x_0 does not affect the fringe spacing and only affects the number of fringes passing past a line in the observation plane. A change in x_0 only changes the fringe width. The successive peaks in $I_R(u, v)$ correspond to piezo-mirror displacement of $\lambda\cos\theta$. In our set-up, a displacement of Δx_0 in the piezo-mirror inclined at an angle θ to the incident light beam, introduces a phase shift ϕ in the reflected beam.

$$\phi = 2\pi\left(\frac{\Delta x_0}{\lambda\cos\theta}\right). \tag{6}$$

For sinusoidal vibrations, the instantaneous displacement Δx_0 of the piezo-mirror is given by

$$\Delta x_0 = A\sin\Omega t, \tag{7}$$

where A is the amplitude and Ω is the angular frequency of vibration.

The intensity distribution at a photodetector or a point of reference, for a mirror displacement is Δx_0 given by

$$I = I_0\cos^2\left(\pi\frac{A\sin\Omega t}{\lambda\cos\theta}\right). \tag{8}$$

Amplitude corresponding to the bright fringes is given by

$$A = N\lambda\cos\theta, \quad \text{where } N \text{ is the fringe number.} \tag{9}$$

Therefore, by counting the number of fringes passing past a line on the detector plane, the vibration amplitude A, for $\theta = \pi/4$ can be calculated by the relation

$$A = \frac{N\lambda}{\sqrt{2}}. \tag{10}$$

Therefore, successive peaks in $I_R(u, v)$ correspond to piezo-mirror displacement of $\lambda/\sqrt{2}$.

3.2. Measurement of vibration frequency

If x_0 is changed by vibrating the object beam, phase difference is introduced between the beam reflecting off the piezo-mirrors and the reference beam, and the fringe pattern starts moving. The intensity distribution at the CCD plane is given by Eq. (5). If one of the piezo-mirrors in the object beam is replaced by an object with unknown vibration, the fringe pattern moves, and becomes stationary when amplitude of both the mirrors are equal and in phase. It is achieved by the vibrating second piezo-mirror, also placed in the object beam, thus compensating the phase difference already introduced due to the first piezo-mirror introducing unknown vibration. If the frequencies of both the vibrating piezo-mirrors are equal, the fringe pattern becomes stationary, and the vibrating mirrors can be said to be in resonance. Alternatively, if one approaches closer to resonance, i.e. displacement of speckles due to vibration in both the mirrors are equal and are in phase, the moving fringe pattern becomes stationary. It indicates that the frequencies of both the beams, reflecting off the vibrating piezo-mirrors are equal. At this moment, the fringe pattern has to be captured to get the information of the frequency matching by calculating the correlation peak height. As mismatch between the frequencies owing to vibration of the two piezo-mirror increases, the contrast of the fringes goes down.

3.3. *Speckle correlation*

Speckle correlation is the Fourier transform of the intensity distribution as witnessed by the CCD camera, and is given by

$$A_C(x',y') = a(x',y')*a(x',y')\otimes[1+\delta(x'+x_0,y)+\delta(x'-x_0,y)]. \tag{11}$$

Thus, there are two peaks at '$x = \pm x_0$'. The symbols $*$ and $\otimes$, respectively, represent the correlation and convolution operations. (x',y') are the correlation plane coordinates.

4. Experimental results and discussion

The experimental arrangement is shown in Fig. 2. Beam from a He–Ne laser ($\lambda = 632.8\,\text{nm}$) in TEM_{00} mode is divided into two parts by a beam splitter BS, providing one beam of low intensity (object beam) and other of strong intensity (pump beam) to enable energy exchange between the two beams. A $BaTiO_3$ crystal measuring $5\times5\times6\,\text{mm}$, with its 6 mm dimension along the *c*-axis direction of the crystals is used to record speckles. To optimize recording geometry for object beam amplification, the angle between object and pump beams is kept at 20°, and the crystal is oriented such that its normal makes an angle of 50° with the pump beam. To introduce vibration in the object beam, we use two piezo-mirrors PM_1 and PM_2 which are set apart and kept parallel so as to keep the direction of the beam unchanged after reflection from the two mirrors. The object beam is then expanded and collimated and passed through a diffuser to generate speckles. Spot size of the beam is kept 6.00 mm in diameter at the diffuser to ensure suitable speckle size. The pump beam is also expanded and collimated to ensure overlap with object beam at the crystal.

The object and the pump beams interact in the volume of the crystal and write a dynamic grating, leading to an energy exchange between the beams. $BaTiO_3$ is a high-gain PR crystal and due to energy exchange, speckles with amplified intensity are observed at the recording plane. Double-exposure method is used, i.e. we record speckles followed by second exposure after giving a known displacement to the diffuser. When the object beam gain saturates, a known in-plane displacement is given to the diffuser to generate Young's fringes. Vibrations are then produced by exciting piezo-mirror PM_1, with a sinusoidal or a saw-tooth voltage from a signal generator. There is interference between the directly transmitted vibrating object speckles and the reconstructed speckles by read out of the previous grating, resulting in straight and parallel moving fringes.

For the measurement of vibration amplitude, piezo-mirror PM_1 is kept stationary and piezo-mirror PM_2 is vibrated. On the observation plane, a reference mark is indented. Now, the amplitude of vibration is varied and the number of fringes passing past the reference mark is counted. This number multiplied by the fringe width gives a direct measurement of the vibration amplitude. By our method measurement of small vibration amplitude can be made down to 0.22 μm. Thus, the method has sensitivity comparable to that of the interferometric method. Table 1 shows the measured values of vibration amplitudes for different values of the voltage applied to the piezo-mirror.

For the measurement of vibration frequency, the piezo-mirror PM_2 is vibrated by exciting with a sinusoidal or saw-tooth voltage from another signal generator. At a certain frequency, the moving fringes become stationary (for the longest duration of time) with maximum contrast. The fringes observed at this stage, are captured by a CCD camera. Observations are made by varying the frequency, and fringes captured at each stage. The value of vibration frequency required to make the moving fringes stationary is the measure of unknown vibration frequency. Due to the nature of PRCs, grating/information recorded in the crystal remains stored for some time. Before writing a new grating for the new set of observations, it is ensured that the previous grating stored in the crystal is completely erased. Fringes detected by the CCD camera

Table 1
Measured values of small vibration amplitude; A-amplitude, λ-wavelength of laser light used and N-number of fringe shifted

Applied voltage to piezo-mirror (V)	No. of fringes passing past a line	Measured value of vibration amplitude (A) $\left[A = N(\text{no. of fringes})\frac{\lambda}{\sqrt{2}}\right]$ (μm)
1.3	0.5	0.22
2.4	1.0	0.45
3.5	1.5	0.67
5.0	2.0	0.89
6.5	2.5	1.12
7.4	3.0	1.34
8.7	3.5	1.57
9.9	4.0	1.79
12.3	5.0	2.24

are displayed onto an EASLM (VGA1, CRL UK) and read-out by an expanded and collimated beam from a mini-YAG laser ($\lambda = 532\,\mathrm{nm}$). Fringes so obtained are then Fourier transformed and subsequently detected by the same CCD camera giving correlation output.

Fig. 3a shows stationary fringe pattern for a known displacement x_0 of the stationary diffuser. Fig. 3b is the corresponding correlation output. Figs. 4a–i and 5a–i show fringe patterns and corresponding correlation outputs depicting variation in the correlation peak height. These figures show that as the mismatch between the frequency of vibration of the object increases compared to the frequency of the stationary object, the contrast of the fringe pattern decreases and the correlation peak height is affected. Correlation peak height gives an idea of the frequency of vibration.

Fig. 6 is a plot between height of the correlation peak and frequency of vibration of PM_1, when frequency of vibration of PM_2 is 3 Hz. It is observed that when the unknown and known vibration frequencies match, a high correlation is achieved. When the object beam vibrates, the speckle pattern at the observation plane starts

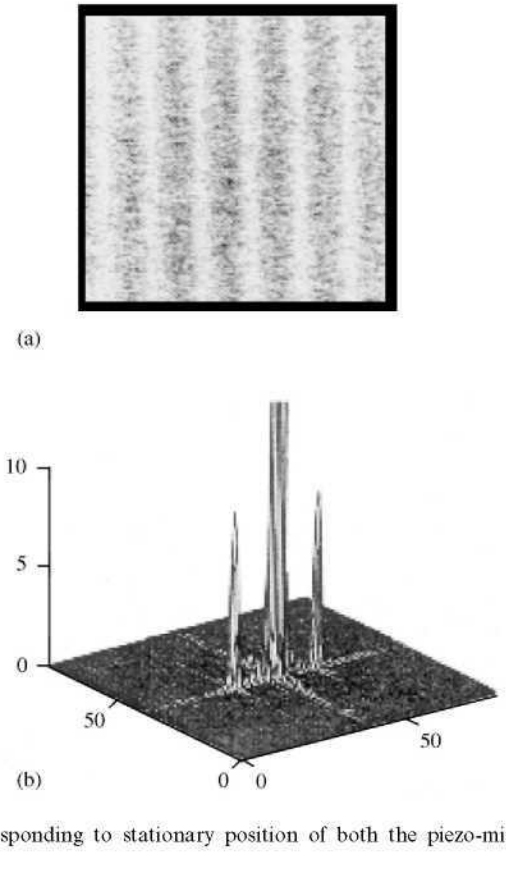

Fig. 3. (a) Fringes corresponding to stationary position of both the piezo-mirrors and (b) respective correlation output.

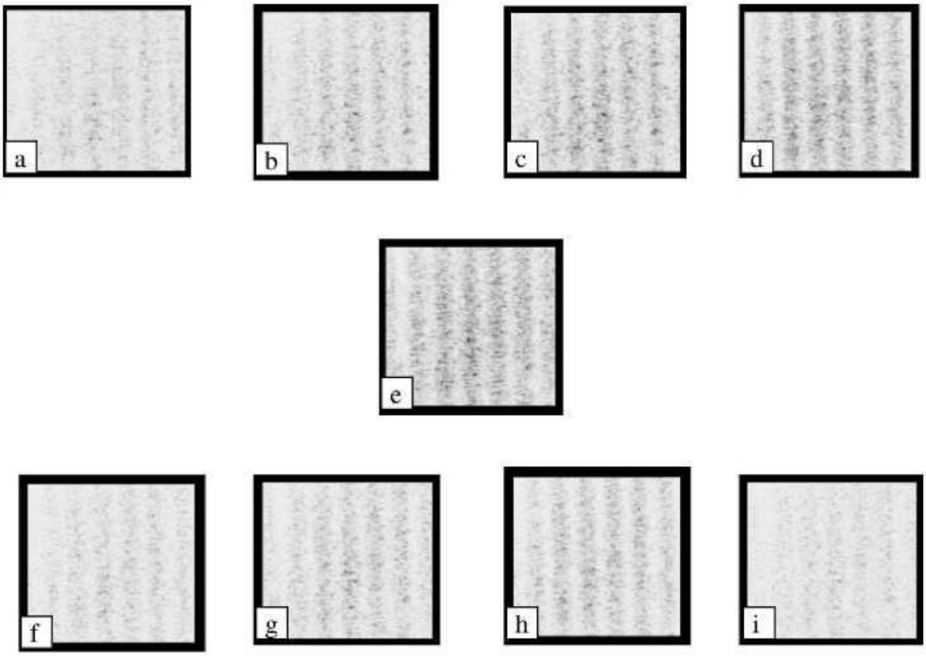

Fig. 4. Fringes showing variation in contrast with variation in frequency. (a–i) for frequencies 0.8, 1.3, 1.8, 2.3, 3.3, 3.8, 4.3, 4.8 and 5.3 Hz.

moving. At high frequencies, the moving speckles are very difficult to detect for the purpose of measurement. If we generate interference fringes out of these speckles, the moving fringes can be easily observed. The slow response of PR $BaTiO_3$ crystal is used to record double exposure. The minimum displacement is decided by the speckle size in the specklegram recording plane in order to generate large numbers of fringes.

The amount of displacement to the diffuser to generate a number of fringes does not affect the result. If the displacement is equal to or greater than the speckle size, the correlation-peaks are separated from the DC term. An appropriate displacement to the diffuser is given to generate fringes with good visibility. Our method facilitates easy and accurate measurement of vibration with small amplitude and frequency. Vibrations with large amplitude can generate their own Young's fringes thus making our technique inapplicable. In this case, we have to use the time average method.

5. Conclusion

A technique has been implemented to analyse small in-plane vibrations. The amplitude of vibration of the order of a wavelength has been measured by detecting the fringe shift. The information about the frequency of vibration has been obtained by analysing the correlation output of captured vibrating Young's fringes. Our

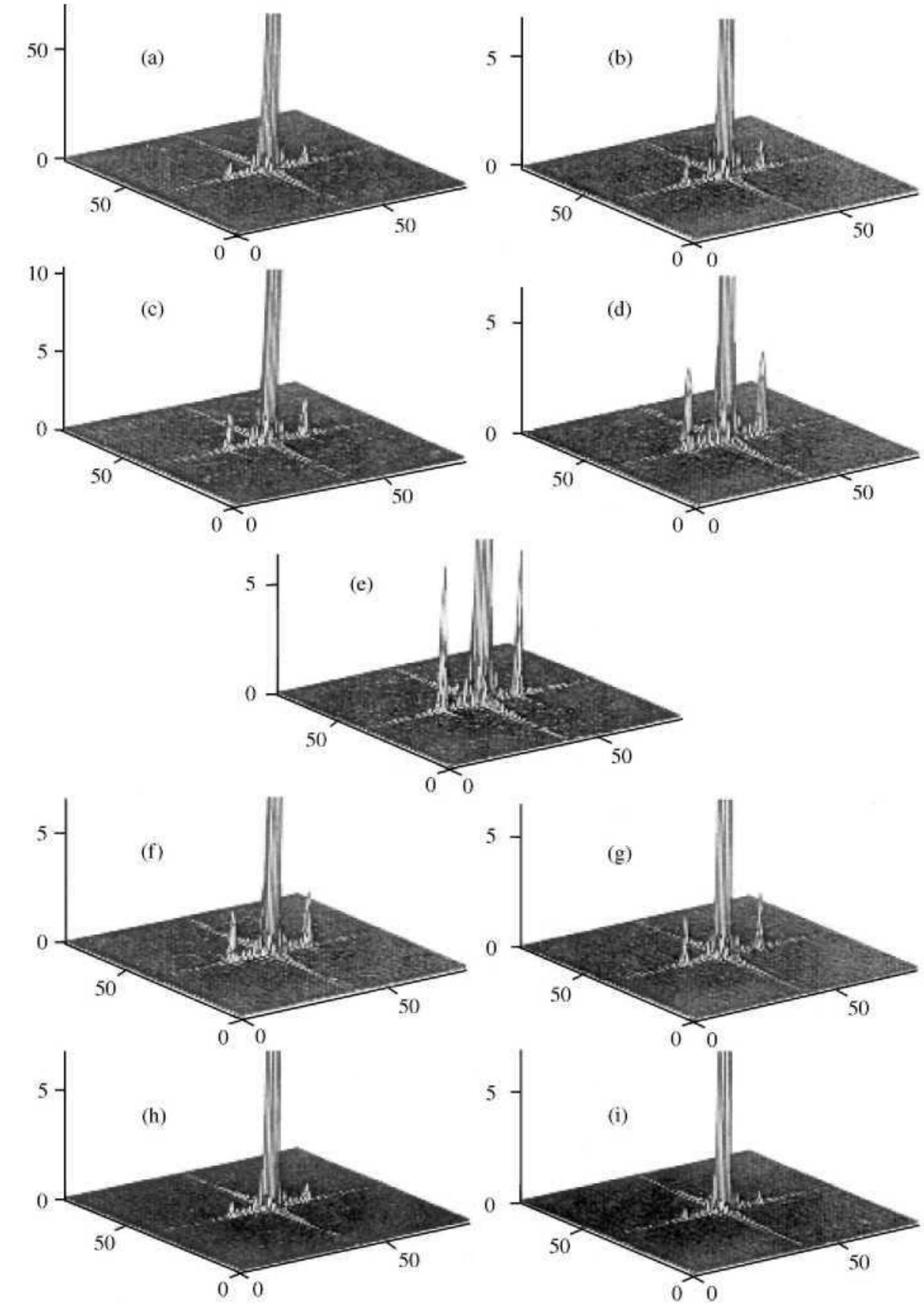

Fig. 5. (a–i) Correlation output corresponding to Fig. 4(a–i).

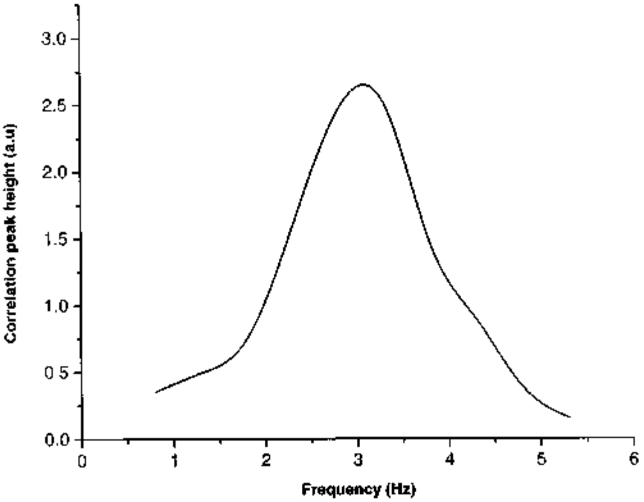

Fig. 6. Variation in correlation peak height with frequencies of vibration of PM_1 when frequency of vibration of PM_2 is 3 Hz.

method using correlation technique gives a direct and quasi-real-time method of measurement of frequency of unknown vibration. Our technique has application in speckle metrology in which vibrations with small amplitude and frequency have to be measured accurately.

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