Equipment flexibility vs. inventory: A simulation study of manufacturing systems

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Received 6 January 1999; accepted 12 April 2000

Abstract

Technological flexibility requires very high initial investments and therefore a decision to acquire it is both risky and strategic in nature. The objective of this paper is to find out the technological flexibility requirements in just-in-time manufacturing under different environmental situations. The simulation results show that the incorporation of flexibility in a manufacturing system is vital. The choice of the extent of flexibility is driven by the market and economic factors.

Keywords: Manufacturing strategy; Just-in-time; Flexibility; Simulation

I. Introduction

In recent years, just-in-time (JIT) production system has drawn considerable attention and proved to be extremely successful, especially in Japan. JIT is a philosophy of eliminating waste in the total manufacturing process – from procurement of raw materials to distribution of final products or services. This makes the manufacturing process more streamlined, cost efficient, quality oriented and responsive to the customer. JIT thus acts as a strategic weapon in a competitive business environment. JIT improves the system design to help manufacturing organizations operate more efficiently by reducing their inventories.

In the present environment characterized by uncertainty, greater customer expectations, competition, and fast developments in technology and products, traditional methods of manufacturing planning and control based on the push system are no longer economical. Inventory under these approaches is pushed through the work stations based on some predetermined plan. One of the weaknesses of the push system is that an error in demand estimation results in large inventories, delayed shipments, long throughput times and shortages. In contrast, the JIT manufacturing system emphasizes production or procurement of parts only when these are needed. Such systems would, however, necessitate flexibility of equipment, multiskilled workers and reliable sources of supply.

Flexibility is the ability of a manufacturing system to cope with changes in the nature, mix, volume or timing of its activities. Flexibility in
a manufacturing system ensures that the production rate can be matched with demand, leading to savings in inventory-related costs, reduced lead times and increased quality.

Flexibility is not free, it comes at a cost. Flexibility more often means extra capacity either in the form of extra machines, technology or in manpower. For developing economies, where the cost of capital is high, the decision to acquire a flexible manufacturing system may not be economically justified, whereas for Japan and other capital rich countries it may be the ideal thing to do. The nature of industry also influences the relative importance of inventory control. For a highly capital intensive factory, machine utilization may be more important than inventory reduction. In a steel wire manufacturing company, a year of inventory may be insignificant as compared to one investment decision on the acquisition of a capital asset. On the other hand, in an automobile assembly, the raw materials are high value added items and the inventories are indeed a major cost component.

Thus JIT is not feasible in developing countries and traditional just-in-case (JIC) has its own demerits. JIC tends to cover up problems existing in the factory. JIC systems are push systems in which control policy gives a production trigger based on forecasted demand and predicted product flow in the production and inventory system. Under these policies inventory is considered as an asset. It protects against forecast error, machine problems and late vendor deliveries.

Non-feasibility of JIT and demerits of JIC lead to an interesting research poster, namely, to determine the best mix of JIT and JIC for these conditions. Certain external and internal factors decide the design of the mix of JIT and JIC for formation of a 'JIT-like' manufacturing system which is economical to operate and meets competition.

The intent in this paper is to investigate the effects of technological flexibility to find out the extent of flexibility required in an environment characterized by different levels of variability. By creating the choice set and evaluating the alternatives on economic criteria, the study suggests how to actually achieve such flexibility. This results in the development of 'JIT-like' manufacturing system to meet the challenges of competition.

2. Literature review

JIT manufacturing has been an area of extensive research in recent years. The benefits of JIT implementation have been acknowledged by many researchers [1-5]. A large variety of approaches have been adopted for the study of JIT systems. Simulation has been widely used as a vehicle to identify and study the internal and external factors that affect the success of pull system in JIT, to determine the number of kanbans required at each work station and to investigate the effect of demand and processing time variability [6-10]. Several researchers [11-14] studied kanban and MRP systems to develop hybrid systems for variable and uncertain manufacturing conditions. Huang et al. [15] and Kimura and Takeda [16] analyzed the pull system for multi-line, multi-stage production systems.

Flexibility of manufacturing systems is an important dimension for the successful operations of the pull system. Flexibility is defined by Gupta and Goyal [17] and Gupta and Gupta [18] as the ability of a manufacturing system to cope with changing circumstances and instability caused by the environment. Gerwin [19] gave mix, changeover, rerouting and volume flexibility as the dimensions of flexibility related to market-oriented uncertainties. Wildemann [20] observed that it is more profitable to store capacity in capital assets, rather than in liquid assets. Atwater and Chakravorty [21] took a very strong stance on the necessity of protective capacity. They asserted that the balanced plant is not only impossible to attain but also undesirable. Nandkeyol and Christy [22] observed that as the product diversity increases, for a given system there is a level of flexibility that will optimize system performance. Fine and Freund [23] illustrated that when the demand is negatively correlated between two products, greater risk stimulates the need for flexible manufacturing technology. Benjaar [24] observed that flexibility is the mechanism in dealing with the disruptive potential of the variability in a dynamic environment. Porteus [25] studied the tradeoff associated with reducing the set-up costs in the classical EOQ model and found that investing in reduced set-up is economical merely on the basis of reduced inventory-related costs.

Studies over the years have established the benefits of the flexibility element of the JIT approach.
Goyal and Deshmukh [26] in their critique of the literature on JIT manufacturing observed that there exists little understanding of the effect of this element of JIT on manufacturing performance beyond this intuitive and rudimentary perception. There is a lack of rigorous models capable of generating clear relationships between the degree of flexibility required under different operating conditions and the desired level of system performance.

3. Description of the manufacturing system

The manufacturing system considered for the investigation has a multi-product, multi-stage series/parallel manufacturing configuration as shown in Fig. 1. A system that relies exclusively on safety stock for meeting demand uncertainties is the JIC system. On the contrary, the JIT is a system that relies on additional capacity and routing flexibility to deal with environmental uncertainty. It is this interplay that is investigated through a simulation model to determine the right mix of JIT and JIC characteristics in a system. This figure broadly summarizes the approach adopted for evaluating alternative policy options and proposing the best strategy with regard to the manufacturing system. In the conventional production distribution system, inventory costs are typically the cost of carrying inventory, cost of shortages and cost of set-up/cost of ordering. In a JIT system, it is desired to reduce all these three types of costs by incorporating flexibility in the manufacturing system. For economic analysis of the 'JIT like' manufacturing system, cost of flexibility should also be considered.

The cost of flexibility is a function of additional capacity cost and routing flexibility cost. The problem investigated in this paper is to minimize the costs associated with providing the desired level of customer service under probabilistic demand conditions by activating both JIT and JIC policies. The trade-off between these two policies is examined to develop a hybrid policy.

The performance of the system is expressed in terms of the total relevant cost (TRC) at a given service level. TRC to be minimized is computed as follows:

\[ TRC = \text{inventory carrying cost} + \text{additional capacity cost} + \text{routing flexibility cost} + \text{additional operational cost}. \]

![Fig. 1. A framework for manufacturing strategy selection.](image-url)
In defining TRC the backlog cost is not included in inventory cost as it implicitly governs the service level. Since the service level is a surrogate and convenient measure of the backlog cost all comparisons are made at a given service level and the impact of varying service levels is also investigated.

Symbols/notations used in this paper are given below in alphabetical order.

\[ a \] discount factor
\[ i \] cost of carrying inventory
\[ t \] discrete time period when demand occurs
\[ A_m(t) \] production trigger of product \( m \) during period \( t \) (units)
\[ AI_m(t) \] available inventory of product \( m \) in period \( t \) (units)
\[ BL_m(t) \] backlog of demand of product \( m \) during period \( t \) (units)
\[ CC_p \] cost of adding capacity of equipment \( p \) (\( Rs. \)/unit)
\[ CI_m \] cost of carrying inventory of product \( m \) (\( Rs. \)/unit yr)
\[ C_m \] penalty cost of shortages of product \( m \) (\( Rs. \)/unit yr)
\[ CT_{m,r,p} \] cycle time of product \( m \) on route \( r \) at equipment \( p \)
\[ CV_m \] coefficient of variation of demand of product \( m \) (units/period)
\[ D_{m2} \] average demand of product \( m \) (units/period)
\[ D_n(t) \] demand of product \( m \) during period \( t \) (units)
\[ DO_n(t) \] dispatch objective of product \( m \) during period \( t \) (units)
\[ H_p \] capacity of the equipment \( p \) (production units)
\[ H'_p \] capacity of the equipment \( p \) (time units)
\[ I_m(t) \] inventory of product \( m \) at the end of period \( t \) (units)
\[ K \] cost ratio index
\[ M \] number of products in the manufacturing system
\[ NPBL_{M} \] number of backlog periods of product \( m \)
\[ NPV \] net present value
\[ P \] number of equipment in the manufacturing system
\[ Q_{m}(t) \] objectivated output of product \( m \) during period \( t \) (units)
\[ R_m \] number of alternate routes for product \( m \)
\[ RG_m \] range of demand of product \( m \)
\[ RN(t) \] pseudo-random number generated in period \( t \)
\[ SD \] standard deviation of demand
\[ SH_m(t) \] shipment of product \( m \) in period \( t \) (units)
\[ SL_m \] service level of product \( m \)
\[ SS_m \] safety stock of product \( m \) (units)
\[ TRC \] total relevant cost

4. The model and assumptions

This manufacturing system has \( P \) number of equipment arranged in flexible lines to process \( M \) products. The \( m \)th product is characterized by \( R_m \) routes, a safety stock of \( SS_m \) and a penalty cost of shortage equal to \( C_m \). The cycle time of product \( m \) on route \( r \) at equipment \( p \) is \( CT_{m,r,p} \). \( CT_{m,r,p} = 0 \) indicates that the equipment \( p \) is not visited by product \( m \) on route \( r \). Demand occurs at discrete intervals and unfulfilled demand is backlogged. Holding and shortage costs are assumed to be linear and occur only for end of period inventory and backlog. The downtime of machines, quality problems and interruptions in supply of raw materials are not considered.

The simulation model for this manufacturing system is developed and applied to three special manufacturing systems as follows:
(a) single product manufacturing system;
(b) two product manufacturing system;
(c) multi-product manufacturing system.

For a single product manufacturing system the following cases are examined:
(i) effect of demand variability on the choice of manufacturing policy;
(ii) effect of desired customer service on the choice of manufacturing policy;
(iii) economic analysis of choice set created in cases (i) and (ii) to select the right policy;
(iv) impact of delays on the choice of manufacturing policy.

The analysis of single product system is extended by considering two products with demand and
resource interactions for the following cases:

(i) impact of manufacturing configuration on system performance;
(ii) effect of general economy expressed by cost of capital and cost of flexibility;
(iii) effect of demand relationships;
(iv) effect of product variety.

A real case of multi-product, multi-stage ancillary unit is examined to determine the suitability of flexible configuration in comparison with existing dedicated lines.

For single product and two product manufacturing systems, it is considered that:

- cost of carrying inventory is 20% and cost of capital is 15%;
- unit cost of product is Rs. 300;
- standard capacity is equal to the average total demand and it costs Rs. 50,000;
- cost of adding capacity is Rs. 1000 per unit;
- life of project is large.

4.1. Simulation model

In the development of the simulation model of this manufacturing system, the objective is to get the desired level of service at minimum cost. For JIT policy, additional capacity and routing flexibility are high, whereas for JIC policy, these are very low. Depending upon the manufacturing policy, suitable safety stock is planned to get the desired service level. Simulation is used to find the average inventory. Objectivized output, defined by Vrat et al. [27] as the optimal level of output which is possible to be attained by a system under the given constraints of input resources and a set of performance objectives is determined as follows:

For a uniformly distributed demand distribution, demand in period \( t \) is generated using Monte Carlo method as shown in Eq. (1). Demand generation with normal distribution and correlated demand of two product manufacturing system is given in the appendix; Deo [28]:

\[
D_m(t) = D_m^{\text{avg}} + (R_m(t) - 0.5)R_G_m. \tag{1}
\]

Despatch objective of product \( m \) in period \( t \), \( \text{DO}_m(t) \) is the quantity to be despatched in period \( t \) if there are no constraints from the manufacturing side. This is equal to the demand in period \( t \) plus the backlog in period \( t - 1 \), if any:

\[
\text{DO}_m(t) = D_m(t) + \text{BL}_m(t - 1). \tag{2}
\]

Production trigger \( A_m(t) \) defined as the production required in the absence of any capacity constraint can be determined from despatch objective, safety stock level and inventory/backlog of finished product in preceding period as follows:

\[
A_m(t) = D_m(t) + \text{BL}_m(t - 1) + SS_m - I_m(t - 1). \tag{3}
\]

As the capacity of the manufacturing system is limited and is not sufficient to produce extreme cases of high demand, it is considered that manufacturing capacity acts as a constraint for determining the objectivized output \( Q_m(t) \). Under capacity constrained conditions, the objective of the planning process is to minimize the penalty cost of backlog by selecting the optimum product mix. This is an optimization problem having the objective function of minimizing the total penalty cost of backlog and is given as

\[
\text{Minimize } Z' = \sum_{m=1}^{M} C_m \text{BL}_m(t), \tag{4}
\]

where

\[
\text{BL}_m(t) = \text{Max}(0, \{[\text{DO}_m(t)] - [Q_m(t) + I_m(t - 1)]\})
\]

\[
= \text{Max}(0, \{[D_m(t) + \text{BL}_m(t - 1)]
\]

\[
- [Q_m(t) + I_m(t - 1)]\}).
\]

Since \( D_m(t), \text{BL}_m(t - 1) \) and \( I_m(t - 1) \) are given parameters and not decision variables in the period \( t \), these are considered as constant for the period \( t \).

Minimize \( Z = \text{Constant} - \sum_{m=1}^{M} C_m Q_m(t), \tag{5} \)

Maximize \( Z = \sum_{m=1}^{M} C_m Q_m(t), \)

subject to the capacity constraints, requirement constraints and non-negativity constraints given by
Eqs. (6), (7) and (8), respectively:

\[
\sum_{m=1}^{M} \sum_{r=1}^{R} C_{m,r,p} Q_{m,r}(t) \leq H_p \quad \text{for all } p, \quad (6)
\]

\[
\sum_{r=1}^{R} Q_{m,r}(t) \leq A_m(t) \quad \text{for all } m, \quad (7)
\]

\[
Q_{m,r}(t) \geq 0 \quad \text{for all } m \text{ and } r. \quad (8)
\]

This formulation is used to study JIT, hybrid and JIC policies. For JIT policy, \( H_p \) and \( R_m \) are high to have additional capacity and routing flexibility whereas for JIC policy \( H_p \) is nearly equal to the average demand and \( R_m \) is equal to 1.

5. Special cases

In this section, two special cases are derived from the general formulation of Section 4.

5.1. Single product single stage

In a single product single stage manufacturing system, \( P, M \) and \( R \) are one each. The objectivated output should satisfy the following constraints deduced from Eqs. (6)-(8):

\[
CT \ Q(t) \leq H' = Q(t) \leq H,
\]

\[
Q(t) \leq A(t),
\]

\[
Q(t) \geq 0.
\]

These constraints are equal to a single constraint as follows:

\[
Q(t) = \text{Min}[H, A(t)].
\]  

(9)

5.2. Two product single stage

In this case \( M = 2 \). The number of equipment and routes for each product depend upon the configuration. Three configurations that are considered are given in Fig. 2. The objective function can be derived from Eq. (5) in Section 4.1:

\[
\text{Maximize } Z = C_1 Q_1(t) + C_2 Q_2(t).
\]

The constraints of the system, number of equipment and routes for each product depend upon the configuration.

5.2.1. Configuration 1: Two dedicated equipments (Fig. 2a)

This configuration has two dedicated equipments each to process one product. Thus \( M = 2, P = 2, R_1 = 1 \) and \( R_2 = 1 \). The constraints of the system are deduced from Eqs. (6)-(8):

\[
CT_1 \ Q_1(t) \leq H_1 \Rightarrow Q_1(t) \leq H_1,
\]

\[
CT_2 \ Q_2(t) \leq H_2 \Rightarrow Q_2(t) \leq H_2,
\]

\[
Q_1(t) \leq A_1(t),
\]

\[
Q_2(t) \leq A_2(t),
\]

\[
Q_1(t) \geq 0,
\]

\[
Q_2(t) \geq 0.
\]

These constraints reduce to the following two constraints to determine the objectivated output of products 1 and 2 as

\[
Q_1(t) = \text{Min}[H_1, A_1(t)].
\]  

(10)

\[
Q_2(t) = \text{Min}[H_2, A_2(t)].
\]  

(11)

5.2.2. Configuration 2: One flexible equipment and one dedicated equipment (Fig. 2b)

In this configuration, equipment 1 is flexible, used mainly for processing product 1 and equipment 2 is dedicated used only for processing product 2. Any surplus capacity of equipment 1 after processing product 1 is used for product 2, as

(a) Dedicated Configuration  (b) Partially Flexible Configuration  (c) Flexible Configuration

Fig. 2. Configuration for two products single stage manufacturing system.
and when required. Thus \( M = 2, P = 2, R_1 = 1, \ R_2 = 2 \). The constraints of the system are deduced from Eqs. (6)-(8):

\[
\begin{align*}
\text{CT}_1 \ Q_1(t) + \text{CT}_{2,1} \ Q_{2,1}(t) & \leq H_1' , \\
\text{CT}_{2,2} \ Q_{2,2}(t) & \leq H_2' , \\
Q_1(t) & \leq A_1(t) , \\
Q_2(t) & \leq A_2(t) , \\
Q_1(t) & \geq 0 , \\
Q_2(t) & \geq 0 .
\end{align*}
\]

**Objectivated output of product 1**: Let product 1 has higher priority as compared to product 2 on the basis of contribution to profit per unit of resource used; therefore, on flexible equipment product 1 is processed first. The constraints for product 1 are as follows:

\[
\begin{align*}
\text{CT}_1 \ Q_1(t) & \leq H_1' \Rightarrow Q_1(t) \leq H_1 , \\
Q_1(t) & \leq A_1(t) , \\
Q_1(t) & \geq 0 .
\end{align*}
\]

These constraints are equivalent to a single constraint as follows:

\[
\begin{equation}
\begin{aligned}
Q_1(t) & = \min[H_1', A_1(t)].
\end{aligned}
\tag{12}
\end{equation}
\]

**Objectivated output of product 2**: After having objectivated output \( Q_1(t) \) for product 1 on equipment 1, the remaining capacity of equipment 1 is \( (H_1' - \text{CT}_1 \ Q_1(t)) \).

The constraints for product 2 are

(i) Capacity constraint of equipment 1:

\[
\begin{align*}
\text{CT}_{2,1} \ Q_{2,1}(t) & \leq H_1' - \text{CT}_1 \ Q_1(t) \\
\Rightarrow Q_{2,1}(t) & \leq \frac{H_1' - \text{CT}_1 \ Q_1(t)}{\text{CT}_{2,1}} .
\end{align*}
\]

(ii) Capacity constraint of equipment 2:

\[
\begin{align*}
\text{CT}_{2,2} \ Q_{2,2}(t) & \leq H_2' \\
\Rightarrow Q_{2,2}(t) & \leq \frac{H_2'}{\text{CT}_{2,2}} .
\end{align*}
\]

The combined capacity constraint of equipments 1 and 2 is as follows:

\[
\begin{align*}
Q_{2,1}(t) + Q_{2,2}(t) & \leq \frac{H_2'}{\text{CT}_{2,2}} + \frac{H_1' - \text{CT}_1 \ Q_1(t)}{\text{CT}_{2,1}} , \\
Q_2(t) & \leq \frac{H_2'}{\text{CT}_{2,2}} + \frac{H_1' - \text{CT}_1 \ Q_1(t)}{\text{CT}_{2,1}} .
\end{align*}
\]

(iii) The requirement constraint is

\[
Q_2(t) \leq A_2(t) .
\]

(iv) The non-negativity constraint is

\[
Q_2(t) \geq 0 .
\]

Combining these constraints, the objectivated output of product 2 is as follows:

\[
\begin{equation}
\begin{aligned}
Q_2(t) & = \min \left[ \frac{H_2'}{\text{CT}_{2,2}} + \frac{H_1' - \text{CT}_1 \ Q_1(t)}{\text{CT}_{2,1}}, A_2(t) \right].
\end{aligned}
\tag{13}
\end{equation}
\]

5.2.3. Configuration 3: One flexible equipment (Fig. 2c)

In this case only one flexible equipment is used; therefore, each product has only one route and \( M = 2, P = 1, R_1 = 1 \) and \( R_2 = 1 \). The constraints of the system are deduced from Eqs. (6)-(8):

\[
\begin{align*}
\text{CT}_1 \ Q_1(t) + \text{CT}_2 \ Q_2(t) & \leq H' , \\
Q_1(t) & \leq A_1(t) , \\
Q_2(t) & \leq A_2(t) , \\
Q_1(t) & \geq 0 , \\
Q_2(t) & \geq 0 .
\end{align*}
\]

**Objectivated output of product 1**: Here also, it is considered that product 1 has a higher priority as compared to product 2. The constraints for product 1 are given below:

\[
\begin{align*}
\text{CT}_1 \ Q_1(t) & \leq H' , \\
Q_1(t) & \leq A_1(t) , \\
Q_1(t) & \geq 0 .
\end{align*}
\]

Combining these constraints, the objectivated output of product 1 is as follows:

\[
\begin{equation}
\begin{aligned}
Q_1(t) & = \min \left[ \frac{H'}{\text{CT}_1}, A_1(t) \right].
\end{aligned}
\tag{14}
\end{equation}
\]
Objectivated output for product 2: After having objectivated output \( Q_1(t) \) for product 1 on this flexible equipment, the remaining capacity of the equipment is as follows:

\[ H' = CT_1 Q_1(t). \]

The constraints for product 2 are as follows:

\[ CT_2 Q_2(t) \leq H' - CT_1 Q_1(t) \]

\[ \Rightarrow Q_2(t) \leq \frac{H' - CT_1 Q_1(t)}{CT_2}. \]

\[ Q_2(t) \leq A_2(t), \]

\[ Q_1(t) \geq 0. \]

Combining these constraints, the objectivated output for product 2 is as follows:

\[ Q_2(t) = \text{Min} \left[ \frac{H' - CT_1 Q_1(t)}{CT_2}, A_2(t) \right] \quad (15) \]

In this section, the objectivated output for products 1 and 2 with three configurations is deduced. These objectivated output equations are used to determine the performance of various policies under different market scenarios of demand variability and service-level expectations. The performance of these policies is compared on economic criteria to select the right manufacturing policy and framing general guidelines for wider application.

6. Single product manufacturing system

In this section, a single product single stage production line has been studied to investigate the effect of probabilistic demand, delays in production system and cost of adding capacity on the choice of manufacturing policy. Level of safety stock and level of additional capacity affect the TRC of this system which can be calculated as

\[ \text{TRC} = \text{cost of carrying inventory} + \text{cost of additional capacity} \]

\[ = \text{CI} \times \text{average inventory} + \text{CC} \times \text{additional capacity.} \quad (16) \]

CI and CC are the cost parameters and are considered to be constant. Cost ratio index \( K \), defined as the ratio of incremental cost of adding capacity to the incremental cost of carrying inventory is used to study the trade-off between adding capacity and reducing inventory:

\[ K = \frac{\text{incremental cost of adding capacity}}{\text{incremental cost of carrying inventory}} = \frac{\text{CC}}{\text{CI}} \quad (17) \]

The system is simulated for 5000 periods. Ten runs were made for each policy and the average values were determined. The simulation results are presented as follows.

6.1. Case 1

In this case, effect of the demand variability on the level of service, average inventory and requirement of additional capacity is considered. Expected values of system performance show that at higher coefficient of variation of demand (CV), policy with more safety stock and additional capacity is required to give the same level of service. Fig. 3 shows additional capacity and safety stock required at three levels of variability (CV = 0.2, 0.3 and 0.4) at 80% service level. Fig. 4 shows the effect of safety stock and additional capacity on the service level. Service level increases with increase in safety stock and additional capacity.

![Fig. 3. Impact of demand variability on additional capacity and safety stock requirements at 80% service level.](image-url)
6.2. Case II

In this case the effect of desired level of service on the choice of manufacturing policy is examined. Six levels of service, 80%, 90%, 95%, 99%, 99.5% and 99.9%, are considered. The desired level of service can be achieved by different policies, i.e. JIT policy of low safety stock or JIC policy of high safety stock as shown in Fig. 5. The six curves are Exchange Curves between safety stock and additional capacity at constant service level. These exchange curves can help the organization in improving its performance. If the organization is operating at $X(A, B)$ as shown in Fig. 6a to give 80% service level, then four actions can be taken to improve the situation as follows:

1. Release a certain amount of additional capacity for other purposes and operate at point $X(A', B)$.
   This will reduce the opportunity cost associated with using additional capacity (Fig. 6a).
2. Reduce the safety stock level and operate at point $X(A, B')$. By reducing safety stock, the inventory carrying cost will decrease (Fig. 6b).
3. Operate at any point between $X(A', B)$ and $X(A, B')$ depending upon the utility of released capacity (Fig. 6c).
4. Operate at higher level of service and earn premium (Fig. 6d).

The outcomes of cases I and II are presented in a decision grid, named “market volatility-market expectation” grid, shown in Fig. 7, for selecting manufacturing policy. Management should first find out, whether the product has a stable or volatile demand. This may be captured by computing CV of demand for the products manufactured. In general, new products, products with short life cycles, having high technological advancement, seasonality, etc. have high demand variability. Variation in demand may also occur due to marketing policies like advertising and price discount of the company and competitors, government fiscal policies and pre-budget/post-budget buying syndrome.

Then it is necessary to find out the expectations of target customers. Products having monopoly, non-critical in manufacturing process generally have low expectations. Since market volatility and market expectation are independent of each other, four types of market situations emerge.

In type 1 market scenario, management can build up their manufacturing operation as flexible. By having flexibility in the manufacturing system, the system is able to respond to small uncertainties of demand more economically without inventory. Automobile ancillaries, having low demand uncertainties and high desired level of service can go for JIT manufacturing system. The market scenario of type 2 is very critical and most challenging as it is characterized by high demand uncertainties and high expectations from customers. Manufacturing
policy under this environment needs high flexibility and high buffer stock so as to provide high level of service under high uncertainties. In type 3 market scenario, manufacturers generally have low manufacturing flexibility and keep low level of inventories. This kind of systems are non-critical, non-challenging and generally practised in public sectors enjoying monopoly of product/service with full government protection. The market scenario of type 4 is volatile with low desired level of service. JIC manufacturing policy of keeping buffer stocks to respond to uncertainties is economical rather than building JIT flexible systems.

6.3. Case III

In this case economic analysis is carried out to select the best policy for the given demand variability and system configuration. TRC of alternate policies is calculated using Eq. (16) with $K = 2.5$ as calculated from Eq. (17). Table 1 gives the TRC of six policy alternatives at 80% service level with average demand of 100 units/period. The results indicate that TRC is minimum for policy II for all the three levels of CV. However, the TRC increases with increase in demand variability. The effect of cost of adding capacity is studied by considering other values of ‘K’. The results (Table 2) indicate that the choice of the policy is dependent upon the value of ‘K’. When the value of $K = 1$, policy IV is the best, whereas policy II is the best for $K = 2−5$ and policy I is the best for $K$ greater than 5. This indicates that the additional capacity becomes less economical at higher $K$ and choice of policy shifts towards JIC.

6.4. Case IV

In this case, impact of delays in the production system on system performance is examined. Delay factor ($z$) is defined as the time lag between the change in demand rate and the change in production rate. Five policies at 99% service level under no delays are subjected to the conditions of different delay factors. The expected level of service for these policies is presented in Fig. 8. As the delay factor increases, the level of service decreases for all
Table 1
Alternate policies for 80% service level

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Additional capacity ($\times D_{AVG} \times CV$)</td>
<td>0.2</td>
</tr>
<tr>
<td>Safety stock ($\times D_{AVG} \times CV$)</td>
<td>3.2</td>
</tr>
<tr>
<td>TRC (CV = 0.2) (Rs.)</td>
<td>18,800</td>
</tr>
<tr>
<td>TRC (CV = 0.3) (Rs.)</td>
<td>27,920</td>
</tr>
<tr>
<td>TRC (CV = 0.4) (Rs.)</td>
<td>38,860</td>
</tr>
</tbody>
</table>

Table 2
Effect of cost ratio index ‘K’ on choice of strategy at CV = 0.4

<table>
<thead>
<tr>
<th>Policy</th>
<th>Additional capacity</th>
<th>Safety stock</th>
<th>Avg. Inv.</th>
<th>Total relevant cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K = 1</td>
<td>K = 2</td>
<td>K = 3</td>
<td>K = 4</td>
</tr>
<tr>
<td>I</td>
<td>8</td>
<td>80.4</td>
<td>88.4</td>
<td>96.4</td>
</tr>
<tr>
<td>II</td>
<td>16</td>
<td>48.3</td>
<td>64.3</td>
<td>80.3</td>
</tr>
<tr>
<td>III</td>
<td>24</td>
<td>45.9</td>
<td>69.9</td>
<td>93.9</td>
</tr>
<tr>
<td>IV</td>
<td>32</td>
<td>43.7</td>
<td>75.7</td>
<td>107.7</td>
</tr>
<tr>
<td>V</td>
<td>40</td>
<td>44.9</td>
<td>84.9</td>
<td>124.9</td>
</tr>
<tr>
<td>VI</td>
<td>48</td>
<td>48.0</td>
<td>96.0</td>
<td>144.0</td>
</tr>
</tbody>
</table>

Fig. 8. Effect of delays on service level at different levels of JIT.

Figures and diagrams are not available in the text.

For high level of JIT policy having high additional capacity and low safety stock, the fall in service level is very steep as compared to very low level of JIT, i.e. JIC policy of low additional capacity and high safety stock.

Thus in a JIT policy, where a major part of flexibility is built in fixed assets and safety stock level is very low, it is essential to reduce the production delays. Quick response manufacturing, where the lead times are very small is the most appropriate manufacturing system for JIT.

7. Two product systems

In this section, issues related to manufacturing of multiple products are addressed. The effective control of multiple products does not require policies different from those for single product, if the products are independent. Multi-product manufacturing decisions are useful when the interactions among the products are recognized.

Product interactions as identified in this study are of two types: (i) interactions resulting from resources and (ii) interactions resulting from customer demand. An example of resource interaction occurs when products to be manufactured compete for scarce resources. For example, the total amount of manufactured products may be limited by the
capacity of CNC machine, a resource needed to manufacture all the products.

Customer demand interaction exists whenever the demand for one product is affected by the demand for other product(s) being manufactured. An example of this interaction is that of models A and B, say of a motorcycle. The demand of these models is negatively correlated, i.e. if the demand of one model is high, it will adversely affect the demand of other model and vice versa. Positive correlation of demand also exists, particularly for vendors manufacturing and supplying two or more components of the same end-product to buyer. Here the demand of the items are inter-related as positively correlated. For the purpose of capacity considerations, scheduling and safety stock determination, these correlated items are treated as one group similar to the single item case.

Three manufacturing configurations, namely, dedicated, partially flexible and fully flexible configuration are considered to investigate in detail the manufacturing systems with two products. The demand of the two products is negatively correlated and CV = 0.4.

7.1. Case I

In this case, the impact of manufacturing configuration and additional capacity on the system performance is studied. Fig. 9 gives the service level achieved with dedicated configuration, partially flexible configuration and fully flexible configuration under different safety stock levels at a given additional capacity. The figure shows that as the safety stock increases, the service level improves in all the three configurations. But it is not possible to achieve 80% service level with dedicated configuration even by having 30% additional capacity and safety stock equal to half the average demand. On the other hand, flexible configuration can give 80% service level with 30% additional capacity and zero safety stock.

7.2. Case II

In this case, the effect of cost of capital and flexibility cost factor, which are governed by the macro-environment of a country is studied.

7.2.1. Effect of flexibility cost factor

Flexibility cost factor 'f' is defined as the ratio of the cost of flexible equipment to the cost of dedicated equipment of similar capacity. f is varied from 1.0 to 2.0 and TRC is calculated for the alternative strategies to give 90% service level at CV = 0.4. The results are plotted in Fig. 10. The choice of the policy depends upon the value of f.

\[ f < 1.58 \quad \text{one flexible equipment (Configuration 2c) based on the combined demand of products 1 and 2.} \]

\[ 1.58 < f < 1.78 \quad \text{one flexible and one dedicated equipment (Configuration 2b)} \]
based on the demands of products 1 and 2, respectively.

\[ f > 1.78 \]

two dedicated equipments (Configuration 2a) based on the demands of products 1 and 2.

Flexible configuration is economical when the flexibility cost factor is low. In developing countries, industry has to import the flexible technology and depend upon the supplier country for the training of manpower and maintenance of equipment. Therefore, the cost of flexible equipment is high, as compared to indigenous available dedicated equipment. In Japan, flexibility in equipment is incorporated not by replacing existing equipment, rather by in-house tooling through kaizen and quality circle.

7.3.2. Effect of cost of capital

The flexible equipment policy is likely to be more attractive when the cost of capital is low. As the cost of capital gets higher, it is more expensive to have flexible equipment and preference is given to inventory for reducing the impacts of the uncertainty in customer demand. Fig. 11 shows the effect of cost of capital and flexibility cost factor on the choice of manufacturing policy.

The general state of economy and the industrial development of a country influence the cost of capital and cost of flexibility. Four scenarios are presented in Fig. 12 and suitable policies are discussed.

Scenario I: low cost of capital and low cost of flexibility. In this kind of scenario, a manufacturer can exploit the full advantages of JIT by going for flexible manufacturing system with capacity cushions. The various kinds of uncertainties like demand variability can be responded with very low or negligible inventories.

Scenario II: high cost of capital and low cost of flexibility. This scenario has low cost of flexibility as state-of-the-art technology is available at nominal additional cost. Because of high cost of capital, it is economical to have partial flexibility with small safety stock.

Scenario III: high cost of capital and high cost of flexibility. This scenario is prevalent in underdeveloped countries, where the cost of capital is high and the technological advancements are at a low level. The cost of flexibility is high in the absence of indigenous know-how and import of flexible equipment. This may further have high maintenance costs due to poor in-house support. In this scenario, the manufacturer can opt for dedicated lines and conventional policy of keeping safety stock.

Scenario IV: low cost of capital and high cost of flexibility. Because of low cost of capital, the present value of inventory carrying cost is high, hence partially flexible configuration with safety stock is most economical to respond to the demand variability.

7.3. Case III

Simulation is conducted to develop the production strategies by considering the demand of the products as independent. For same demand variability, higher service level is achieved when the demand is independent as compared to the case of negatively correlated demand. The difference in service level is more in case of dedicated configuration as compared to partially flexible configuration. In
Table 3
Effect of product variety on service level

<table>
<thead>
<tr>
<th>Manufacturing configuration</th>
<th>Demand distribution</th>
<th>Service level 2</th>
<th>Service level 3</th>
<th>Service level 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dedicated</td>
<td>Independent</td>
<td>80.4</td>
<td>80.4</td>
<td>80.4</td>
</tr>
<tr>
<td>Dedicated</td>
<td>Correlated</td>
<td>55.5</td>
<td>55.5</td>
<td>50.8</td>
</tr>
<tr>
<td>Flexible</td>
<td>Independent</td>
<td>92.7</td>
<td>96.5</td>
<td>98.7</td>
</tr>
<tr>
<td>Flexible</td>
<td>Correlated</td>
<td>80.0</td>
<td>89.4</td>
<td>92.8</td>
</tr>
</tbody>
</table>

not affected by the number of products in the system;
- in the case of dedicated configuration and correlated demand of products, the service level decreases as the number of products increases. Alternatively, more safety stock is required to achieve the desired service level;
- in manufacturing systems with flexible configuration, the performance of the system improves as the number of products increases.

8. Multi-product multi-stage production system: A case study

FLEXO AUTO LTD. (FAL), is the name given, in this case study, to a medium-sized automobile ancillary unit established in 1987. This unit (referred to as FAL, hereafter) is a part of a group having other units and the raw material to this unit is supplied by another unit of the group. During 1997-1998, the group turnover was approximately Rs. 750 millions and FAL turnover was Rs. 450 million. The company is ISO 9002 certified and is manufacturing high-precision auto components. The quality policy of the company is “To achieve complete customer satisfaction through technological excellence, continuous improvement, adherence to system and procedures with active employee participation”.

FAL’s production functions are organized with a view to have a minimum dependence on external

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1 The name has been disguised as requested by the management of the company.
sources for supplies of inputs. The group has its own foundry to meet the requirements of castings, its own tool room to cater to the needs of cutting tools. The company has well-established quality systems and quality control facilities. Manufacturing facilities of the FAL include majority of the general purpose machines with few special purpose machines. For some operations CNC machining centres and CNC machines are also used. The machines are arranged in five production lines. The lines act as dedicated lines for five products, which constitute more than 95% of the company’s turnover. The management is keen to seek an optimum utilization of the facilities, besides maximization of use of labour. The variability in demand rate of these products does affect the idleness and overload on these lines.

FAL has been able to introduce a successful incentive scheme which covers direct and indirect workmen together with supervisory staff connected with production. As a policy, the management does not generally permit overtime and restricts it to real emergencies. Repetitiveness of jobs has also helped the production function to introduce standardization of tooling and manufacturing practices. Raw material supplies from group’s own unit helps FAL in getting dependable supply of raw materials. Regular supplies also help in obtaining a high utilization of its facilities. There are only a few failures of supply leading to machine stoppages.

8.1. Product profile

As mentioned earlier, the company is manufacturing various components for different buyers. Five components, namely COMP1, COMP2, COMP3, COMP4 and COMP5 constitute more than 95% of the sales turnover. $\chi^2$ test at 0.05 level of significance shows that the given demand data fit the uniform distribution. From the past demand data, average, standard deviation and coefficient of variation of demand are calculated. After discussion with management, targets of average demand for coming years were set. This information is presented in Table 4. For accounting purpose, company uses 25% as cost of carrying inventory and 15% cost of capital.

<table>
<thead>
<tr>
<th>Product</th>
<th>Unit cost (Rs.)</th>
<th>Average demand (units/week)</th>
<th>CV</th>
<th>Standard deviation (units/week)</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMP1</td>
<td>200</td>
<td>9000</td>
<td>0.145</td>
<td>1305</td>
</tr>
<tr>
<td>COMP2</td>
<td>470</td>
<td>540</td>
<td>0.110</td>
<td>69</td>
</tr>
<tr>
<td>COMP3</td>
<td>400</td>
<td>540</td>
<td>0.205</td>
<td>159</td>
</tr>
<tr>
<td>COMP4</td>
<td>200</td>
<td>1560</td>
<td>0.274</td>
<td>427</td>
</tr>
<tr>
<td>COMP5</td>
<td>500</td>
<td>900</td>
<td>0.650</td>
<td>585</td>
</tr>
</tbody>
</table>

8.2. Manufacturing configuration

The existing manufacturing configuration consists of five dedicated lines for five products. These lines are designated as LNA for COMP1, LNB for COMP2, LNC for COMP3, LND for COMP4 and LNE for COMP5. The number of stages in these lines are 20, 17, 17, 10 and 9 in LNA, LNB, LNC, LND and LNE, respectively. This configuration is designated as ‘Configuration-I’. To reduce the inventory of the system and give the predefined level of service, existing dedicated system is modified to make it flexible. Three flexible configurations were achieved by making changes in the dedicated configuration.

Configuration-II is achieved from the existing dedicated configuration by making investments in tooling. The similarity of equipment of LNB and LNC lines and LND and LNE lines is exploited. By having additional tools, it is possible to set LNB line for COMP3 and LNC line for COMP2. This flexibility helps in increasing the production rate of COMP2 or COMP3 up to two times. Similarly with additional tools, LND and LNE lines can be made flexible for production of COMP5 and COMP4, respectively. The additional cost required to implement is in the form of cost of tooling, jigs and fixtures, which is estimated by the engineering department at Rs. 9 million.

Configuration-III is achieved by having an extra line, consisting of CNC machining centres capable of processing all the five products. With this extra line, named LNF, all five products have an alternate route. The total capacity of the system is also increased. The cost of establishing this line is Rs. 14 million including cost of equipment and tooling.
Table 5
Alternate manufacturing configurations

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Description</th>
<th>Flexibility cost (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5 dedicated lines</td>
<td>—</td>
</tr>
<tr>
<td>II</td>
<td>COMP1 on LNA; LNB and LNC lines flexible to take COMP3 and COMP2; LND and LNE lines flexible to take COMP5 and COMP4</td>
<td>9 million</td>
</tr>
<tr>
<td>III</td>
<td>COMP1–COMP5 on their dedicated lines as well as on flexible line LNF</td>
<td>14 million</td>
</tr>
<tr>
<td>IV</td>
<td>Configuration II plus flexible line to process all 5 products</td>
<td>22 million</td>
</tr>
</tbody>
</table>

Equipment cost is determined by the representatives of machinery manufacturer and tooling costs by the engineering department.

In Configuration-IV an extra flexible line, as in Configuration-III and additional tooling, as in Configuration-II are incorporated. This results in one additional route for COMP1 and two additional routes for each of the remaining products. The total capacity of the system is also increased. The cost of implementation is equal to the cost of Configurations II and III. Some cost is saved due to common tooling in these configurations. The estimated cost is Rs. 22 million. The information for these four configurations is summarized in Table 5.

From the study of the existing system it was observed that the average utilization of equipment is around 50% and similar machines are installed in different lines. By having conveyored material handling, it is possible to reroute the product at different lines for different operations, resulting in increased service level. Some of the equipment can be removed to improve the utilization of machines, reduction in manpower, utilities, space requirements and other overheads. In this study, this option of flexibility is not examined since the production lines are not under one shed and the distance is quite high. Further, this may also result in increased WIP, lead time, and material handling.

Table 6
Average inventory and inventory carrying cost for demand characteristics of Table 4

<table>
<thead>
<tr>
<th>Configuration</th>
<th>COMP1</th>
<th>COMP2</th>
<th>COMP3</th>
<th>COMP4</th>
<th>COMP5</th>
<th>ICC (Rs.)</th>
<th>NPV_{icc} (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service level = 80%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>480</td>
<td>0</td>
<td>36</td>
<td>216</td>
<td>372</td>
<td>84,900</td>
<td>566,000</td>
</tr>
<tr>
<td>II</td>
<td>480</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>228</td>
<td>52,500</td>
<td>350,000</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>180</td>
<td>22,500</td>
<td>150,000</td>
</tr>
<tr>
<td>IV</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>84</td>
<td>19,500</td>
<td>70,000</td>
</tr>
<tr>
<td>Service level = 90%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>696</td>
<td>0</td>
<td>96</td>
<td>420</td>
<td>588</td>
<td>138,500</td>
<td>926,000</td>
</tr>
<tr>
<td>II</td>
<td>696</td>
<td>0</td>
<td>0</td>
<td>216</td>
<td>456</td>
<td>102,600</td>
<td>684,000</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>84</td>
<td>360</td>
<td>49,200</td>
<td>328,000</td>
</tr>
<tr>
<td>IV</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>288</td>
<td>36,000</td>
<td>70,000</td>
</tr>
<tr>
<td>Service level = 95%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>1080</td>
<td>0</td>
<td>120</td>
<td>660</td>
<td>924</td>
<td>214,500</td>
<td>1,430,000</td>
</tr>
<tr>
<td>II</td>
<td>1080</td>
<td>0</td>
<td>0</td>
<td>504</td>
<td>704</td>
<td>175,600</td>
<td>1,170,667</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>312</td>
<td>528</td>
<td>84,900</td>
<td>560,000</td>
</tr>
<tr>
<td>IV</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>396</td>
<td>52,500</td>
<td>350,000</td>
</tr>
<tr>
<td>Service level = 99%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>1824</td>
<td>48</td>
<td>216</td>
<td>1248</td>
<td>1560</td>
<td>375,840</td>
<td>2,505,600</td>
</tr>
<tr>
<td>II</td>
<td>1824</td>
<td>22</td>
<td>144</td>
<td>1440</td>
<td>1140</td>
<td>322,685</td>
<td>2,151,233</td>
</tr>
<tr>
<td>III</td>
<td>0</td>
<td>8</td>
<td>48</td>
<td>1128</td>
<td>756</td>
<td>156,640</td>
<td>1,044,267</td>
</tr>
<tr>
<td>IV</td>
<td>0</td>
<td>0</td>
<td>14</td>
<td>276</td>
<td>576</td>
<td>87,200</td>
<td>581,333</td>
</tr>
</tbody>
</table>
Table 7
Benchmark-cost analysis of flexible configurations

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Investment (Rs.)</th>
<th>Saving in inventory carrying cost (Rs.) at service level of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>80%</td>
</tr>
<tr>
<td>II</td>
<td>9 million</td>
<td>0.216 million</td>
</tr>
<tr>
<td>III</td>
<td>14 million</td>
<td>0.416 million</td>
</tr>
<tr>
<td>IV</td>
<td>22 million</td>
<td>0.496 million</td>
</tr>
</tbody>
</table>

8.3. Performance measurement of alternative configurations

The four manufacturing configurations presented in Table 5 are simulated to determine the average inventory of the products to give the desired level of service and the results are presented in Table 6. At 80% service level, no safety stock is required for COMP2 in Configuration-I and for COMP2, COMP3 and COMP4 with Configuration-II. With Configurations III and IV, only for COMP5 safety stock is required to give the desired level of service. To compare the recurring savings in inventory with the initial investments in flexibility, NPV of inventory carrying cost is calculated. The effect of service level and flexibility on the inventory carrying cost is given in Table 6. The results show that inventory carrying cost increases at higher level of service. It also shows that inventory carrying cost is minimum with the most flexible configuration and maximum with dedicated configuration.

The savings in inventory carrying cost by implementing flexible configurations II, III and IV in place of dedicated configuration I is compared with the additional investments in these configurations and presented in Table 7. The results show that none of the flexible configurations is economical and present configuration of five dedicated lines for five products is the most economical. The results justify our stand that the implementation of JIT is situation specific and must be evaluated on economic criteria. In this case study the manufacturing process is capital intensive and cost of carrying inventory is very less as compared to the total investments in the plant.

9. Conclusions

This paper has presented the results of a simulation study to investigate the requirements of technological flexibility for the implementation of JIT manufacturing system. It is found that the performance measure of JIT systems is very much affected by the incorporation of the flexibility in the system under probabilistic demand conditions. Therefore, incorporation of flexibility is vital. The choice of the extent of flexibility is driven by the market volatility, market expectations, economic and industrial development and other internal and external factors.

Desired customer service from a manufacturing system can be achieved through flexible equipment or through inventories. The choice is based on economics. JIT systems with high flexibility and low safety stock are very sensitive to manufacturing delays. On the other hand, flexibility is more suitable for multiple product manufacturing system with correlated demand. This simulation study reveals that there can be no unique answer to the flexibility; inventory mix and alternative strategies have to be evaluated to determine the best option for a given situation.

Appendix. Demand generation

(a) Normal distribution: For a normal distribution, Box Muller transformation can be used. The equation is as follows:

\[ D_m(t) = D_m^{\text{exp}} + \sigma(m(t)S(t)) \]

where \( \sigma_m(t) \) is the standard deviation of demand of product \( m \) and

\[ S(t) = \sqrt{-2 \log(R_1(t)) \cos(2\pi R_2(t))} \]

\( R_1(t) \) and \( R_2(t) \)
are two independently generated uniform pseudo-random numbers.

(b) Correlated demand: Multi-product manufacturing systems have demand-related interactions, where the demand for one product can be affected by the demand for other products being manufactured.

For two product manufacturing system, the negatively correlated demand is calculated by using the following equations:

\[ D(t) = D^{\text{ave}} + (R_1(t) - 0.5)RG, \]
\[ D_1(t) = D(t)R_2(t), \]
\[ D_2(t) = D(t) - D_1(t), \]

where

\[ D(t) = D_1(t) + D_2(t), \]
\[ D^{\text{ave}} = D_1^{\text{ave}} + D_2^{\text{ave}}, \]
\[ RG^2 = RG_1^2 + RG_2^2. \]

The same logic can be used for generating correlated demand of manufacturing systems having more than two products.

References